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Forecasting Electricity Consumption in Sulaimani Province by Using Spectral Analysis Approach

التنبؤ باستهلاك الكهرباء باستخدام منهج التحليل الطيفي في محافظة السليمانية

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Abstract:

In the modern world, with the extensive use of electricity in homes and businesses, studying and forecasting electrical consumption is crucial for its management and distribution. This research aims to forecast the electricity consumption in Sulaimani province using spectral analysis on time series data. For this purpose, hourly data within the time range of July to September 2022 has been utilized, and a new model has been proposed, consisting of a sinusoidal explanatory variable and a nonlinear quadratic variable. In this model, the most significant periodic cycles present in the time series under study were identified by employing the periodogram and Fourier expansion method, capable of identifying hidden cycles in the data. Subsequently, each of these periodic components was added to the model based on the principle of parsimony. The proposed composite model includes a quadratic component and two harmonic components with 24 and 12-hour periods. This model demonstrates notable efficiency and accuracy as it outperforms the classical Autoregressive Moving Average (ARMA) model based on the MSE=0.0009 rate and MAE=23.819 when compared.

Keywords: forecasting electrical consumption, Fourier method, Periodogram, Spectral Analysis, ARIMA model.

الملخص:

في العالم الحديث، مع الاستخدام المكثف للكهرباء في المنازل والشركات، تعد دراسة استهلاك الكهرباء والتنبؤ به أمرًا بالغ الأهمية لإدارته وتوزيعه. يهدف هذا البحث إلى التنبؤ باستهلاك الكهرباء في محافظة السليمانية باستخدام التحليل الطيفي على بيانات السلاسل الزمنية. ولهذا

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الغرض، تم استخدام البيانات بالساعة ضمن النطاق الزمني من يوليو إلى سبتمبر 2022، وتم اقتراح نموذج جديد يتكون من متغير توضيحي جيبي ومتغير تربيعي غير خطي. في هذا النموذج تم تحديد أهم المولفات الدورية الموجودة في السلاسل الزمنية قيد الدراسة من خلال استخدام مخطط الدورة وطريقة توسيع فورييه القادرة على تحديد الدورات المخفية في البيانات. وبعد ذلك، تم إضافة كل من هذه المكونات الدورية إلى النموذج بناءً على مبدأ الامساك. يتضمن النموذج المركب المقترح عنصرا تربيعيا واثنين من المكونات التوافقية مع فترات 24 و 12 ساعة. يُظهر هذا النموذج كفاءة ودقة ملحوظتين لأنه يتفوق على نموذج المتوسط المتحرك للانحدار الذاتي الكلاسيكي (ARMA) استنادًا إلى معدل MSE=0.0009 و MAE=23.819 عند المقارنة. **كلمات مفتاحية:** التنبوء استهلاك الكهرباء، طريقة فوريه، البيريودوكرام، تحليل طيفي، نموذج ARMA

1.Introduction:

Given the relatively high growth of energy consumption worldwide, ensuring its supply is considered a necessity in industrial production and a final commodity in residential and commercial sectors. It's worth mentioning that due to the high cost involved in its production, distribution, and transmission, accurate planning based on the consumption of this energy is necessary. This is because any disruption in the balance between demand in different sectors and its supply will result in irreparable damage to the country's economy. However, accurate prediction of future electricity consumption can prevent such damages.

Like most behavioral climatic factors, average power consumption is characterized by a wave, mainly regular, alternating up and down movements. Employing sinusoidal functions provides for the approximate estimation and forecasting of alternating behaviors at various frequencies suitably and acceptably. One of the modeling approaches used in spectral analysis, the Fourier model, is a convenient and effective tool. The theoretical and mathematical foundations related to Fourier models were examined and explained by Spiegel and Murray (1974). Fourier series were introduced through a discussion of trigonometric functions and pulse functions, which are essential tools in Fourier analysis. It is worth noting that the decomposition of complex signals using a combination of simple trigonometric functions is achievable through Fourier series. Based on this, the transfer of signals from the time domain to the frequency domain, according to the concept of the Fourier transform, will be facilitated. Brown and Churchill (1993) delved into the fundamental and physical concepts related to Fourier series, trigonometric functions, the characteristics of these series, and their applications in boundary value problems. They elaborated on how Fourier series could assist in analyzing and studying various functions and signals in physics in detail. Wei (2019) and Box et al (2018) contributed to a deeper understanding and analysis of signals and time series by analyzing and developing signal analysis tools and techniques such as wavelet transform and Fourier analysis. Many of these methods have evolved to provide effective solutions in various scientific disciplines.

Modeling the behavior of cyclic data has received increasing attention in the scientific community in recent years due to the importance and consequences of cyclic data to life situations and their role in influencing government decisions. Abass I. Taiwo et al (2019) considers periodic data modeling as one of the most crucial phases of statistical modeling. In order to fit the model to the studied data, Iwok (2016) employed Fourier models. They demonstrated that there is a good agreement between the estimated and observed. Yi-Chung Hu (2018) proposed the fractional grey prediction model with Fourier that offers high prediction accuracy. Also, Lange (2021) believes that the key to understanding optimization issues is to define goals in the frequency domain, where, according to the computational capabilities of the Fourier transform, scalable and computationally effective solutions are found. Sadek (2020) states the obtained results as that Fourier modeling is suitable for the practice of developing forecasts of cyclical activity such as truck parking, despite a number of limitations .In this study, an attempt has been made to introduce hybrid Fourier models based on fast Fourier transform algorithms and the periodogram method. These models are proposed as an acceptable approach for analyzing hidden components in the average hourly electricity consumption during a 90-day period in Sulaimani province.

2.Methodology

Repetitive patterns in time are the main characteristic of periodic events. As expected from periodic behavior, after a complete cycle, the initial configuration will reoccur. The speed of these behaviors is sometimes gradual and gentle, and at other times, it is very fast and unstable. However, they always have a stable and continuous trend. Therefore, some oscillatory behaviors are observed in unknown and longtime intervals, and some of them are observable in specific time intervals, such as daily and even weekly periods (Iwok 2016). In this province, due to the cheap and easy use of electricity, it is used as the main source of cooling and heating in hot and cold seasons. In general, it can be said that electricity consumption is closely related to meteorological changes, which have a periodic behavior. In such a way, electricity consumption is almost equal in different time periods and shows similar consecutive conditions. Modeling these recurring behaviors is of importance. At this scale, wave behavior and a type of periodic pattern with continuous fluctuations towards higher and lower values can be observed. Based on the studied data, it is observed that this wave behavior in electricity consumption is a natural response to the periodic distribution of air temperature as well as the daily lifestyle of consumers.

2-1 Fourier Series representation

Fourier series is a fundamental concept in mathematics that proves to be highly efficient in describing functions with periodic and repetitive characteristics. This series has the capability to decompose complex functions into a combination of simple sinusoidal waves. In other words, any periodic function can be represented as the sum of sinusoidal waves with different frequencies and amplitudes (Percival and Walden 2020).

Let $\{y_t\}_{t=1}^{n} = \{y_1, y_2, \dots, y_n\}$ is the time series can be written as follows

$$y_t = a_0 + \sum_{i=1}^{n} (a_i \cos(wt) + b_i \sin(wt)) \qquad (1)$$

$$w = 2\pi f_i \quad f_i = i/n \qquad (2)$$

Where

$a_0 = \frac{1}{n}\sum_{t=1}^{n} y_t$, (w) is the angular of frequency. (y_t) is the element under investigation in time (t). (f_i) is frequency of repetition of observations, (i=1, 2, 3, …, p) and (a_i, b_i) are constant parameters, known as Fourier coefficients, are calculated by Eq.3 as follows:

$$\begin{cases} a_i = \frac{2}{n}\sum_{t=1}^{n} y_t \cos(wt) \\ b_i = \frac{2}{n}\sum_{t=1}^{n} y_t \sin(wt) \end{cases} \qquad (3)$$

If the length of the statistical period (n) is even, that number is obtained from ($p = \frac{n}{2}$) and if (n) is odd then ($p = \frac{n-1}{2}$), and its coefficients are obtained from Eq.3, except (a_p and b_p) in even period. which are obtained from Eq.4 as follows:

$$a_p = {}^1/_n \sum_{t=1}^{n}(-1)y_t \qquad (4)$$

$$b_p = 0$$

The sine and cosine components ($a_i \cos(wt) + b_i \sin(wt)$) around a fixed mean ($a_0$) in equation (1) determine the behavior of the time series and play a crucial role in describing periodic phenomena. In other words, these sinusoidal components are harmonics that have a significant impact on shaping the behavior of periodic series (Marple 2019). For this reason, equation (1) can be expressed as a collection of these harmonics as follows.

$$y_t = a_0 + \sum_{i=1}^{p} h_p(t) \qquad (5)$$

In which.

$$h_i(t) = a_i \cos(wt) + b_i \sin(wt) \qquad (6)$$

It is called (ith) hormonic. The number of observations in a complete cycle of each harmonic is referred to as its periodicity, which is equal to($\frac{1}{f_i} = \frac{n}{i}$), and it can be easily shown in Eq.7 as follows:

$$h_i(t) = A_i \sin(2\pi f_i\, t + \emptyset_i) \qquad (7)$$

Phase is ($\emptyset_i$) , refers to the relative temporal position of the signal at a specific point within its repetitive cycle and is typically expressed as an angular measurement from a reference point, and represent in Eq.8 as follows:

$$\emptyset_i = Arctan\left({}^{a_i}/_{b_i}\right) \qquad (8)$$

(A_i) is Amplitude representing the maximum height of the wave (peak height) and indicates the power and intensity of the signal at various points in time. which is described in Eq.9.

$$A_i = \sqrt{a_i^2 + b_i^2} \qquad (9)$$

To clarify the issue, the mentioned specifications are shown in Fig.1

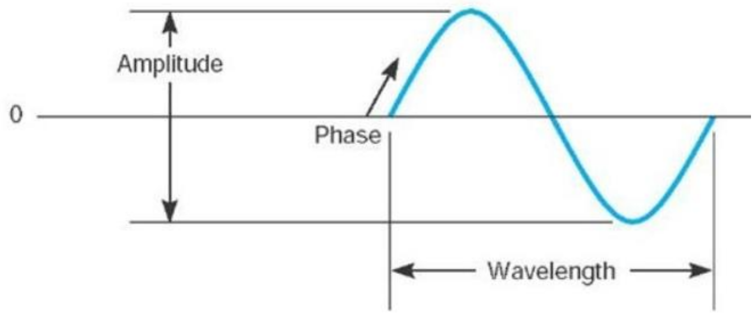

Figure 1. The harmonic component

Source: internet

2.1.1. Fourier modeling

If a time series is composed of the sum of a random component and a periodic component, under general conditions that are usually met, adding a finite number of (P) harmonics can provide a suitable approximation for describing that time series (Rahman M. et al 2020). For example, if the (J th) harmonic alone is a good approximation for the time series ($\{y_t\}_{t=1}^{n}$), in that case, Eq.1 can be written as follows:

$$y_t = a_0 + a_J \cos(wt) + b_J \sin(wt) + e_t \qquad (10)$$

where (e_t) is the sum of (p-1) other harmonics which are not considered in the model. The Eq.10 is called Fourier model, and in statistics, Fourier modeling is about determining a finite number of harmonics that play a significant role in the variations of a time series. Eq.11 represent the Fourier model for the ($\{y_t\}_{t=1}^{n}$) as follows:

$$y_t = a_0 + \sum_{i=1}^{p} (a_i \cos(2\pi i t/n) + b_i \sin(2\pi i t/n)) \qquad i = 1,2,3, \dots, p \qquad (11)$$

It should be noted that although this Fourier model is a purely mathematical sequence, determining the limited number of harmonics that are statistically significant and can provide a good approximation, and also is very important in determining the general behavior of any time series data.

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The Fourier model can also be considered as a linear regression model in which the number of regression coefficients is equal to the length of the series (Wei 2019). And it can be easily shown that the estimation of the least squared error of a_0 , a_P and b_P can be obtained from Eq. (3,4). It should be mentioned that although the Fourier model is a regression model, since each harmonic has independent information about the series changes, the problem of collinearity is not raised.

It should be noted that the Eq.11 is suitable for a group of time series that exhibit periodic behavior around the horizontal axis. If a time series, in addition to this periodic behavior, displays non-linear or trend behavior, appropriate parameters should be added to the model based on the type of observed behavior. For example, if a time series exhibits non-linear behavior, parameters (α_0) and (α_1) should be added to the model through component$(\alpha_0 + \alpha_1 t)$. And if the series includes a quadratic exhibit, parameters (α_0), $(\alpha_1)$and $(\alpha_{2)}$ should be added to the model through component $(\alpha_0 + \alpha_1 t + \alpha_2 t^2)$. It is also suggested that if the time series contains a trend, the residuals of the model should be fitted by an ARIMA model or the time series should be stationary by differentiation and then a Fourier model should be fitted on it (Percival and Walden 2020).

2.1.2. Periodogram

Periodogram is a valuable and effective tool in Fourier modeling. The periodogram comprises (1, 2, …, p) values, such as $I(f_1), I(f_2), \dots, I(f_p)$ (Yakimov 2020). so that if the length of the statistical period is odd, then the Eq.12 will be used.

$$I(f_i) = {}^{n}/_{2}\,(a_i^2 + b_i^2) \qquad i = 1,2,3, \dots,p \qquad (12)$$

And if the length of the statistical period is even, the Eq.12 for (1, 2, …, p-1) has been used and Eq.13 for $(i = p)$.

$$I(f_p) = n a_p^2 \qquad (13)$$

Where

a_i and b_i are Fourier coefficients

A graph that shows the value of $(I(f_i))$ versus (i) is called a periodogram. In the periodogram, the order of importance of harmonics is determined. The meaning of the importance of a harmonic is the contribution that a harmonic play in the total variability of the series (Percival and Walden 2020). So that it can be easily shown that the total value of the periodogram is equal to the variance of the series, which is shown in Eq.14, and in a table of analysis of variance related to a Fourier pattern $(I(f_i))$, is actually equal to the sum of the second power corresponding to the (i th) harmonic.

$$\sum_{t=1}^{n}(y_t - \bar{y})^2 = \sum_{i=1}^{p} I(f_i) \qquad (14)$$

2.2. Autoregressive Moving Average (ARMA) models

Autoregressive Moving Average (ARMA) models are powerful tools in statistics and econometrics for modeling time series data. These models assist in analyzing and forecasting time series trends (Box et al 2018). ARMA models have two main components: 'AR' (Autoregressive), used for its relationship with past time series values, and the 'MA' (Moving Average) component, based on the average of past values. The Eq.15, represent the ARIMA model as follows:

$$y_t = C + \varphi_1 y_{t-1} + \cdots + \varphi_p y_{t-p} + a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} + \varepsilon_t \qquad (15)$$

where (y_t) represents the time series variable at time (t). (C) is a constant term. $(\varphi_1, \varphi_2, \dots, \varphi_p)$are the autoregressive (AR) coefficients of order (1, 2, …, p). (a_t)represents the noise at time (t). $(\theta_1, \theta_2, \dots, \theta_q)$ are the moving average (MA) coefficients of order (1, 2, …, q). (ε_t) is error. (p) indicates the order of AR (the number of AR coefficients), and (q)indicates the order of MA (the number of MA coefficients) in the ARMA model.

One of the significant advantages of ARMA models is their ability to describe complex data variations using simple mathematical functions. This capability assists researchers and data analysts in describing various phenomena and making more accurate predictions the average of past values (Wei 2019).

3. Results and discussion

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The studied data has been prepared from the electricity control and distribution department of the Sulaimani province, these data are measured and available on an hourly, daily, weekly and monthly basis according to the special devices in this regard. According to Table.1 as follows:

Table 1. Description of data

Minimum	Maximum	Mean	Std. Deviation
441.55	911.73	691.4144	94.98524

Source: Authors 'calculation based on consumption data

which is a description of the investigated data, it should be noted that the lowest amount of electricity consumption was measured on 8/19/2022 at 5:00 p.m. and the highest consumption amount was measured on 7/24/2022 at 9:00 p.m. The average value of consumption in this time period is 691.41 and the standard deviation value is 94.98.

3.1. Data behavior

Before conducting normality and stationarity tests, data can be fitted with linear and nonlinear models. This approach has several advantages, the study data are fitted with various models, including linear models, exponential growth models, quadratic models, and S-curve models. These models are fitted to the data, and the best model that demonstrates the highest fitting capability is chosen as the study model. Based on Table 2, it can be observed that the results obtained suggest that the quadratic model appears to perform better than the other models, and these findings are also clearly visible in Figure 2.

Table.2. Represent the Fitted models values

Accuracy measurement	Linear	Exp. Growth	S-Curve	Quadratic
MADE	7.6	7.7	7.42	7.39
MAD	49.93	51	48.91	48.21
MSE	3848.89	3968	3749.76	3695

Source: Authors 'calculation based on consumption data

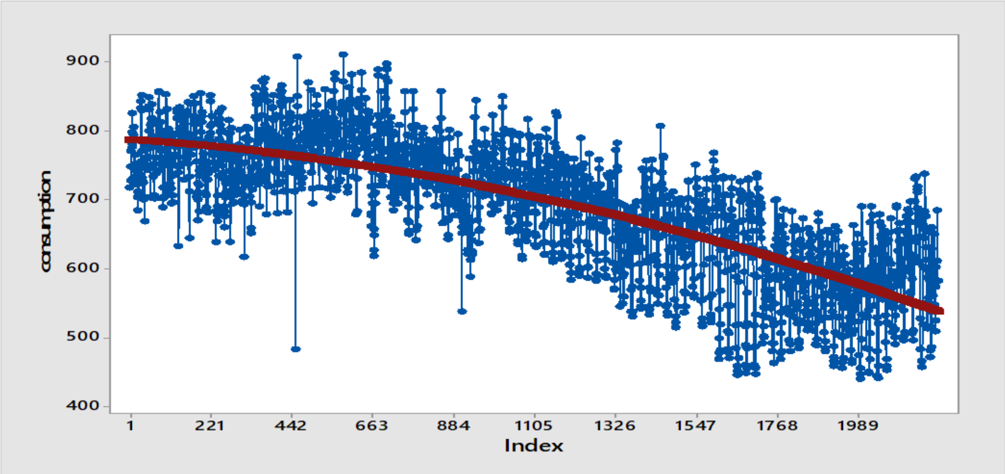

Figure 4-2. Trend Analysis Plot for consumption Quadratic Trend Model

Source: Minitab Software, Authors 'result based on consumption data in

Considering the results provided in the table above, it can be inferred that the studied data follows a quadratic pattern, and the fitted model can be represented as follows, where the parameter values are 788.257, -0.0385, and -0.000034, corresponding to α_0, α_1 and α_1 respectively.

$$y_t = 788.257 - 0.0385t - 0.000034t^2 \quad (16)$$

A test for the normality of the time series under study was conducted using the Anderson-Darling test. This value of AD = 0.734 is greater than the specified significance level (0.05), indicating that the data under investigation are significantly close to a normal distribution.

The stationarity of the data was also examined using the Augment Dickey-Fuller (ADF) test, and the results obtained, as shown in table.3, indicate the data's stationarity.

Table 3. Stationary test

ADF test	Score	P-value	C.V	Stationary?
Quadratic	-11.2	%0.000	-2.3	TRUE

Source: Authors 'calculation based on consumption data

3.2. Pattern recognition and probabilistic modeling

Using Eq.12 and NUMXL toll in Excel software, the values related to the electricity consumption pattern have been calculated for 1104 data points. Part of the calculation results related to the electricity consumption pattern can be observed in Table 4 and Figure.3 as follows:

Table4. frequency and periodogram of electricity consumption

	1	2	3	4	5
Fi	0.042	0.083	0.125	0.167	0.170
1/fi	24	12	8	6	6
n*fi	93	183	276	368	376
Ifi	915438.02	499.945.45	412562.17	14462.34	506.31

Source: Authors ‘calculation based on consumption data

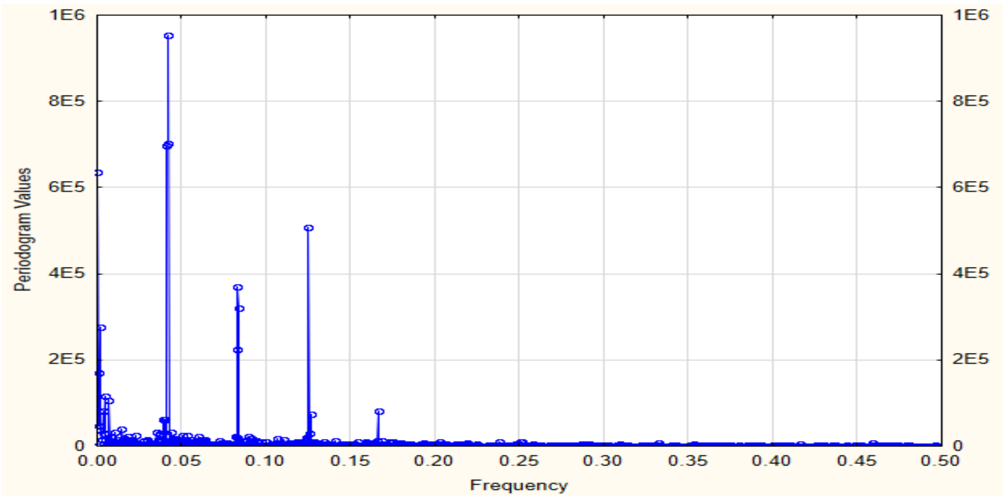

Figure 3. Represent the value of periodogram verses frequency

Source: Minitab Software, Authors ‘result based on consumption data in

Utilizing the information from Table 1 and Figure 3, it has been deduced that the first 5 significant harmonics are, in order of importance, the 24th, 12th, 8th, and 6 (doubled) harmonics. Considering that the examined data also demonstrate quadratic behavior, parameters $(\alpha_0, \alpha_1 \; and \; \alpha_2)$ should be incorporated into the model. In other words, the desired model can be expressed in Equation.17 as follows.

$$\hat{y_t} = \text{Quadratic model} + \sum sinusoildal\ component \qquad (17)$$

Based on the obtained model, in order to ensure the accuracy and efficiency of the model and in accordance with the parsimony principle in parameter selection, the selected harmonics are added to the model in order of importance. This process continues until reaching a harmonic that does not have a significant impact on the model.

$$\hat{y}_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + A_{93} \sin\left(2\pi \frac{93}{2208} t + \emptyset_{93}\right) + A_{184} \sin\left(2\pi \frac{184}{2208} t + \emptyset_{184}\right) \quad (18)$$

Where α_0, α_1 *and* α_2 are the quadratic models' coefficients. $A_{93}, A_{184}, \emptyset_{93}$ *and* $\emptyset_{184}$ are the amplitude and phases of 93th and 184th harmonic components were calculated by equations (8,9).

3.3. ARMA model representation

The ARMA statistical model, as specified in Equation 6, was evaluated, and its necessary parameters were obtained using STATGRAPHIC software. Based on the table (12), the results for this model are presented in Eq.19 as follows:

Tab.12 ARIMA (2,0) Model Summary

Parameter	Estimate	S. E	t	P-value
AR (1)	1.0351	0.02111	49.0217	0.000
AR (2)	-0.1296	0.02112	-6.1398	0.000
Mean	691.143	8.4446	81.8443	0.000

Source: Authors 'calculation based on consumption data

$$\hat{y}_t = 691.143 + 1.03512 y_{t-1} - 0.1296 y_{t-2} \quad (19)$$

3.4. Comparison between models

In order to check the forecast power of the used methods, two common criteria in this field were used.

The first criterion is the average absolute value of errors (x). In this criterion, the average absolute value of the errors is used for each of the forecasts. that's mean

$$MAE = \sum {|e_i|}/{N} \quad (20)$$

where N is the number of forecasts and (e_i) is the difference between the forecasted and its actual value.

The second criterion is the mean squared error (MSE) which is expressed as follows.

$$MSE = {\sum e_i^2}/{N} \qquad (21)$$

In order to compare the forecasted and actual values of each time series, the criteria mentioned above were used and in table.13 the value of the discussed criteria is compared for the models used.

Table.13. the results of comparison between models

	Quadratic + sinusoidal components	ARMA (2,0)
MAE	23.81978	29.3987
MSE	0.000942	37.7547

Source: Authors ‘calculation

Considering that various criteria have been used to measure the power of forecasting and the model that has the lowest value makes a better forecast. Therefore, according to the results obtained in table (13), it is concluded that the model with 2-sinusoid component is a more suitable model for forecasting. And the approved model according to (17) can be shown by using the first and second harmonics and removing other additional harmonics as follows.

$$\hat{y}_t = 788.25 - 0.0385t - 0.000034t^2 + 28.783\,Sin(2\pi 0.042t - 0.016) + 19.323\,Sin(2\pi 0.083t - 0.268) \qquad (22)$$

5. conclusion

Due to the importance and impact of electricity consumption on living and economic conditions, modeling electricity consumption modeling has gained significant attention in scientific communities. In this study, the electricity consumption patterns in Suleimani province were examined through the analysis of hourly time series data. The dataset included values ranging from a minimum of 441.55 to a maximum of 911.73. By fitting the mentioned data to various models, the behavior of the data was identified as a quadratic model. This model was compared with linear and exponential models, and it demonstrated the lowest Mean Squared Error (MSE) with parameter values of 788.257, -0.0385, and -0.000034. Then, the Anderson-Darling and Dickey-Fuller tests were used for assessing the normality and stationarity of the study data, and based on the results obtained, the data were employed for modeling. To identify hidden components in the data, a periodogram and Fourier-based spectral analysis were utilized. After applying the principle of parsimony, five hidden components were identified. From their total, two harmonic components with frequencies of 0.04 and 0.083 in 24 and 12-hour periods were added to the model. The proposed model consists of a polynomial component and two sinusoidal components.

Finally, by comparing the proposed hybrid model with the classical ARMA (2,0) model obtained from the study data with parameters 1.0351 and -0.1286, and considering the comparison of average absolute value of errors and mean squared error criteria, the proposed model was selected as the superior model with MAE=23.8197 and MSE=0.00094.

The results indicate that the proposed model demonstrates a very high accuracy in forecasting electricity consumption, and considering the obtained error values, it suggests a minimal difference between the forecasted results and the actual data. It is worth noting that the studied data are in the form of discrete and periodic time series. For continuous time series, by dividing the time axis into equal intervals, continuous time series can be converted and introduced into discrete time series. This method can be utilized for modeling.

Authors' contributions statement:

F.A and M.A suggested the idea, F.A suggested the outline of the proposal. Then F.A did each part, extracted the results and M.A supervised this article as a consultant professor.

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