Journal of Kufa for Mathematics and Computer Vol.10, No.1, Mar., 2023, pp 13-17

Bayesian Binary reciprocal LASSO quantile regression (with practical application)

Mohammed T. Kahnger
Faculty of computer science and Maths
University of Kufa
Najaf Al-Ashraf, Iraq
mohammedt.azooz@uokufa.edu.iq
https://orcid.org/0009-0001-2898-3219

Prof. Dr. Ahmad Naeem Flaih
Faculty of Administratration and Economics
University of Al-Qadisiyah
Al Diwaniyah, Iraq
ahmad.flaih@qu.edu.iq

DOI: http://dx.doi.org/10.31642/JoKMC/2018/100102

Received Oct. 22, 2022. Accepted for publication Feb. 1, 2023

Abstract : *Quantile regression is one of the methods that has taken a wide space in application in the previous two decades because of the attractive features of these methods to researchers, as it is not affected by outliers values, meaning that it is considered one of the robust methods, and it gives more details of the effect of explanatory variables on the dependent variable.In this paper, a Bayesian hierarchical model for variable selection and estimation in the context of binary quantile regression is proposed. Current approaches to variable selection in the context of binary classification are sensitive to outliers, heterogeneous values, and other anomalies. The proposed method in this study overcomes these problems in an attractive and direct way.*

Keywords— Quantile regression ; variable selection; binary quantile regression.

I. INTRODUCTION

The quantile regression (QR) is one of the regression models that measures the effect of the independent variable on the dependent variable with infinite regression lines at certain divisions confined between (0.1) and this type of regression model is the robust methods because it is not affected by outliers, due to the passage of One of the regression lines near these values(23 , 24)

The quantile regression model can be expressed as in the following equation (5):

$$y_i = x_i'\beta_p + \varepsilon_i$$

y_i: the real observation value of the variable of order i

x_i: vector of degree (n×p) ; β_p: Parameters vector with quantity $0 < p < 1$, (p) .

The process of estimating the parameters of a quantile regression model β_p It is by minimizing the following loss function or check function.

$$\min_{\beta_p} \sum_{i=1}^{n} \rho_p(y_i - x_i'\beta_p)$$

$$\rho_p(y_i - x_i'\beta_p) = \begin{cases} p(y_i - x_i'\beta_p) & if \;\; y_i \geq x_i'\beta_p \\ -(1-p)(y_i - x_i'\beta_p) & if \;\; y_i < x_i'\beta_p \end{cases}$$

In our study, we will study the Bayes method in estimating the quantile regression model in the case that the response variable is binary, and we will rely on reciprocal functions that are included in the regression equation for the parameters of the studied model. Regression models are among the widespread statistical methods in statistical analyzes, and it can be expressed by the mathematical formula of the binary quantitative regression model, as follows:

$$y_i^* = x_i'\beta_p + \varepsilon_i$$

$$y_i = \begin{cases} 1 \;\; if & y_i^* \geq 0 \\ 0 \;\; if & otherwise \end{cases}$$

II- AIM OF SEARCH

The aim of the research is to propose a new estimation formula by employing a proposed prior distribution to reach the best estimates for choosing the most important variables in an estimation method with high-accuracy results and building an easy and attractive algorithm in shortening the estimation time

III- theo. Appli.

III.1 Quantile Regression

Quantile regression is one of the important methods of regression analysis, and it is considered an extension of linear regression in the event that the conditions for linear regression are not met.

Assuming that the normal linear regression model is given according to the following mathematical formula (9):

$y = X\beta + \varepsilon$; y : vector of responses of degree $(n \times 1)$; x: matrix of degree $(n \times p)$; $\boldsymbol{\varepsilon}$: vector of random errors of degree $(n \times 1)$

The classical estimator in linear regression is the Ordinary Least Squares (OLS) estimator if the conditions of this method , such that:

$\{\hat{\beta}_{ols} = (X'X)^{-1}X'Y\}$

It is known or observed that the (OLS) estimator is unstable in the case of a multicollinearity problem, and also when it is $(p > n)$ it results in the production of a non-singular estimator because the order of the regression matrix is less than the full order (less than full mark) and that the method of least squares (Least square ; LS) is sensitive in the event that there are anomalous or extreme data, or when the random error is not distributed normally. Standard linearity (4) as it does not assume a normal distribution of random error, unlike linear models, and is also not affected by extreme observations, because there are regression lines that pass close to these values, and therefore it can be said that it provides a more comprehensive statistical model than the classical models. (OLS, so it provides more robust solutions in many real applications, and for these reasons, the quantitative regression model is considered one of the robust models. Assuming that $\{(x_i, y_i); i = 1,2, \ldots\ldots, n\}$where y_i indicates The The dependent variable (response variable) and that $(x_i \in R^p)$represents the vector of explanatory variables (independent variables) . Therefore, the quantile regression model is expressed as in the following equation(5):

$$y_i = x_i'\beta_p + \varepsilon_i \quad ; \quad (i = 1,2, \ldots\ldots, n) \ldots\ldots.\ldots\ldots (3-1)$$

III.2 USING THE INVERSE (LASSO) METHOD IN LINEAR REGRESSION

Some suggestions and improvements have been made by a group of researchers over the years [13,10,,3,2] on the Lasso estimation method proposed by the researcher [14] Tibshirani; 1996 in estimating linear regression models. In recent years, both researchers [12] resented some developments or improvements in the method of Estimation ((Lasso), where the new formula is called the reciprocal Lasso[Least absolute shrinkage and selection operator], which is abbreviated as (rLasso), and the estimation of the parameters of the linear regression model using the inverse of Lasso is by minimizing the following mathematical equation:

$$\min_{\beta} \sum_{i=1}^{n} (y_i - x_i'\beta)^2 + \lambda \sum_{j=1}^{k} \frac{1}{|\beta_j|} \; I(\beta_j \neq 0) \ldots\ldots.(3-2)$$

To estimate the parameters of the quantile regression model by using the inverse Lasso, it is by minimizing the following equation as follows [12]:

$$Q(B) = \min_{\beta} \sum_{i=1}^{n} \rho_p(y_i - x_i'\beta_p) + \lambda \sum_{j=1}^{k} \frac{1}{|\beta_j|} \ldots\ldots(3-3)$$

$(\lambda > 0)$: parameter of Regularization ; y_i: vector of responses of degree $(n \times 1)$; x: matrix of degree $(n \times p)$.

III.3 Binary reciprocal lasso quantile regression

In this paper, we extend the Bayesian quantum regression referred to by the researcher [8] to the Bayesian quantile regression using the reciprocal lasso of the dichotomous response data. We think this approach is of interest, because by doing so we take advantage of both the desirable properties of binary quantitative regression as well as the excellent properties of the inverse Lasso penalty function (rlasso). [1, 7]. The mathematical formula of the binary quantile regression model is:

$$\boldsymbol{y_i^* = x_i'\beta_p + \varepsilon_i}$$

$$\boldsymbol{y_i} = \begin{cases} \mathbf{1} & \boldsymbol{if} \quad \boldsymbol{y_i^* \geq 0} \\ \mathbf{0} & \boldsymbol{if} \quad \boldsymbol{otherwise} \end{cases}$$

And the estimation of the parameters of the quantile regression model by using the reciprocal Lasso (rlasso) is according to the numbered equation (3-3). An important feature of quantile regression is that it is able to accommodate the distribution of unnormal random errors.

The researcher [11] Mallick and others proposed in the year 2021 a proposal to represent the previous (prior) distribution of the parameter β, as this proposal assumed that the inverse Laplace distribution of the parameter β can be represented as follows:

$$\boldsymbol{\frac{\lambda}{2\beta^2} e^{-\frac{\lambda}{|\beta|}} = \int_0^\infty \frac{\eta}{2\beta^2} I\{|\beta| > \eta\} \frac{\lambda^2}{\Gamma 2} \eta^{-2-1} e^{-\frac{\lambda}{\eta}} d\eta \ldots\ldots\ldots.(3-4)}$$

In this research, the following transformation was performed on the representation mentioned in equation (3-4):

$$Z = \frac{\lambda}{\eta} \Rightarrow \eta = \frac{\lambda}{Z}$$

$$\Rightarrow d\,\eta = \lambda\left(\frac{-1}{Z^2}\right)dz$$

$$\eta = 0 \Leftarrow Z = \infty\,; \; \eta = \infty \Leftarrow Z = 0$$

Thus, the terms of integration can be inverted and the negative sign removed from the derivative above, and we get the following:

$$\frac{\lambda}{2\beta^2} e^{-\frac{\lambda}{|\beta|}} = \int_0^\infty \frac{\frac{\lambda}{Z}}{2\beta^2} I\left\{|\beta| \frac{\lambda}{Z}\right\} . \frac{\lambda^2}{\Gamma(2)} \left(\frac{\lambda}{Z}\right)^{-2-1} . e^{-Z} \lambda \left(\frac{-1}{Z^2}\right) dz$$

$$= \int_0^\infty \frac{\lambda}{2\beta^2} I\left\{|\beta| > \frac{\lambda}{Z}\right\} . e^{-Z} dz$$

Such that : $Z \sim standard\ exponential$

$$\beta \sim \text{Inverse uniform } \left(-\frac{1}{\lambda}, \frac{1}{\lambda}\right) \ldots\ldots..(3-5)$$

This means that : $\beta \sim Inverse\ uniform\ (-\frac{1}{\lambda}, \frac{1}{\lambda})$

It has the following probability density function:

$$f(\beta|\lambda) = \frac{1}{\beta^2}\ Uniform\ \left(-\frac{1}{\lambda}, \frac{1}{\lambda}\right)$$

$$= \frac{1}{\beta^2} \cdot \frac{1}{1/_\lambda + 1/_\lambda} = \frac{1}{\beta^2}\ \frac{1}{2/_\lambda} = \frac{\lambda}{2\beta^2}$$

It is the inverse uniform distribution or called the double Pareto distribution. Thus from the relationship (3-5) it can be seen that the distribution of the inverse of the regular distribution is a uniform distribution function $[-\frac{1}{\lambda}, \frac{1}{\lambda}]$ multiplied by $\frac{1}{\beta^2}$, and thus the following hierarchical model of prior distributions can be linked (inverse Lasso) with the hierarchical model of prior distributions with the presence of the prior distribution of the Laplace distribution (Lasso regression method) (9) (Mallick and Yi, 2014).

III.4 The proposed Bayesian hierarchical model of the prior distributions:

The hierarchical model of a prior distributions can be assumed as follows:

$$y^* = X_i'\beta + e_i\ ,$$

$$y_i = \begin{cases} 1 & if \quad y_i^* \geq 0 \\ 0 & if \quad y_i^* < 0 \quad i = 1, \ldots., n \end{cases} ,$$

$$y_{n\times 1}^* | X, \beta, \sigma^2 \sim N_n(X\beta, \sigma^2 I_n),$$

$$\beta^{p\times 1} |\ \lambda \sim \prod_{j=1}^{p} Inverse\ uniform\ \left(-\frac{1}{\lambda}, \frac{1}{\lambda}\right),$$

$$\lambda|\theta \sim \prod_{j=1}^{p} Inverse\ Gamma\ (2, \theta),$$

$$\sigma^2 \sim Inverse\ Gamma\ (c, d),$$

$$Z \sim standard\ exponential$$

III.5 The full conditional posterior distribution

Relying on the hierarchical model of the prior distributions mentioned in paragraph 3-4, it is now possible to derive the posterior distributions for each variable, as shown below:

First, you must write the joint full for all the variables:

$$\boldsymbol{f(y^*|\beta, X, \sigma^2).\pi(\sigma^2).\pi(\lambda).\prod_{j=1}^{p}\pi(\beta_j|\lambda_j).\pi(z_j) I\left\{|\beta_j| > \frac{\lambda_j}{Z_j}\right\}}$$

$$= \frac{\mathbf{1}}{(\mathbf{2}\pi\sigma^2)^{\frac{n}{2}}} e^{-\frac{1}{2\sigma^2}}(\boldsymbol{y^*} - \boldsymbol{X\beta})'(\boldsymbol{y^*} - \boldsymbol{X\beta}) * \frac{\boldsymbol{d^c}}{\Gamma(\boldsymbol{c})}(\boldsymbol{\sigma^2})^{-c-1} e^{-\frac{d}{\sigma^2}} * \prod_{j=1}^{p}\frac{\boldsymbol{\lambda_j}}{\mathbf{2}\boldsymbol{\beta^2}} * \frac{\boldsymbol{\theta^2}}{\Gamma(\mathbf{2})}(\boldsymbol{\lambda})^{-2-1} e^{-\frac{\theta}{\lambda}} * \prod_{j=1}^{p} e^{-Z_j} * \prod_{j=1}^{p} \boldsymbol{I}\{|\boldsymbol{\beta_j}| > \frac{\boldsymbol{\lambda_j}}{\boldsymbol{Z_j}}\} \ldots\ldots\ldots\ldots\ldots\ldots.(\mathbf{3}-\mathbf{6})$$

1-The complete conditional distribution of the variable $\boldsymbol{y_i^*}$is a normal distribution:

$$\boldsymbol{y_i^*|y_i, \beta, \sigma^2} = \begin{cases} \boldsymbol{N(x_i'\beta, \sigma^2 I_n) I\{y_i^* > 0\}, if\ y_i = 1} \\ \boldsymbol{N(x_i'\beta, \sigma^2 I_n) I\{y_i^* \leq 0\}, if\ y_i = 0} \end{cases}$$

2- The complete conditional distribution of (β) is:

As long as the Gibbs Sampler algorithm does not need more than proportional or unnormalized distribution, which expresses the product of the possibility function with the predistribution function. Therefore, we will need only the parts that fit or contain the variable β of the common distribution in equation (3-6), and we will delete any part that does not contain β and using the relationship we have indicated by connecting between the lasso method and the inverse lasso method, we will get:

$$\boldsymbol{\pi(\beta|y^*, X, \lambda, \sigma^2) \propto \pi(y^*|X, \beta, \sigma^2).\pi(\beta|\lambda)}$$

$$\propto \boldsymbol{exp\left\{-\frac{1}{2\sigma^2}(y^* - X\beta)'(y^* - X\beta)\right\}\prod_{j=1}^{p} I\{|\beta_j| > \frac{\lambda_j}{Z_j}\}}$$

$$\propto \boldsymbol{exp\{-\frac{1}{2\sigma^2}[\beta'(X'X)\beta - 2y^*X\beta + y^*y^{*\prime}]\prod_{j=1}^{p} I\{|\beta_j| > \frac{\lambda_j}{Z_j}\}}$$

$$\propto \boldsymbol{exp\{-\frac{1}{2\sigma^2}[\beta'A\beta - 2y^*X\beta + y^*y^{*\prime}]} \ldots\ldots\ldots..(\mathbf{3}-\mathbf{7})$$

Such that $= (\boldsymbol{X'X})$, The following square expression can be used.

$$\boldsymbol{(\beta - A^{-1}X'y^*)'A(\beta - A^{-1}X'y^*) = \beta'A\beta - 2y^*X\beta + y^{*\prime}(XA^{-1}X')y^*}$$

Thus, the relationship (3-7) can be written as follows:

$$\propto \boldsymbol{exp\{-\frac{1}{2\sigma^2}[(\beta - A^{-1}X'y^*)'A(\beta - A^{-1}X'y^*) + y^{*\prime}(I_n - XA^{-1}X')y^*]\}} \ldots\ldots..(\mathbf{3}-\mathbf{8})$$

The second part of the relationship (3-8) does not contain a β, so the joint distribution that contains β will be abbreviated as follows:

$$\propto \boldsymbol{exp\{-\frac{1}{2\sigma^2}[(\beta - A^{-1}X'y^*)'A(\beta - A^{-1}X'y^*)\}}$$

Returning to the multivariate normal distribution $\boldsymbol{X \sim N(M, \Sigma)}$, that is:

$$\boldsymbol{f(x; M, \Sigma)} = \frac{\boldsymbol{e^{-\frac{1}{2}(X-M)'\Sigma^{-1}(X-M)}}}{(\mathbf{2}\boldsymbol{\pi})^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}}$$

It can be concluded that β has a multivariate normal distribution with mean $\boldsymbol{A^{-1}X'y^*}$and variance $\boldsymbol{\sigma^2A^{-1}}$.

3- The complete conditional distribution of $\boldsymbol{\sigma^2}$is:

Samples for the variable $\boldsymbol{\sigma^2}$ will be generated using the Gibbs Sampler algorithm by taking all the parts that include $\boldsymbol{\sigma^2}$in the joint distribution (3-6), that is:

$$\boldsymbol{\pi(\sigma^2|y^*,x,\beta) \propto \pi(y^*|x,\beta,\sigma^2).\pi(\sigma^2)}$$

$$\propto \frac{\boldsymbol{1}}{\boldsymbol{\sigma^{2\frac{n-1}{2}}}}[-\frac{\boldsymbol{1}}{\boldsymbol{2\sigma^2}}\boldsymbol{(y^*-X\beta)'(y^*-X\beta)}$$

$$*\frac{\boldsymbol{d^c}}{\Gamma(\boldsymbol{c})}\boldsymbol{(\sigma^2)^{-c-1}*e^{-\frac{d}{\sigma^2}}}$$

$$\propto \boldsymbol{\sigma^{2^{-\left(\frac{n-1}{2}\right)-c-1}}exp[-\frac{1}{2\sigma^2}(y^*-X\beta)'(y^*-X\beta)}$$

$$-\frac{\boldsymbol{d}}{\boldsymbol{\sigma^2}} \qquad \dots\boldsymbol{(3-9)}$$

Using an inverse gamma distribution, we find that:

$$\boldsymbol{f(x;\alpha,\beta)=\frac{\beta^\alpha}{\Gamma(\alpha)}x^{-\alpha-1}\,exp\left\{-\frac{\beta}{x}\right\}}\dots\dots\dots\dots\boldsymbol{(3-10)}$$

Where α represents the shape parameter and β represents a scale parameter, and using the function (3-10) with the relationship (3-9) it can be concluded that $\boldsymbol{\sigma^2}$ has a gamma inverse distribution with a shape parameter $(\boldsymbol{\frac{n-1}{2}+c})$ and a scale parameter $\boldsymbol{(y^*-X\beta)'(y^*-X\beta)+d}$

4- The complete conditional distribution of $\boldsymbol{\lambda}$ is:

Samples for the variable λ will be generated from the joint distribution in the relationship (3-6) after taking the limits that include only the variable λ and using the Gibbs sampler algorithm as follows:

$$\boldsymbol{\pi(\lambda|\theta,\beta)\propto\pi(\beta|\lambda)*\pi(\lambda)}$$

$$\propto\prod_{j=1}^{p}\frac{\boldsymbol{\lambda_j}}{\boldsymbol{2\beta^2}}\frac{\boldsymbol{\theta^2}}{\Gamma(\boldsymbol{2})}\boldsymbol{(\lambda)^{-2-1}e^{-\frac{\theta}{\lambda}}.I\{|\beta_j|>\frac{\lambda_j}{Z_j}\}}$$

$$\propto\boldsymbol{\lambda^{-(p+2)-1}e^{-\frac{\theta}{\lambda}}.I\{\lambda_j<Z_j|\beta_j|\}}\dots\dots\dots.\boldsymbol{(3-11)}$$

Where it can be said that the relationship (3-11) produced the distribution of the variable λ to be .

$$\boldsymbol{inverse\ gamma(p+2,\theta)}$$

5- The complete conditional distribution of $\boldsymbol{Z}$ is:

$$\boldsymbol{\pi(z|\,\lambda)\propto\pi(z)*\pi(\beta|z)}$$

$$\propto\prod_{j=1}^{p}\boldsymbol{e^{-z_j}.I\left\{Z_j>\frac{\lambda_j}{|\beta_j|}\right\}}$$

$$\propto\prod_{j=1}^{p}\boldsymbol{exponential(1).I\left\{Z_j>\frac{\lambda_j}{|\beta_j|}\right\}}$$

IV- Application

In this aspect, real data taken from a hospital from the department for diagnosing corona disease (Covid-19) was relied upon in order to analyze the relationship between the patient's condition upon discharge from the hospital (the dependent variable) and a set of diagnoses (independent variables) specified by the specialist doctors For the patient's case, where the data set obtained from the hospital by the researcher was 250 observations, where the number of independent variables was equal to 25 variables, training data equals 20 and test data equals 230. The (quantile) equal to (p = 0.50) and (p = 0.95) were chosen in the analysis of this relationship. The reason for choosing these ratios is that the data is always centered around the center line, i.e. when (p = 0.50), and as for the other value, which is an extreme value for (quantile), i.e. when (p = 0.95), and the reason for choosing this value is because if the regression line is good at This value is sure to be good in the quantile values below or close to it

IV-1 data characterization

The variables were categorized as follows:

Y: the patient's discharge status, which represents the dependent variable, X1: gender, X2: educational attainment, X3: occupation, X4: age in years, X5: marital status, X6: place of residence, X7: patient's residence, X8: patient's duration of stay in days , X9: smoking status, X10: blood sugar, X11: blood pressure, X12: person's weight, X13: blood urea rate, X14: creatine percentage in the blood, X15: (LDH) lung-related enzyme, X16: (CRP) (An analysis specific to the presence of infections, X17: an analysis representing iron stores in the blood, X18: (ESR) an analysis specific to the presence of infections, X19: (HGB) representing the blood percentage, X20: (WBC) the percentage of white blood cells, X21: (NEU) representing An analysis that detects immunity in humans, X22: (LYM) an analysis that represents the presence of a virus in the body, X23: (PLT) an analysis that represents the number of blood platelets that benefit clotting, X24: (D-DIMER) an analysis that detects clotting in the blood, X25: (SPO2) Analysis indicates the percentage of oxygen concentration in the blood.

IV-2 Algorithm implementation results

When implementing the algorithm that was written by the researcher, the results were as follows:

Table (4-1) shows the results of the algorithm at (p = 0.5)

	BBqr	BrLBqr
Intercept	0.009	-1.828
x1	0.232	-6.864
x2	0.237	-0.144
x3	-0.006	2.673
x4	0.018	-0.719
x5	0.302	2.911
x6	0.022	-1.981
x7	0.093	-4.649
x8	-0.015	-0.244
x9	0.211	-3.516
x10	-0.107	-5.468
x11	-0.008	-0.606
x12	-0.007	-1.388
x13	0.005	0.107
x14	-0.031	-1.829
x15	-0.002	-0.05
x16	-0.122	-1.154
x17	0	0.017
x18	-0.004	0.268
x19	0.056	1.354

x20	-0.009	0.492
x21	-0.049	1.967
x22	-0.048	2.348
x23	0	0.046
x24	-0.054	3.482
x25	0.039	-0.674

Table (4-2) shows the results of the algorithm at (p = 0.95)

	BBqr	BrLBqr
Intercept	0.457	15.317
x1	-0.031	-2.536
x2	0.072	0.671
x3	-0.064	3.149
x4	0.01	0.195
x5	0.237	7.044
x6	-0.066	3.347
x7	0.063	1.398
x8	0.005	0.239
x9	0.039	4.053
x10	-0.367	0.829
x11	-0.102	0.611
x12	-0.001	-0.53
x13	-0.006	-0.015
x14	0.353	-1.299
x15	-0.001	-0.024
x16	-0.284	0.196
x17	0	0
x18	-0.003	-0.085
x19	-0.027	-2.339
x20	-0.007	-0.144
x21	0.03	0.058
x22	0.028	-0.164
x23	0.002	0.019
x24	-1.077	0.498
x25	0.05	0.392

IV-3 *Interpretation of results*

1- The results of Table No. (4-1) showed superior performance of the proposed method(BrLBqr)[Bayesian reciprocal Lasso Binary quantile regression] at the level (p = 0.5), where the proposed method BrLBqr obtained 217 results of the correct classification out of a total of 230 observations, while the other Bayesian method[Bayesian Binary quantile regression](BBqr) got 202 correct rating out of 230 total views.

2- The results of Table No. (4-2) also showed an outperformance in the performance of the proposed method(BrLBqr) at the level (p = 0.95), where it got 205 of the correct classification out of a total of 230 observations, while the other methods (BBqr) got 194 of the correct classification out of a total of Views 230.

CONCLUSIONS

In this paper, we present a Bayesian approach to binary quantitative regression coupled with the technique of selection variables, that is, Binary reciprocal lasso quantile regression. The main advantages of this approach were the estimation of the parameters of the model and the selection of predictive variables affecting the dependent variable without sensitivity to abnormal values, unlike other methods such as the method of ordinary least squares (OLS) and other methods, and therefore this approach is considered one of the fortified methods. Also, this method can identify variables that are important predictors of different quantities of the response variable distribution. The proposed method (BrLBqr) was applied to real data and compared with (BBqr) method. Using the Gibbs sampler algorithm, the results showed that the proposed method (BrLBqr) outperformed the other method (BBqr).

REFERENCES

[1] Benoit DF, Van den Poel D (2012) Binary Quantile Regression: A Bayesian Approach Based on the Asymmetric Laplace Distribution. J App Econom 27:1174{1188.

[2] Efron, B., T. Hastie, I. Johnstone, R. Tibshirani, et al. (2004). Least angle regression. The Annals of statistics 32 (2), 407–499.

[3] Fan, J. and R. Li (2001). Variable selection via nonconcave penalized likelihood and its oracle properties. Journal of the American statistical Association 96 (456), 1348–1360.

[4] Koenker, R., Bassett G. J., 1978, “Regression quantiles”, Econometrica, 46, pp.33-50.

[5] Keming Yu& Rana A.Moyeed (2001)" Bayesian Quantile Regression" Statistical & Probability Letters 54:437-447.

[6] Keming Yu, Zudi Lu and Julian Stander (2003) “Quantile regression :applications and current research areas” The Statistician 52, Part 3, pp. 331-350.

[7] Kordas G (2006) Smoothed Binary Regression Quantiles. J Appl Econom 21:387- 407.

[8] Li Q, Xi R, Lin N (2010) Bayesian Regularized Quantile Regression. Bayesian Analysis 5:1-24.

[9] Mallick, H., Yi, N., 2014, “A new Bayesian lasso”, Statistics and its interface, 7: pp. 571-582.

[10] Meinshausen, N. (2007). Relaxed lasso. Computational Statistics & Data Analysis 52 (1), 374–393.

[11] Mallick, H., Alhamzawi, R., Paul, E., & Svetnik, V. (2021). The reciprocal Bayesian lasso. Statistics in Medicine, 40(22), 4830-4849.

[12] Qifan Song and Faming Liang. High-dimensional variable selection with reciprocal 1-Regularization.Jornal of the American Statistical Association, 110(512):1607-1620,2015.

[13] Radchenko, P. and G. M. James (2008). Variable inclusion and shrinkage algorithms. Journal of the American Statistical Association 103 (483), 1304–1315.

[14] Tibshirani (1996)"Regression Shrinkage and selection via lasso" J. R.Statis.Soc.58,No.1,pp. 267-288.