Fuzzy Differential Equations in Pseudo BH-algebra

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Abstract—*We introduce the notions of an algebra-BH , Fuzzy of an algebra-BH , Pseudo algebra-BH , Solve Differential Equations in pseudo BH-algebra , Solve Differential Equations in fuzzy pseudo BH-algebra . Also we prove and stated some properties , theorems and examples with some Remarks that explain the relationship with the definitions.*

Keywords—Pseudo algebra-BH, Differential Equations of a pseudo algebra-BH, Fuzzy Differential Equations of a pseudo algebra-BH.

INTRODUCTION

We will give some notions for this paper, in 1966, "K. Iseki" with "Y. Imai" introduced BCK [2]. In 1998, "Y. Jun", with "E. Rogh" and "H.. Kim" introduced (some basic of algebra of BH) [1]. In 2015, "Y. Jun, E. Roh" with "H. Kim" introduced the basic (a pseudo of BH in algebra) [3]. In 2011,"H. Abbas" with "H..Saeed" introduced the notions of respect to an element with a fuzzy closed ideals of algebras of BH [4]. And we will introduce of this paper, some notions in abstract.

1. Basic Concepts About pseudo BH-algebra or algebra-BH

In this section, we give some basics definitions for an algebra of BH and definitions for pseudo an algebra of BH, with a basics of BH for a fuzzy an algebra and definitions of a homomorphism in a mapping with proposition with some theorems and some examples to explain the relation this definitions.

Definition (1.1)[3]: An algebra of BH be not empty subsets or sets Z with (0) be a constant with operation $*$ and satisfy the conditions of binary operation, such that called An algebra of BH if holding $\forall g1, g2 \in Z$

i. $g1* g1=$ (constant 0).
ii. $g1*$ (constant 0) $= g1$.
iii. $g1* g2= g2 * g1 =$ (constant 0) imply $g1 = g2$,.

Definition (1.2)[3]: A pseudo of an algebra of BH is a not-empty set Z with (0) be a constant with a binary operations "$*$","#" holding: $\forall g1, g2 \in Z$

i. $g1* g1= g1\# g1=0$,
ii. $g1* g2= g2 \# g1 =$ (constant 0) imply $g1 = g2$,
iii. $g1* 0 = g1\# 0 = g1$

Definition (1.3)[4]: If Z be any not – empty subset. or set, then a fuzzy subset or set of Z is defined be a mapping f: Z → [0,1] and [0,1] be a real numbers ®,

Definition ***(*****1.4)[4]**: Let be C1 with C2 two a sets and hold fuzzy subsets or sets in Z, then :

1. $(C1\cap C2)(g1)=\min\{C1(g1),C2(g1)\}$, for all $g1\in Z$.
2. $(C1\cup C2)(g1)=\max\{C1(g1),C2(g1)\}$, for all $g1\in Z$.

$C1\cap C2$ and $C1\cup C2$ are all fuzzy subsets or sets in Z,

s.t, if $\{ C^{\alpha}, \alpha\in C\}$ is a family of fuzzy subsets or sets in Z, such that, :

$(\bigcap C_k)$ (g1)= inf $\{C_k (g1), k\in\Gamma\}$ and $(\bigcup C_k)$ (g1) =

sup$\{ C^k (g1),k\in\Gamma\}$, for every $g1\in Z$
For all " fuzzy subsets or set" in Z.

Definition **(1.5)[4]**: Let a fuzzy subsets or set C on a set Z, in interval closed [0,1]. The subsets or set

$C^{\alpha} =\{ g1\in X, C(g1) \geq \alpha \}$ is called "fuzzy level set" of Z .

THE MAIN RESULTS

we will introduce the some concepts and some types of pseudo algebra and a Differential Equations in pseudo-algebra of BH- with a fuzzy. Also we state and prove some examples and theorems about the concepts. In general "algebra differential equation (first-order)" (y1 = y1(x) be a function) is written as $Y^{(1)} = H1(x1, y1)$
When H1(x1, y1) be any function
H1 independent on x and dependent y

2. Solve Differential Equations by Pseudo BH-Algebra

we introduce the definitions Differential Equations in Pseudo-algebra of BH-. Also we state and prove some relations , theorems with examples about these definitions.

Definition **(2.1):** A Differential Equation (denoted by D. E.) in Pseudo-algebra of BH (denoted by S-BH) **,** let H1(x1,y1) be a function and $(*)$,$(\#)$ be any two binary operations , (0) be a constant and denoted by (D. E. in S-BH) :
let y1 = 1 to has f(x1,1) = H1(x1)……….(1)
let x1 = 1 to has f(1,y1) = H1(y1)……….(2)
and must be hold three condition

i. $H1(x1) * H1(x1) = H1(x1) \# H1(x1) = (0 \text{ constant})$
ii. $H1(x1) * (0 \text{ constant}) = H1(x1) \# (0 \text{ constant}) = H1(x1)$
iii. $H1(x1) * H1(y1) = H1(y1) \# H1(x1) = (0 \text{ constant})$ imply x1= y1 or H1(x1) = H1(y1)

Example **(2.2):** Let $(R ,\backslash ,\times,1)$ be a differential equation and a function define as H1(x,y) = 1
is hold pseudo-algebra of BH- and D. E. in S-BH
Solve: Let H1(x1) = 1 , H1(y1) = 1
i. H1(x1) (\operation1)H1(x1)
= H1(x1) (× operation2) H1(x1)
Such that $1\backslash 1= 1\times 1= 1$
ii. H1(x1) (\operation1)1→ $1\backslash 1 = 1$ = H1(x1)
and H1(x1)×1 imply $1\times 1 = 1$ = H1(x1)
iii. H1(x1) (× operation2) H1(y1) = H1(y1) (\ operation1)H1(x1) imply $1\backslash 1 = 1\times 1 = 1$ imply x1= y1
It is (D. E. in S-BH).

Example **(2.3):** Let $(R ,\backslash ,\times,4)$ be a differential equation and a function define as H1(x1,y1) = 2. Is it (D. E. in S-BH) ?
Solve: Let H1(x1) = 2 , H1(y1) = 2
i. H1(x1) (× operation2) H1(x1) = H1(x1)(\ operation1)H1(x1) → $2\backslash 2 \neq 4$ and $2\times 2 = 4$
ii. H1(x1) (\operation1)4 → $2\backslash 4 \neq 2$ = H1(x1) and H1(x1) (× operation2) 4 → $2\times 4 \neq 2$ = H1(x1)
It is not (D. E. in S-BH)

Theorem **(2.4):** Every a pseudo-algebra of BH- of Z iff a D. E. in S-BH of Z
Proof: Let Z be a D. E. in S-BH of Z to prove is a pseudo-algebra of BH-
Since H1(x1) = x1 and H1(y1) = y1 , $\forall$ x1, y1 $\in Z$
i. $x1 * x1 = H1(x1) * H1(x1) = 0$ and $x1 \# x1= H1(x1) \# H1(x1) = 0$
ii. $x1 * 0 = H1(x1) * 0 = H1(x1) = x1$ and $x1 \# 0 = H1(x1) \# 0 = H1(x1) = x1$
iii. $x1 * y1 =H1(x1) * H1(y1) = 0$ and $y1\# x1 = H1(y1) \# H1(x1) = 0$ imply H1(x1) = H1(y1) → x1= y1
Then it is a pseudo-algebra of BH-
Conversely
if Z be a pseudo-algebra of BH- of Z.

To prove it is a D. E. in S-BH
Since H1(x1) = x1 and H1(y1) = y1
i. H1(x1) $*$ H1(x1) = x1$*$x1 = 0 and H1(x1) # H1(x1) = x1 # x1 = 0
ii. H1(x1) $*$ 0 = x1 $*$ 0 = x1 = H1(x1) and H1(x1) # 0 = x1 # 0 = x1= H1(x1)
iii. H1(x1) $*$ H1(y1) = x1 $*$ y1 = 0 and H1(y1) # H1(x1) = y1 # x1 = 0 imply x1= y1 → H1(x1) = H1(y1)
Then it is a D. E. in S-BH

Proposition (2.5): If Z be a pseudo-algebra of BH- and { $H1_m(x1)_i$, x1∈Z, m∈λ} be a D. E. in S-BH. Then $\bigcap_{m\in\lambda}$ $H1_m$(x1) is a D. E. in S-BH.

Proof: To prove $\bigcap_{m\in\lambda}$ $H1_{im}$(x1) is a D. E. in S-BH.
Since $H1_i$(x1) = x1 and $H1_i$(y1) = y1
i. $\bigcap_{m\in\lambda}$ $H1_{im}$ (x1) $*$ $\bigcap_{m\in\lambda}$ $_i$ $H1_{im}$ (x1) = $\bigcap_{m\in\lambda}$ ($H1_{im}$ (x1) $*$ $H1_{im}$ (x1)) = 0
And $\bigcap_{m\in\lambda}$ $H1_{im}$ (x1) # $\bigcap_{m\in\lambda}$ $H1_{im}$ (x1) = $\bigcap_{m\in\lambda}$ ($H1_{im}$ (x1) # $H1_{im}$ (x1)) = 0
ii. $\bigcap_{m\in\lambda}$ $H1_{im}$ (x1) $*$ 0 = $\bigcap_{m\in\lambda}$ ($H1_{im}$ (x1) $*$ 0 = $\bigcap_{m\in\lambda}$ $H1_{im}$ (x1)
And $\bigcap_{m\in\lambda}$ $H1_{im}$ (x1) # 0 = $\bigcap_{m\in\lambda}$ ($H1_{im}$ (x1) # 0 = $\bigcap_{m\in\lambda}$ $H1_{im}$ (x1)
iii. $\bigcap_{m\in\lambda}$ $H1_{im}$ (x1) $*$ $\bigcap_{m\in\lambda}$ $H1_{im}$ (y1) = $\bigcap_{m\in\lambda}$ ($H1_{im}$ (x1) $*$ $H1_{im}$ (y1)) = 0
And $\bigcap_{m\in\lambda}$ $H1_{im}$ (y1) # $\bigcap_{m\in\lambda}$ $H1_{im}$ x1) = $\bigcap_{m\in\lambda}$ ($H1_{im}$ (y1) # $H1_{im}$ (x1)) = 0
→ $\bigcap_{m\in\lambda}$ $H1_{im}$ (x1) = $\bigcap_{m\in\lambda}$ $H1_{im}$ (y1) imply x1= y1
Then, it is a D. E. in S-BH.

Example (2.6):
If Z = R and R be a real numbers, a binary operations $*$, # and (R ,$*$, #,0) defined by :

$$H1(x1) * H1(y1) = \begin{cases} 0, if, H1(x1) = H1(y1) \\ H1(x1), O.W \end{cases}$$

$$H1(x1) \# H1(y1) = \begin{cases} 0, if, H1(x1) = H1(y1) \\ H1(x1) - H1(y1), O.W \end{cases}$$

For all x1, y1 be a real numbers
Then, Z is a D. E. in S-BH of X .

Proposition (2.7):
Let { $H1_m$(x1) m∈λ} be a chain of a D. E. in S-BH. Then $\bigcup_{m\in\lambda}$ $H1_m$(x1) is a D. E. in S-BH of Z.

Proof: to prove $\bigcup_{m\in\lambda}$ $H1_m$ (x1) is a D. E. in S-BH.
i. $\bigcup_{m\in\lambda}$ $H1_m$ (x1) $*$ $\bigcup_{m\in\lambda}$ $H1_m$ (x1) = $\bigcup_{m\in\lambda}$ ($H1_m$ (x1) $*$ $H1_m$ (x1)) = 0
And $\bigcup_{m\in\lambda}$ $H1_m$ (x1) # $\bigcup_{m\in\lambda}$ $H1_m$ (x1) = $\bigcup_{m\in\lambda}$ ($H1_m$ (x1) # $H1_m$ (x1)) = 0
ii. $\bigcup_{m\in\lambda}$ $H1_m$ (x1) $*$ 0 = $\bigcup_{m\in\lambda}$ ($H1_m$ (x1) $*$ 0 = $\bigcup_{m\in\lambda}$ $H1_m$ (x1)
And $\bigcup_{m\in\lambda}$ $H1_m$ (x1) # 0 = $\bigcup_{m\in\lambda}$ ($H1_m$ (x1) # 0 = $\bigcup_{m\in\lambda}$ $H1_m$ (x1)
iii. $\bigcup_{m\in\lambda}$ $H1_m$ (x1) $*$ $\bigcup_{m\in\lambda}$ $H1_m$ (y1) = $\bigcup_{m\in\lambda}$ ($H1_m$ (x1) $*$ $H1_m$ (y1)) = 0
And $\bigcup_{m\in\lambda}$ $H1_m$ (y1) # $\bigcup_{m\in\lambda}$ $H1_m$ (x1) = $\bigcup_{m\in\lambda}$ ($H1_m$ (y1) # $H1_m$ (x1)) = 0
→ $\bigcup_{m\in\lambda}$ $H1_m$ (x1) = $\bigcup_{m\in\lambda}$ $H1_m$ (y1) imply x1= y1
Then it is a D. E. in S-BH.

3. Solve Differential Equations in a Fuzzy Pseudo-Algebra of BH-
we introduce the definitions Differential Equations in fuzzy Pseudo-algebra of BH- . Also we state and

prove some relations, examples about these definitions.

Definition (3.1): Let f(x,y) or f(b1,b2) be a function the domain is R and a rang [0,1] and (∗),(#) be any tow binary operations and let (0) be a constant with a fuzzy function A(x)

let y = b1 = 1 to have f(x,1) = f(b1,1) = f(b1) = f(x) ……….(1)

let x = b2 = 1 to have f(1,y) = f(1,b2) = f(b2) = f(y) ……….(2)

and must be hold three condition

1. $A(\int_0^1 f(b1)db1 * \int_0^1 f(b1)db1) = A(\int_0^1 f(b1)db1 \# \int_0^1 f(b1)db1) = A(0)$
2. $A(\int_0^1 f(b1)db1 * 0 = A(\int_0^1 f(b1)db1)$, $A(\int_0^1 f(b1)db1 \# 0 = A(\int_0^1 f(b1)db1)$
3. $A(\int_0^1 f(b1)db1 * \int_0^1 f(b2)db2) = A(\int_0^1 f(b2)db2 \# \int_0^1 f(b1)db1) = A(0)$ imply $A(\int_0^1 f(b1)db1) = A(\int_0^1 f(b2)db2)$

Example (3.2): Let (R ,×, \ ,1) be a differential equation and defined as f(b1,b2) = 1 is hold fuzzy pseudo-algebra of BH-

$$A(x) = \begin{cases} 0.5 & \text{if } x \neq 1 \\ 0.7 & \text{if } x = 1 \end{cases}$$

Solve: Let f(xb1) = 1 , f(b2) = 1

$\int_0^1 f(b1)db1 = \int_0^1 1db1 = 1$

$\int_0^1 f(b2)db2 = \int_0^1 1db2 = 1$

1. $A(\int_0^1 f(b1)db1 \times \int_0^1 f(b1)db1) = A(\int_0^1 f(b1)db1 \backslash \int_0^1 f(b1)db1) \rightarrow A(\int_0^1 1db1 \times \int_0^1 1db1) = A(\int_0^1 1db1 \backslash \int_0^1 1db1) \rightarrow A(\frac{1}{1}) = A(1\times1) = A(1) = 0.7$
2. $A(\int_0^1 f(b1)db1 \times 1) \rightarrow A(\int_0^1 1db1 \times 1) \rightarrow A(1\times1) = A(1) = 0.7$

$A(\int_0^1 f(x)dx \backslash 1) \rightarrow A(\int_0^1 1db1 \backslash 1) \rightarrow A(\frac{1}{1}) = A(1) \rightarrow A(\int_0^1 1db1) = 0.7$

3. when $A(\int_0^1 f(b1)db1 \times \int_0^1 f(b2)db2) \rightarrow A(\int_0^1 1db1 \times \int_0^1 1db2) \rightarrow A(1\times1) = A(1) = 0.7$

And $A(\int_0^1 f(b2)db2 \backslash \int_0^1 f(b1)db1) \rightarrow A(\int_0^1 1db2 \backslash \int_0^1 1db1) \rightarrow A(\frac{1}{1}) = A(1) = 0.7$

s.t $A(\int_0^1 f(b1)db1 \times \int_0^1 f(b2)db2) = A(\int_0^1 f(b2)db2 \backslash \int_0^1 f(b1)db1) = A(1) = 0.7$

imply $A(\int_0^1 f(b1)db1) = A(\int_0^1 f(b2)db2) \rightarrow A(1) = A(1) = 0.7$

It is hold a fuzzy pseudo-algebra of BH-

Example (3.3): Let (R ,\ ,×,4) be a differential equation and a function define as f(b1,b2) = 2 is hold pseudo-algebra of BH-

$$A(x) = \begin{cases} 0.7 & \text{if } x = 1,4,8,1/2 \\ 0.5 & \text{if } x \neq 1,4,8,1/2 \end{cases}$$

Solve: f(b1) = 2 , f(b2) = 2

1. $A(\int_0^1 f(b1)db1 \times \int_0^1 f(b1)db1) = A(\int_0^1 f(b1)db1 \backslash \int_0^1 f(b1)db1) \rightarrow A(\int_0^1 2db1 \times \int_0^1 2db1) = A(\int_0^1 2db1 \backslash \int_0^1 2db1) \rightarrow A(2\times2) = A(\frac{2}{2}) \rightarrow A(4) = A(1) = 0.7$
2. $A(\int_0^1 f(b1)db1 \times 4) \rightarrow A(\int_0^1 2db1 \times 4) \rightarrow A(2\times4) = A(8) \rightarrow A(\int_0^1 f(b1)db1) = 0.7$

$A(\int_0^1 f(b1)db1 \backslash 4) \rightarrow A(\int_0^1 2db1 \backslash 4) \rightarrow A(\frac{2}{4}) = A(\frac{1}{2}) \rightarrow A(\int_0^1 f(b1)db1) = 0.7$

3. when $A(\int_0^1 f(b1)db1 \times \int_0^1 f(b2)db2) \rightarrow$

$A(\int_0^1 2db1 \times \int_0^1 2db2) \rightarrow A(2\times2) = A(4) = 0.7$

And $A(\int_0^1 f(b2)db2 \backslash \int_0^1 f(b1)db1) \rightarrow A(\int_0^1 2db2 \backslash$

$\int_0^1 2db1) \rightarrow A(\frac{2}{2}) = A(1) = 0.7$

s.t $A(\int_0^1 f(b1)db1 \times \int_0^1 f(b2)db2) = A(\int_0^1 f(b2)db2$

$\backslash \int_0^1 f(b1)db1) = A(4) = A(1) = 0.7$ imply

$A(\int_0^1 f(b1)db1) = A(\int_0^1 f(b2)db2) \rightarrow A(2) = A(2) =$

0.5

It is hold a fuzzy pseudo-algebra of BH- but is not hold pseudo-algebra of BH- because f(b1)\f(b1) must be equal to f(b1)×f(b1) = 4

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