The Cyclic Decomposition of $\mathbf{cf}(\mathbf{Q}_{2q} \times \mathbf{C}_{10})/\ \overline{\mathbf{R}}(\mathbf{Q}_{2q} \times \mathbf{C}_{10})$

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Abstract— *In this paper, we propose the cyclic decomposition of the factor group* $\boldsymbol{cf}(\boldsymbol{Q}_{2q} \times \boldsymbol{C}_{10}, \boldsymbol{Z})/\overline{\boldsymbol{R}}(\boldsymbol{Q}_{2q} \times \boldsymbol{C}_{10})$*, and the group* $\boldsymbol{cf}(\boldsymbol{Q}_{2q} \times \boldsymbol{C}_{10}, \boldsymbol{Z})$ *is Z-valued class functions of the direct product group* $((\boldsymbol{Q}_{14} \times \boldsymbol{C}_{10}))$ *under the operation of addition, and* $\boldsymbol{R}((\boldsymbol{Q}_{14} \times \boldsymbol{C}_{10}))$ *is the subgroup of the generalized characters of the group* $\boldsymbol{cf}(\boldsymbol{Q}_{2q} \times \boldsymbol{C}_{10}), \boldsymbol{Z})$*.Then* $\boldsymbol{cf}(\boldsymbol{Q}_{2q} \times \boldsymbol{C}_{10}, \boldsymbol{Z})/\overline{\boldsymbol{R}(}\boldsymbol{Q}_{2q} \times \boldsymbol{C}_{10}))$ *is an abelian factor group denoted by* $\boldsymbol{K}(\boldsymbol{Q}_{2q} \times \boldsymbol{C}_{10}))$ *where* $(\boldsymbol{Q}_{14}$ *is the quaternion group of order* $\mathbf{28}$ *and* $\boldsymbol{C}_{10}$ *is the cyclic group of order* $\mathbf{10}$*. Also, we find the rational valued characters table of the group* $(\boldsymbol{Q}_{2q} \times \boldsymbol{C}_{10})$ *when* $\boldsymbol{p}$*, and* $\boldsymbol{q}$ *prime numbers and* $\boldsymbol{s} \in \boldsymbol{Z}^{+}$ *is given as follows :*

$$\equiv^{*}(\boldsymbol{Q}_{2q} \times \boldsymbol{C}_{10})) = \equiv^{*}((\boldsymbol{Q}_{14}) \otimes \equiv^{*}(\boldsymbol{C}_{10}) \qquad (1)$$

and find the cyclic decomposition of group $(\boldsymbol{Q}_{2q} \times \boldsymbol{C}_{10})$ *in this paper and prove that*

$$\boldsymbol{K}(\boldsymbol{Q}_{2q} \times \boldsymbol{C}_{10}) = \oplus_{i=1}^{4}[\boldsymbol{K}\boldsymbol{Q}_{2q}] \oplus_{i=1}^{5} \boldsymbol{K}(\boldsymbol{C}_{10}) \qquad (2)$$

Keywords— matrix representation, character tables, Quaternion groups, the cyclic decomposition of group)

I. INTRODUCTION

Γ -conjugate contains all elements of a finite group that generate equivalent conjugate cyclic subgroups in G, which defines an equivalence relation on G; its classes are called Γ-classes

"The number of $\boldsymbol{\Gamma - classes}$ of G is equal to the rank of $\mathrm{cf}(\mathrm{G.Z})$ is the intersection of $\mathrm{cf}(\mathrm{G.Z})$ with the group $\mathrm{R}(\mathrm{G})$ which is a normal subgroup of $\mathrm{cf}(\mathrm{G.Z})$, then, $\mathrm{cf}(\mathrm{G.Z})/\overline{\mathrm{R}}(\mathrm{G})$ is a finite abelian factor group denoted by $\mathrm{K}(\mathrm{G})$".

"Each element in $\overline{\mathrm{R}}((\mathrm{Q}_{14} \times \mathrm{C}_{10}))$ can be written as" $\mathrm{v1\theta1 + v2\theta2 + \cdots + vr\theta r}$,"where r is the number of $\Gamma - \mathrm{classes.v1.v2.\ldots.vr} \in \mathrm{Z}$ and , where χi is an irreducible character of the group G and σ is any element in Galios group Gal $(\mathrm{Q}(\chi\mathrm{i})/\mathrm{Q})$".

"In 1982, M. S. Kirdar [4] studied the $\mathrm{K}(\mathrm{C_n})$. In 1994, H. H. Abass [2] studied the $\mathrm{K}(\mathrm{D_n})$ and found $\equiv^{*}(\mathrm{D_n})$. In 1995, N. R. Mahmood [6] studied the factor group $\mathrm{K}(\mathrm{Q_{2m}})$ and foun $\equiv^{*}(\mathrm{Q_{2m}})$.

"The aim of this paper to find" $\equiv^{*}(\mathbf{Q}_{2q} \times \mathbf{C}_{10})$ "and to determine the $\mathrm{K}(\mathbf{Q}_{2q} \times \mathbf{C}_{10}))$, when p and q are primes numbers and s∈ z^{+} .

The number of conjugacy classes of G and number of deferent k-irreducible representations of G ,are a finite if the group G is a finite character of a group representation is constant on a conjugacy class $\mathrm{CL} \propto. (1 \leq \propto \leq \mathrm{k})$, the values of the characters can be written as a table known as the characters table which is denoted by ≡(G)." Let $\mathrm{T_a}: \mathrm{G_a} \rightarrow \mathrm{GL}(\mathrm{n.K})$ and $\mathrm{T_b}: \mathrm{G}_b \rightarrow \mathrm{GL}(\mathrm{m.K})$ are two matrix representations of the groups $\mathrm{G_a}$ and G_b . χ_a and χ_b be two characters of T_a and T_b respectively, then the character of $\mathrm{T_a} \otimes \mathrm{T_b}$ is $\chi_\mathrm{a}\chi_b$.

II. BASIC CONCEPTS

Definition (2-1):[1] A rational valued character θ of G is a character whose values are in Z, which is θ (g)∈ Z. for all g

Proposition (2-2):[4]

The rational valued characters

$$\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i) \qquad (3)$$

form basis for $\overline{R}(G)$, where χ_i are the irreducible characters of G and their numbers are equal to the number of all distinct Γ- classes of G.

Proposition (2-3): [4]

The rational valued characters table of the cyclic group C_p of the rank 2 where p is a prime number which is denoted by $(\equiv^* (C_p))$, is given as follows:

Table 1. The Rational Valued Characters Table Of The Cyclic Group $\mathbf{C_p}$

Γ − classes	$[I]$	$[r]$
θ_1	1	1
θ_2	$p-1$	-1

where its rank 2 represents the number of all distinct Γ- classes.

Definition (2-4): [6]

The generalized Quaternion Group Q_{2m} of order 4m, the Q2m generator bay x and y satisfies

$x^m = y^2, \; y^1 x^m y^{-1} = x^{-m}$ which implies

$x^m = y^4 = 1$

then∀ d ∈Q_{2m} can be expressed uniquely in the form

$Q_{2m} = \{d = x^h y^k \mid 0 \le h \le 2m-1, k = 0,1\}$

Proposition (2.5): [6]

If m is an odd number the table $\equiv^* (Q_{2m})$ is given as follows:

	$\Gamma -$ *classes of* C_{2m}								
	x^{2r}				x^{2r+1}				$[y]$
θ1	1 1 …… 1				1 1 …… 1				1
θ2	$\equiv^* (C_m)$				$\equiv^* (C_m)$				0
:									:
$\theta_{(I/2)-1}$									0
$\theta_{(I/2)}$									0
$\theta_{(I/2)+1}$	1 1 …… 1				1 1 …… 1				-1
$\theta_{(I/2)+2}$	$\equiv^* (C_m)$				H				0
:									:
θ_{I-1}									0
θ_{I}									0
θ_{I+1}			… …		2	2	… …	2	0

Table (1)

When $0 < r < m-1$, I is the number of Γ- classes of, θ_j such that $1 < j < I+1$ are the rational valued characters of group Q_{2m} and if we denote c_{ij} the elements $\equiv^* C_m$ and $h_{ij}$the elements of H as defined by : $h_{ij} = \begin{cases} c_{ij} & if \quad i = 1 \\ -c_{ij} & if \quad i \neq 1 \end{cases}$

and where I is the number of $\Gamma -$ classes of C_{2m}.

Theorem (2.7) :- [6]

If m is an odd number , then $K(Q_{2m}) = K(C_{2m}) \oplus C4$.

III. THE MAIN RESULTS

We study the rational valued characters table of $(\mathbf{Q}_{2q} \times \mathbf{C_{10}})$. and find the cyclic decomposition of $K(\mathbf{Q}_{2q} \times \mathbf{C_{10}})$.

The group(of($\mathbf{Q}_{2q} \times \mathbf{C_{10}}$) is the direct product of the Quaternion group$Q_{14}$of order 28 (number), and the group C_{10}. which is cyclic of order 10 the order of$(\mathbf{Q}_{2q} \times \mathbf{C_{10}})$.is $= 40q$

Theorem (3.1)

$\equiv^* (\mathbf{Q}_{2q} \times \mathbf{C_{10}})$ has the following form

$$\equiv^* (\mathbf{Q}_{2q} \times \mathbf{C_{10}}) = \equiv^* (\mathbf{Q}_{2q}) \otimes \equiv^* (\mathbf{C_{10}})$$

Proof

For each element $g_{khl} \in (\mathbf{Q}_{2q} \times \mathbf{C_{10}})$, we have $g_{khl} = g_k g_h g_l$, such that $g_d \in Q_{14}$,$g_h \in C_{2,} = \prec r_2 \succ = \prec -1 \succ$, $g_l \in C_5 = \prec r \succ$.and $r = e^{2\pi/5}$ Then $g_k = x^t y^f, 0 \le t \le 14, f = 0,1$, and each irreducible character of $\mathbf{Q}_{2q} \times \mathbf{C_{10}}$ is $\chi_{ijk} = \chi_i \chi'_j \chi''_k$, where χ_i is an irreducible character of $\mathbf{Q}_{2q}$, χ'_j is an irreducible character of C_2, and χ''_k is an irreducible character of C_5

By Proposition (2-2) we get

$$\theta_{ijk=}\theta_{ijk(\mathrm{g}_{dhl})=}\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(\mathrm{g}_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(\mathrm{g}_{dhl}))\right\rrbracket$$

$$=\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(\mathrm{g}_d))/\mathcal{Q})} \sigma\left(\chi_i(\mathrm{g}_d)\right)\left[\begin{matrix}\displaystyle\sum_{\sigma\epsilon\, Gal\left(\frac{\mathcal{Q}\left(\acute{\chi}_{\mathrm{j}}(\mathrm{g}_h)\right)}{\mathcal{Q}}\right)} \sigma\left(\acute{\chi}_{\mathrm{j}}(\mathrm{g}_h)\right)\\ \left\{\displaystyle\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi''_k(\mathrm{g}_l))/\mathcal{Q})} \sigma(\chi''_k(\mathrm{g}_l))\right\}\end{matrix}\right]\right\rrbracket$$

1) If j = 1, and k = 1, then for all $\mathrm{g}_h \in C_2$and for all $\mathrm{g}_l \in C_5$ such that $\theta'_j(\mathrm{g}_h) = \sum \chi'_j(\mathrm{g}_h) = 1, \theta''_k(\mathrm{g}_l) = \sum \chi''_k(\mathrm{g}_l) = 1.$ Then

$$\theta_{ijk(\mathrm{g}_{dhl})=}\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(\mathrm{g}_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(\mathrm{g}_{dhl}))\right\rrbracket$$

$$=\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(\mathrm{g}_d))/\mathcal{Q})} \sigma\left(\chi_i(\mathrm{g}_d)\right)[1\{1\}]\right\rrbracket$$

$$= \theta_i(\mathrm{g}_d) * \theta'_j(\mathrm{g}_h) * \ \theta''{}_k(\mathrm{g}_l)$$

2) If j = 1, and k = 2,3,4,5, for Avery $\mathrm{g}_h \in C_2$and g_l is the identity of C_5 that $\theta'_j(\mathrm{g}_h) = \sum \chi'_1(\mathrm{g}_h) = 1, \theta''_k(\mathrm{g}_l) = \sum_{k=2}^{5} \chi''_k(\mathrm{g}_l) = 4$ Then

$$\theta_{ijk(\mathrm{g}_{dhl})=}\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(\mathrm{g}_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(\mathrm{g}_{dhl}))\right\rrbracket$$

$$=\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(\mathrm{g}_d))/\mathcal{Q})} \sigma\left(\chi_i(\mathrm{g}_d)\right)[1\{4\}]\right\rrbracket$$

$$= \ \theta_i(\mathrm{g}_d)(1)(4) = \theta_i(\mathrm{g}_d) * \theta'_j(\mathrm{g}_h) * \ \theta''_k(\mathrm{g}_l)$$

If j = 1, and k = 2,3,4,5, for Avery $\mathrm{g}_h \in C_2$and g_l is not identity of C_5 that $\theta'_1(\mathrm{g}_h) = \sum \chi'_1(\mathrm{g}_h) = 1, \theta''_k(\mathrm{g}_l) = \sum_{k=2}^{5} \chi''_k(\mathrm{g}_l) = (\varepsilon + \varepsilon^2 + \varepsilon^3 + \varepsilon^4)_{=}$ -1

$$\theta_{ijk(\mathrm{g}_{dhl})=}\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(\mathrm{g}_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(\mathrm{g}_{dhl}))\right\rrbracket$$

$$=\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(\mathrm{g}_d))/\mathcal{Q})} \sigma\left(\chi_i(\mathrm{g}_d)\right)[1\{-1\}]\right\rrbracket$$

$$\theta_{ijk} = (1)(-1) = \ \theta_i(\mathrm{g}_d)(1)(-1)$$
$$= \ \theta_i(\mathrm{g}_d) * \theta'_j(\mathrm{g}_h) * \ \theta''_k(\mathrm{g}_l)$$

If $j = 2$ and $k = 1$, g_h is the identity of C_2 and for all $\mathrm{g}_l \in C_5$

$$\theta_i(\mathrm{g}_d) = \sum \chi_i(\mathrm{g}_d)\ ,\ \ \theta'_j(\mathrm{g}_h) = \chi'_2(\mathrm{g}_h) = \ 1\,,$$
$$\theta''_k(\mathrm{g}_l) = \ \chi''_k(\mathrm{g}_l) = 1$$

$$\theta_{ijk(\mathrm{g}_{dhl})=}\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(\mathrm{g}_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(\mathrm{g}_{dhl}))\right\rrbracket$$

$$=\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(\mathrm{g}_d))/\mathcal{Q})} \sigma\left(\chi_i(\mathrm{g}_d)\right)[1\{1\}]\right\rrbracket$$

$$= \ \theta_i(\mathrm{g}_d)(1)(1) = \ \theta_i(\mathrm{g}_d) * \theta'_j(\mathrm{g}_h) * \ \theta''_k(\mathrm{g}_l)$$

If $j = 2$ and $k = 2{,}3{,}4{,}5$, g_h is the identity of C_2 and g_l is the identity of C_5

$$\theta_i(\mathrm{g}_d) = \sum \chi_i(\mathrm{g}_d)\,,\ \ \theta'_j(\mathrm{g}_h) = \sum_{k=2}^{2} \chi'_j(\mathrm{g}_h) = \ 1\,,\ \theta''_k(\mathrm{g}_l)$$
$$= \sum_{k=2}^{5} \chi''_k(\mathrm{g}_l) = \sum_{k=2}^{5} 1 \ = 4$$

$$\theta_{ijk(\mathrm{g}_{dhl})=}\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(\mathrm{g}_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(\mathrm{g}_{dhl}))\right\rrbracket$$

$$=\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(\mathrm{g}_d))/\mathcal{Q})} \sigma\left(\chi_i(\mathrm{g}_d)\right)[1\{4\}]\right\rrbracket$$

$$= \ \ \theta_i(\mathrm{g}_d)(1)(4) = \ \theta_i(\mathrm{g}_d) * \theta'_j(\mathrm{g}_h) * \ \theta''_k(\mathrm{g}_l)$$

If $j = 2$and $k = 2{,}3{,}4{,}5$, g_h is the identity of C_2 and g_l is not identity of C_5

$$\theta_i(\mathrm{g}_d) = \sum \chi_i(\mathrm{g}_d),\ \ \ \theta'_j(\mathrm{g}_h) = \sum_{k=2}^{2} \chi'_j(\mathrm{g}_h) \ = \ 1\,,$$
$$\theta''_k(\mathrm{g}_l) = \sum_{k=2}^{5} \chi''_k(\mathrm{g}_l)$$
$$= \ (\varepsilon + \varepsilon^2 + \varepsilon^3 + \varepsilon^4) = -1$$

$$\theta_{ijk(\mathrm{g}_{dhl})=}\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(\mathrm{g}_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(\mathrm{g}_{dhl}))\right\rrbracket$$

$$=\left\llbracket \sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(\mathrm{g}_d))/\mathcal{Q})} \sigma\left(\chi_i(\mathrm{g}_d)\right)[1\{-1\}]\right\rrbracket$$

$$= \theta_i(\mathrm{g}_d)(1)(-1) = \theta_i(\mathrm{g}_d) * \theta'_j(\mathrm{g}_h) * \theta''_k(\mathrm{g}_l)$$

If $j = 2$and $k = 1$, g_h is the not identity of C_2 and for all $\mathrm{g}_l \in C_5$

$$\theta_i(\mathrm{g}_d) = \sum \chi_i(\mathrm{g}_d), \quad \theta'_j(\mathrm{g}_h) = \sum_{k=2}^{2} \chi'_j(\mathrm{g}_h) = -1,$$

$$\theta''_k(\mathrm{g}_l) = \sum_{k=2}^{5} \chi''_k(\mathrm{g}_l) = 1$$

$$\theta_{ijk(\mathrm{g}_{dhl})} = \left[\!\left[\sum_{\sigma \epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(\mathrm{g}_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(\mathrm{g}_{dhl})) \right]\!\right]$$

$$= \left[\!\left[\sum_{\sigma \epsilon\, Gal(\mathcal{Q}(\chi_i(\mathrm{g}_d))/\mathcal{Q})} \sigma\left(\chi_i(\mathrm{g}_d)\right)[1\{1\}] \right]\!\right]$$

$$= \theta_i(\mathrm{g}_d)(-1)(1) = \theta_i(\mathrm{g}_d) * \theta'_j(\mathrm{g}_h) * \theta''_k(\mathrm{g}_l)$$

If $j = 2$ and $k = 2,3,4,5$, g_h is not identity of C_2 and g_l is the identity of C_5

$$\theta_i(\mathrm{g}_d) = \sum \chi_i(\mathrm{g}_d), \quad \theta'_j(\mathrm{g}_h) = \chi'_2(\mathrm{g}_h) = -1,$$

$$\theta''_k(\mathrm{g}_l) = \sum_{k=2}^{5} \chi''_k(\mathrm{g}_l) = \sum_{k=2}^{5} 1 = 4$$

$$\theta_{ijk(\mathrm{g}_{dhl})} = \left[\!\left[\sum_{\sigma \epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(\mathrm{g}_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(\mathrm{g}_{dhl})) \right]\!\right]$$

$$= \left[\!\left[\sum_{\sigma \epsilon\, Gal(\mathcal{Q}(\chi_i(\mathrm{g}_d))/\mathcal{Q})} \sigma\left(\chi_i(\mathrm{g}_d)\right)[-1\{4\}] \right]\!\right]$$

$$= (-1)(4)\, \theta_i(\mathrm{g}_d) = \theta_i(\mathrm{g}_d) * \theta'_j(\mathrm{g}_h) * \theta''_k(\mathrm{g}_l)$$

If $j = 2$and $k = 2,3,4,5$, g_h is not identity of C_2 and g_l is not identity of C_5

$$\theta_i(\mathrm{g}_d) = \sum \chi_i(\mathrm{g}_d), \quad \theta'_j(\mathrm{g}_h) = \chi'_2(\mathrm{g}_h) = -1,$$

$$\theta''_k(\mathrm{g}_l) = \sum_{k=2}^{5} \chi''_k(\mathrm{g}_l) = (\varepsilon + \varepsilon^2 + \varepsilon^3 + \varepsilon^4) = -1$$

$$\theta_{ijk(\mathrm{g}_{dhl})} = \left[\!\left[\sum_{\sigma \epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(\mathrm{g}_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(\mathrm{g}_{dhl})) \right]\!\right]$$

$$= \left[\!\left[\sum_{\sigma \epsilon\, Gal(\mathcal{Q}(\chi_i(\mathrm{g}_d))/\mathcal{Q})} \sigma\left(\chi_i(\mathrm{g}_d)\right)[-1\{-1\}] \right]\!\right]$$

$$\theta_i(\mathrm{g}_d)(-1)(-1) = \theta_i(\mathrm{g}_d) * \theta'_j(\mathrm{g}_h) * \theta''_k(\mathrm{g}_l)$$

We have $\theta_{ijk}(\mathrm{g}_{dhl}) = \theta_i(\mathrm{g}_d) * \theta'_j(\mathrm{g}_h) * \theta''_k(\mathrm{g}_l)$ for all i, j, k and for all $\mathrm{g}_d, \mathrm{g}_h, \mathrm{g}_l$ where $\theta_{ijk}(\mathrm{g}_{dhl}), \theta_i(\mathrm{g}_d), \theta'_j(\mathrm{g}_h)$ *and* $\theta''_k(\mathrm{g}_l)$ are the rational valued character of the group $(\mathbf{Q}_{2q} \times \mathbf{C_{10}})$,$(Q_{2\mathrm{q}})$,$(\mathrm{C}_2)$and,$(\mathrm{C}_5)$ respectively and e is the identity of group

$$\theta_{ijk}(\mathrm{g}_{dhl}) = \begin{cases} \theta_i(\mathrm{g}_d), \text{if } j = 1, \wedge k = 1, \wedge \forall \mathrm{g}_h \in C_2 \wedge \forall \mathrm{g}_l \in \mathrm{C}_5 \\ (4)\chi_i(\mathrm{g}_d), \text{if } j = 1, \wedge k = 2,3,4,5, \wedge \forall \mathrm{g}_h \in C_2 \wedge \mathrm{g}_l = e \\ -\theta_i(\mathrm{g}_d), \text{if } j = 1, \wedge k = 2,3,4,5, \wedge \forall \mathrm{g}_h \in C_2 \wedge \mathrm{g}_l \neq e \\ \theta_i(\mathrm{g}_d), \text{if } j = 2, \wedge k = 1, \mathrm{g}_h = e \wedge \forall \mathrm{g}_l \in \mathrm{C}_5 \\ (4)\chi_i(\mathrm{g}_d), \text{if } j = 2, \wedge k = 2,3,4,5, \wedge \mathrm{g}_h = e \wedge \mathrm{g}_l = e \\ -\chi_i(\mathrm{g}_d), \text{if } j = 2, \wedge k = 2,3,4,5, \wedge \mathrm{g}_h = e \wedge \mathrm{g}_l \neq e \\ -\chi_i(\mathrm{g}_d), \text{if } j = 2, \wedge k = 1, \mathrm{g}_h \neq e \wedge \forall \mathrm{g}_l \in \mathrm{C}_5 \\ (-4)\chi_i(\mathrm{g}_d), \text{if } j = 2, \wedge k = 2,3,4,5, \neq e \wedge \mathrm{g}_l = e \\ \chi_i(\mathrm{g}_d), \text{if } j = 2, \wedge k = 2,3,4,5, \mathrm{g}_h \neq e \wedge \mathrm{g}_l \neq e \end{cases}$$

then

$$\equiv^* (\mathbf{Q}_{2q} \times \mathbf{C_{10}}) = \equiv^* (\mathbf{Q}_{2q}) \otimes \equiv^* (\mathbf{C_{10}}) \quad \square$$

Then, $\equiv^* (\mathbf{Q}_{2q} \times \mathbf{C_{10}})$ is given in the following table :

Γ-classes	[I , I]	[I , r2]	I , r]	[I , r r2]	[x2, I]	[x2, r2]	[x2, r]	[x2, r r2]	[xq, I]	[xq, r2]	[xq , r]	[xq , r r2]	q -1	[x , r2]	[x , r]	[x , r r2]	[y , I]	y , r2]	y , r]	[y , r r2]
| clα |	1	1		4	q -1	q -1	4 (q-1)	4 (q-1))	1	1	4	4	q -1	q -1	4 (q-1)	4 (q-1)	2 q	q	q	8 q
θ 11	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1			1
θ 13	1	1 -		1 -	1	1 -	1	1 -	1	1 -	1	1 -	1	1 -	1	1 -	1	-		1 -
θ 13	4	4	1	- 1	4	4	- 1	- 1	4	4	- 1	- 1	4	4	- 1	- 1	4		1	- 1
θ 14	4	- 4	1	1	4	- 4	- 1	1	4	- 4	- 1	1	4	- 4	- 1	1	4	4	1	1
θ 21	q -1	q -1	-1	q -1	- 1	- 1	- 1	- 1	q -1	q -1	q -1	q -1	- 1	- 1	- 1	- 1	0			0
θ 22	q -1	1 -q	-1	1 -q	- 1	1	- 1	1	q -1	1 -q	q -1	1 -q	- 1	1	- 1	1	0			0
θ 23	4 (q-1)	4 (q-1)	-q	1 -q	- 4	- 4	1	1	4 (q-1)	4 (q-1)	1 -q	1 -q	- 4	- 4	1	1	0			0
θ 24	4 (q-1)	4 (1-q)	-q	q -1	- 4	4	1	- 1	4 (q-1)	4 (1-q)	1 -q	q -1	- 4	4	1	- 1	0			0
θ 31	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	- 1	1	1	- 1
θ 32	1	1 -		1 -	1	1 -	1	1 -	1	1 -	1	1 -	1	1 -	1	1 -	- 1		1	1
θ 33	4	4	1	- 1	4	4	- 1	- 1	4	4	- 1	- 1	4	4	- 1	- 1	- 4	4		1
θ 34	4	- 4	1	1	4	- 4	- 1	1	4	- 4	- 1	1	4	- 4	- 1	1	- 4			- 1
θ 41	q -1	q -1	-1	q -1	- 1	- 1	- 1	- 1	1 -q	1 -q	1 -q	1 -q	1	1	1	1	0			0
θ 42	q -1	1 -q	-1	1 -q	- 1	1	- 1	1	1 -q	q -1	1 -q	q -1	1	- 1	1	- 1	0			0
θ 43	4 (q-1)	4 (q-1)	-q	1 -q	- 4	- 4	1	1	4 (1-q)	4 (1-q)	q -1	q -1	4	4	- 1	- 1	0			0
θ 44	4 (q-1)	4 (1-q)	-q	q -1	- 4	- 4	1	- 1	4 (1-q)	4 (q-1)	q -1	1 -q	4	- 4	- 1	1	0			0
θ 51	2	2		2	2	2	2	2	- 2	- 2	- 2	- 2	- 2	- 2	- 2	- 2	0			0
θ 52	2	- 2		- 2	2	- 2	2	- 2	- 2	2	- 2	2	- 2	2	- 2	2	0			0
θ 53	8	8	2	- 2	8	8	- 2	- 2	- 8	- 8	2	2	- 8	- 8	2	2	0			0
θ 54	8	- 8	2	- 2	8	- 8	- 2	2	- 8	8	2	- 2	- 8	8	2	- 2	0			0

$$\equiv^* (Q_{14} \times 10) = \equiv^* (Q_{14}) \otimes \equiv^* (\mathrm{C}_{10})$$

Theorem (3.2)

the cyclic decomposition of the group $(Q_{14} \times C_{10})$is given by

$$K\left(\mathbf{Q}_{2q} \times \mathbf{C}_{\mathbf{10}}\right) =\oplus_{i=1}^{4} K(C_{14}) \oplus_{i=1}^{4} C_4 \oplus_{i=1}^{10} C_2 \oplus_{i=1}^{10} C_5$$

$$K\left(\mathbf{Q}_{2q} \times \mathbf{C}_{\mathbf{10}}\right) =\oplus_{i=1}^{4} K(Q_{14}) \oplus_{i=1}^{5} K(C_{10})$$

Proof : Let A and B be matrices defining as follows:

$$\mathrm{A}=\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad , B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & A & A & A \\ 0 & A & 0 & A \\ 0 & 0 & A & A \\ 0 & 0 & 0 & A \end{bmatrix} \equiv^* \left(Q_{2q} \times C_{10}\right) \begin{bmatrix} B & 0 & 0 & 0 \\ B & B & 0 & 0 \\ 4B & 0 & B & 0 \\ 4B & 4B & B & B \end{bmatrix} = \begin{bmatrix} E_3 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 \\ 0 & 0 & E_1 & 0 \\ 0 & 0 & 0 & E_0 \end{bmatrix}$$

Such that $E_{0,}E_1 , E_2, E_3$ of degree A and B the invariant factors of the matrix$\equiv^* \left(\mathbf{Q}_{2q} \times \mathbf{C}_{\mathbf{10}}\right)$ it's the same invariant factors of the matrices $E_{0,}E_1 , E_2, E_3$, so as will the proof

$$E_0 = \begin{bmatrix} -4 & -4 & 0 & 0 & 0 \\ 2 & 2-2q & 0 & 0 & 0 \\ 0 & 0 & -2 & -2 & 0 \\ 0 & 0 & 1 & 1-q & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

such as $(E_3 = -10E_0), (E_2 = -5E_0), (E_1 = 2E_0)$ And we will defined two matrices Lp and Wp , such that :

$$L_q = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, W_q = \left[\begin{bmatrix} q-1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & q-1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}\right]$$

Such that; $L_q . E_0 . W_q = diag(-4q, 2, -2q, 1, -1)$

$$L_q . E_1 . W_q = diag(-8q, 4, -4q, 2, -2)$$

$$L_q . E_2 . W_q = diag(20q, -10,10q, -5,5)$$

$$L_q . E_3 . W_q = diag(40q, 20,20q, -10,10)$$

Then: $K\mathbf{Q}_{2q} \times \mathbf{C}_{\mathbf{10}} = \oplus_{i=1}^{4} K(\mathbf{C}_{2q}) \oplus_{i=1}^{4} C_4 \oplus_{i=1}^{10} C_5 \oplus_{i=1}^{10} C_2$

$$K\left(\mathbf{Q}_{2q} \times \mathbf{C}_{\mathbf{10}}\right) =\oplus_{i=1}^{4} K(\mathbf{Q}_{2q}) \oplus_{i=1}^{5} K(C_{10})$$

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