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Restrict Nearly Primary Finitely Compactly Packed Modules

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***Abstract*—** In this work, all rings under consideration will be Supposedly to be commutative with nonzero identity and all modules will be nonzero unite. Restrict Nearly Primary Finitely Compactly Packed Modules is a generalization of Restrict Nearly Primary Compactly Packed Modules that we introduce. We identify the requirements that result in the modules being Restrict Nearly primary finitely compactly packed. Also; proved are several results regarding the Restrict Nearly Primary Finitely Compactly Packed Modules. Investigated are also the pre-and post-requisites for a $\mathcal{R}$ -module $\mathcal{W}$ to be Restrict Nearly Primary Finitely Compactly Packed. We denoted to the submodule by s−mod

Keywords*— *Restrict Nearly Primary submodules, Restrict Nearly primary compactly packed modules, Restrict Nearly primary finitely compactly packed modules.

I. INTRODUCTION

A proper s−mod $\mathcal{A}$ of $\mathcal{W}$ is Restrict Nearly Primary (RNPr) if whenever $\mathcal{S}\omega \in \mathcal{A}$, for $\mathcal{S} \in \mathcal{R}$, $\omega \in \mathcal{W}$ it means that $\omega \in \mathcal{A} + (Soc(\mathcal{W}) \cap J(\mathcal{W}))$ or $\mathcal{S}^{\kappa}\mathcal{W} \subseteq \mathcal{A} + (Soc(\mathcal{W}) \cap J(\mathcal{W}))$, $\exists \kappa \in \mathbb{Z}^{+}$[3]. In [1], the concept of compactly packed modules was introduced. The above concept has been generalized in [2]. To the idea of Restrict Nearly Primary Compactly Packed Modules, we generalize Previous Concepts. Restrict Nearly Primary Compactly Packed (RNPcp) refers to a correct s−mod $\mathcal{A}$ of an $\mathcal{R}$- module $\mathcal{W}$ is assumed to Restrict Nearly Primary Compactly Packed (RNPcp) if $\forall\{\mathcal{A}_\alpha\}_{\alpha\in\Lambda}$ of RNPr s−mods of $\mathcal{W}$ with $\mathcal{A} \subseteq \bigcup_{\alpha\in\Lambda}\mathcal{A}_\alpha$, $\exists\delta \in \Lambda$ such that $\mathcal{A} \subseteq \mathcal{A}_\delta$. A module $\mathcal{W}$ is named RNPcp if every s−mod is RNPcp. Then we generalized the previous concept to Restrict Nearly finitely primary compactly packed modules. A proper s−mod $\mathcal{A}$ of an $\mathcal{R}$-module $\mathcal{W}$ is named Restrict Nearly Primary finitely Compactly Packed (RNPfcp) if for each family $\{\mathcal{A}_\alpha\}_{\alpha\in\Lambda}$ of RNPr s−mods of $\mathcal{W}$ with $\mathcal{A} \subseteq \bigcup_{\alpha\in\Lambda}\mathcal{A}_\alpha$, $\exists$ $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n \in \Lambda$ such that $\mathcal{A} \subseteq \bigcup_{i=1}^{n} \mathcal{A}_{\alpha i}$. . A module $\mathcal{W}$ is assumed to be RNPfcp if each proper s−mod of $\mathcal{W}$ is RNPfcp.

In the second section of this work, we provide a few RNPfcp module examples and explore the connection between RNPcp and RNPfcp modules. We also discover the prerequisites for an RNPfcp module becoming RNPcp. In part 3: We are looking into some of the RNPfcp modules' properties. Additionally, we discover the prerequisites for any $\mathcal{R}$-module $\mathcal{W}$ to be RNPfcp.

II.COMPACTLY PACKED AND FINITELY COMPACTLY PACKED SUBMODULES

We present the next definition of compactly packed and finitely compactly packed s−mod.

Definition 2.1

A proper s−mod $\mathcal{A}$ of an $\mathcal{R}$-module $\mathcal{W}$ is named Restrict Nearly Primary Compactly Packed (for short RNPcp) s−mod if for each family $\{\mathcal{A}_\alpha\}_{\alpha\in\Lambda}$ of RNPr s−mods of $\mathcal{W}$ with $\mathcal{A} \subseteq \bigcup_{\alpha\in\Lambda}\mathcal{A}_\alpha$, $\exists\delta \in \Lambda$ such that $\mathcal{A} \subseteq \mathcal{A}_\delta$.

Remark 2.2

A module $\mathcal{W}$ is referred to as RNPcp if each of its s−mods is RNPcp.

Definition 2.3

A proper s−mod $\mathcal{A}$ of an $\mathcal{R}$-module $\mathcal{W}$ is said to Restrict Nearly Primary finitely Compactly Packed (for short RNPfcp) s−mod if for each family $\{\mathcal{A}_\alpha\}_{\alpha\in\Lambda}$ of RNPr s−mods of $\mathcal{W}$ with $\mathcal{A} \subseteq \bigcup_{\alpha\in\Lambda}\mathcal{A}_\alpha$, $\exists\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n \in \Lambda$ such that $\mathcal{A} \subseteq \bigcup_{i=1}^{n} \mathcal{A}_{\alpha i}$.

Remark 2.4

A module $\mathcal{W}$ is referred to as RNPfcp if each of its s−mods is RNPfcp.

Proposition 2.5

Every RNPcp s−mod is an RNPfcp s−mod but not conversely.

Proof

It is clear by the definitions RNPcp s−mod and the RNPfcp s−mod.

For the converse consider the folowing example

Example 2.6

Assume that P be a vector space of dimeinsion greater than 2 over the field $F = {}^{Z}/_{2Z}$ then every subspace of P is prime, so every subspace of P is primary and we know P is RNPr [3, def, (2.1)]. Let d_1 and d_2 be distinct vector of a besis for P, $P_1 = d_1 F$, $P_2 = d_2 F$, $d_3 - (d_1 + d_2) F$ and $L = P_1 + P_2$. Then $L = \{0, d_1, d_2, d_1 + d_2\} = P_1 \cup P_2 \cup P_1$ is an efficient union of three RNPr s−mod with $\sqrt{[P_i + (Soc(\mathcal{W}) \cap J(\mathcal{W}) : \mathcal{W}]} = 0$ but $L \nsubseteq P_i + (Soc(\mathcal{W}) \cap J(\mathcal{W})$ for all $i = 1, 2, 3$.

In the following, we look at the behavior of a certain module over an RNPfcp ring.

Proposition 2.7

Let $\mathcal{R}$ be a RNPfcp ring. Let be a semi simple $\mathcal{R}$-module such that for every submodule $\mathcal{A}$ and N of $\mathcal{W}$ with $\mathcal{A} \subseteq N$, whenever $\sqrt{[\mathcal{A} :_{\mathcal{R}} \mathcal{W}]} \subseteq \sqrt{[N :_{\mathcal{R}} \mathcal{W}]}$. Then $\mathcal{W}$ is an RNPcp module.

Proof:

Assume that $\mathcal{A} \subseteq \bigcup_{\alpha \in \Lambda} \mathcal{A}_\alpha$, where $\mathcal{A}$ is a proper s−mod of $\mathcal{W}$, and $\mathcal{A}_\alpha$ be RNPr s−mods of $\mathcal{W}$ for each $\alpha \in \Lambda$. Then $\sqrt{[\mathcal{A} + (Soc(\mathcal{W}) \cap J(\mathcal{W}) :_{\mathcal{R}} \mathcal{W}]} \subseteq \sqrt{[\bigcup_{\alpha \in \Lambda} \mathcal{A}_\alpha + (Soc(\mathcal{W}) \cap J(\mathcal{W}) :_{\mathcal{R}} \mathcal{W}]} = \bigcup_{\alpha \in \Lambda} \sqrt{[\mathcal{A}_\alpha + (Soc(\mathcal{W}) \cap J(\mathcal{W}) :_{\mathcal{R}} \mathcal{W}]}$. Since $\sqrt{[\mathcal{A}_\alpha :_{\mathcal{R}} \mathcal{W}]}$ is a prime ideal of $\mathcal{R}$ and $\mathcal{W}$ is semi simple, then $\sqrt{[\mathcal{A}_\alpha + (Soc(\mathcal{W}) \cap J(\mathcal{W}) :_{\mathcal{R}} \mathcal{W}]}$ is a prime ideal of $\mathcal{R}$ and $\mathcal{R}$ is an RNPfcp ring, $\exists \delta \in \Lambda$ such that $\sqrt{[\mathcal{A} + (Soc(\mathcal{W}) \cap J(\mathcal{W}) :_{\mathcal{R}} \mathcal{W}]} \subseteq \sqrt{[\mathcal{A}_\delta + (Soc(\mathcal{W}) \cap J(\mathcal{W}) :_{\mathcal{R}} \mathcal{W}]}$. Then by hypothesis $\mathcal{A} \subseteq \mathcal{A}_\delta$. Therefore is RNPcp module.

III. IMPORTANT RESULTS ON PRIMARY FINITELY COMPACTLY PACKED MODULES

The folowing proposition shows that RNPfcp mod ule which has

$J(\mathcal{W}) \neq \mathcal{W}$, satisfies a certain kind of ascending chain condition.

Proposition 3.1

Let $\mathcal{W}$ be an RNPfcp $\mathcal{R}$-module with $J(\mathcal{W}) \neq \mathcal{W}$, then $\mathcal{W}$ satisfies the ascending chain condition for RNPr s−mods.

Proof:

Assume that $\mathcal{A} \neq \mathcal{W}$ and "$\mathcal{A}_1 \subseteq \mathcal{A}_2 \subseteq \mathcal{A}_3 \subseteq \dots,$ be ascending chain condition for RNPr s−mods of $\mathcal{W}$. Let $\mathcal{A} = \bigcup_i \mathcal{A}_i$. If $\mathcal{A} = \mathcal{W}$ and N is a maximal s−mod of $\mathcal{W}$, then $N \subsetneq \bigcup_i \mathcal{A}_i$, but $\mathcal{W}$ is RNPfcp module, then $\exists \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ such that $N \subseteq \bigcup_{i=1}^{n} \mathcal{A}_{\alpha i}$ and since "$\mathcal{A}_1 \subseteq \mathcal{A}_2 \subseteq \mathcal{A}_3 \subseteq \dots,$ be asceending chain, so, $\exists m \in \{1, 2, 3, \dots, n\}$ such that $\bigcup_{i=1}^{n} \mathcal{A}_{\alpha i} = \mathcal{A}_{\alpha_m}$ then $N \subseteq \mathcal{A}_{\alpha_m}$, and N is a maximal s−mod, then $N = \mathcal{A}_{\alpha_m}$, and so $\mathcal{W} = \bigcup_i \mathcal{A}_i = \mathcal{A}_{\alpha_m}$ which is a contradiction. So $\mathcal{A}$ is a proper s−mod of $\mathcal{W}$, thus, $\exists \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ such that $\mathcal{A} \subseteq \bigcup_{i=1}^{n} \mathcal{A}_{\alpha i}$ and since "$\mathcal{A}_1 \subseteq \mathcal{A}_2 \subseteq \mathcal{A}_3 \subseteq \dots,$ be asceending chain, then, $\exists m \in \{1, 2, 3, \dots, n\}$ such that $\bigcup_{i=1}^{n} \mathcal{A}_{\alpha i} = \mathcal{A}_{\alpha_m}$ then $\bigcup_i \mathcal{A}_i \subseteq \mathcal{A}_{\alpha_m}$, so "$\mathcal{A}_1 \subseteq \mathcal{A}_2 \subseteq \mathcal{A}_3 \subseteq \dots \subseteq \mathcal{A}_{\alpha_m}$. Therefore satisfies the asceending chain condition on RNPr s−mods.

Since the finitely generated module has a maximal s−mod the following corollary follows directly from the previous proposition.

Corollary 3.2

Let $\mathcal{W}$ be an RNPfcp finitely generated module, then $\mathcal{W}$ satisfies the ascendiing chain condition for RNPr s−mods.

Also, since multiiplication module has a maximal s−mod, the folowing corollary follows directly from the previous proposition.

Corollary 3.3

Let $\mathcal{W}$ be an RNPfcp multiplication module, then $\mathcal{W}$ satisfies the ascending chain condition for RNPr s−mods.

Proposition 3.4

Let $\mathcal{W}$, $\mathcal{W}'$ are two $\mathcal{R}$-modules, f:$\mathcal{W}\longrightarrow\mathcal{W}'$ be an $\mathcal{R}$-epimorphism such that

$ker\, f \subseteq W$, with $ker\, f$ is small for each RNPr W of $\mathcal{W}$. Then $\mathcal{W}$ is a RNPfcp iff $'$ is a RNPfcp.

Proof

$\Longrightarrow$ Assume that $\mathcal{W}$ is RNPfcp $\mathcal{R}$-modules and $' \subseteq \bigcup_{\alpha\in\Lambda}K_\alpha$, where $\mathcal{A}'$ is a proper s$-$mod of $\mathcal{W}'$ and K_α is an RNPr s$-$mod of $\mathcal{W}'$ for all $\alpha\in\Lambda$. Then $f^{-1}(\mathcal{A}') \subseteq f^{-1}(\bigcup_{\alpha\in\Lambda}K_\alpha)$, thus $f^{-1}(\mathcal{A}') \subseteq \bigcup\, f^{-1}(K'_\alpha)$. But K_α is an RNPr s$-$mod of $\mathcal{W}'$, then by [3, prop, (4.18)] $f^{-1}(K_\alpha)$ is the RNPr s$-$mod of $\mathcal{W}$ for all $\alpha\in\Lambda$. Since $\mathcal{W}$ is RNPfcp, then there exists $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n \in \Lambda$ such that $f^{-1}(\mathcal{A}') \subseteq \bigcup_{i=1}^{n} f^{-1}$ $(K'_{\alpha i})$ it means that $f^{-1}(\mathcal{A}') \subseteq f^{-1}(\bigcup_{i=1}^{n}K'_{\alpha i})$. But f is an epimorphism, then $' \subseteq \bigcup_{i=1}^{n}K'_{\alpha i}$. Therefore $\mathcal{A}'$ is the RNPfcp s$-$mod of $\mathcal{W}'$. Hence $\mathcal{W}'$ is RNPfcp $\mathcal{R}$'modules.

$\Longleftarrow$ Assume that $\mathcal{W}'$ is RNPfcp $\mathcal{R}$-modules and $ker\, f \subseteq W$ for all RNPr s$-$mod W of $\mathcal{W}$. Let $\mathcal{A}$ be a proper s$-$mod of $\mathcal{W}$ such that $\subseteq \bigcup_{\alpha\in\Lambda}W_\alpha$, where W_α is an RNPr s$-$mod of $\mathcal{W}$ for all $\alpha\in\Lambda$. Then $f(\mathcal{A}) \subseteq f(\bigcup_{\alpha\in\Lambda}K_\alpha)$ it means that $f(\mathcal{A}) \subseteq \bigcup_{\alpha\in\Lambda}f(K_\alpha)$. But $ker\, f \subseteq W_\alpha$ for all α. Then by [3, prop, (4.17)], we have $f(\, W_\alpha)$ is RNPr s$-$mod of $\mathcal{W}'$ for all $\alpha\in\Lambda$. Since $\mathcal{W}'$ is RNPfcp $\mathcal{R}$-modules then $\exists$ $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\in\Lambda$ such that $f(\mathcal{A}) \subseteq \bigcup_{i=1}^{n}f(W_{\alpha i})$. Now, let $x\in\mathcal{A}$, then $f(x)\in f(\mathcal{A}) \subseteq \bigcup_{i=1}^{n}f(W_{\alpha i})$, then there exists $m\in\{1, 2, \ldots, n\}$such that $f(x)\in f(W_{\alpha_m})$ it means that $\exists$ b $\in W_{\alpha_m}$ such that $f(x)\in f(\text{b})$, then $f(x)-f(\text{b}) = 0$, then $f(x-\text{b}) = 0$ so $x-\text{b}\in ker\, f \subseteq W_{\alpha_m}$. That is $\in K_{\alpha_m}$. Hence $\subseteq \bigcup_{i=1}^{n}W_{\alpha i}$, that is $\mathcal{A}$ is the RNPfcp s$-$mod. Thus $\mathcal{W}$ is RNPfcp $\mathcal{R}$-modules.

Proposition 3.5

Let $\mathcal{W}$ be an $\mathcal{R}$'modules and S multiplicatively closed subset in $\mathcal{R}$, If $\mathcal{A}$ is an RNPr s$-$mod of the $\mathcal{R}_S$'module $\mathcal{W}_S$ then $f^{-1}(\mathcal{A})$ is an RNPr of $\mathcal{W}$.

Proof

Assume that is an RNPr s$-$mod of $\mathcal{W}_S$, to proof $f^{-1}(\mathcal{A})$ is a proper s$-$mod of $\mathcal{W}$, it is sufficient to show $[f^{-1}(\mathcal{A})$: $\mathcal{W}] \cap$ S $= \emptyset$. Assume that $h\in[f^{-1}(\mathcal{A})$: $\mathcal{W}] \cap$ S, then

$h\in$ S and $hm\in f^{-1}(\mathcal{A})$ for all $m\in\mathcal{W}$, $f(hm) = {hm}/_{1} \in \mathcal{A}$. Assume that ${\text{r}}/_{c} \in \mathcal{W}_S$, so ${\text{r}}/_{c} = {h\text{r}}/_{hc} = {h\text{r}}/_{1}.{1}/_{hc} \in \mathcal{A}$, thus $\mathcal{W}_S \subseteq \mathcal{A}$ which is a contradiction. To prove $f^{-1}(\mathcal{A})$ is an RNPr submodule, if h$m\in f^{-1}(\mathcal{A})$ for all $h\in\mathcal{R}$, $m\in\mathcal{W}$, so $f(\text{h}m)\in\mathcal{A}$, ${hm}/_{1}= {\text{h}}/_{1}.{m}/_{1} \in\mathcal{A}$, but $\mathcal{A}$ is an RNPr submodule of $\mathcal{W}_S$. Thus ${m}/_{1} \in \mathcal{A}+(Soc(\mathcal{W})\cap J(\mathcal{W}))$ then $f(m)={m}/_{1} \in \mathcal{A}+(Soc(\mathcal{W})\cap J(\mathcal{W}))$ this implies $m\in f^{-1}(\mathcal{A}+(Soc(\mathcal{W})\cap J(\mathcal{W}))) \subseteq f^{-1}(\mathcal{A})+f^{-1}(Soc(\mathcal{W})\cap J(\mathcal{W}))$ or ${\text{h}}/_{1} \in \sqrt{[\mathcal{A}+(Soc(\mathcal{W})\cap J(\mathcal{W})):\mathcal{W}_S]}$ so, $\exists\,\kappa$ is a positive integer such that ${h^\kappa}/_{1}.{m}/_{1} \in \mathcal{A}$ for all ${m}/_{s} \in \mathcal{W}_S$. Thus

$f(h^\kappa m) = {h^\kappa m}/_{1} = {h^\kappa ms}/_{s} = {h^\kappa m}/_{s}.{\text{s}}/_{1} \in \mathcal{A}$. Hence $h^\kappa m \in f^{-1}(\mathcal{A})$, from this $h^\kappa m \in f^{-1}(\mathcal{A}) + (Soc(\mathcal{W})\cap J(\mathcal{W}))$ thus $h \in \sqrt{[f^{-1}(\mathcal{A}):\mathcal{W}]}$, then $f^{-1}(\mathcal{A})$ is a RNPr.

Proposition 3.6

Let $\mathcal{W}$ be an $\mathcal{R}$-module, and S multiplicatively closed subset in $\mathcal{R}$. If $\mathcal{W}$ is RNPfcp mod ule then $\mathcal{W}_S$ is RNPfcp module.

Proof

Assume that $\mathcal{W}$ is an RNPcp $\mathcal{R}$-modules, and $\mathcal{A}$ is a proper s$-$mod of $\mathcal{W}_S$ such that $\subseteq \bigcup_{\alpha\in\Lambda}K_\alpha$, where K_α is an RNPr s$-$mod of $\mathcal{W}_S$ for all $\alpha\in\Lambda$. Define f: $\mathcal{W}\rightarrow\mathcal{W}_S$ by $f(m) = \frac{m}{1}$ for all $m\in\mathcal{W}$. Thus f is an epimorphism, Then $f^{-1}(\mathcal{A}) \subseteq f^{-1}(\bigcup_{\alpha\in\Lambda}K_\alpha)$, thus $f^{-1}(\mathcal{A}) \subseteq \bigcup_{\alpha\in\Lambda} f^{-1}(K_\alpha)$. Now $f^{-1}(K_\alpha)$ is an RNPr s$-$mod of $\mathcal{W}$ (**3.5**). Since $\mathcal{W}$ is RNPfcp module, then $\exists$ $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\in\Lambda$ such that $f^{-1}(\mathcal{A}) \subseteq \bigcup_{i=1}^{n}$ f^{-1} $(K_{\alpha i})$ it means that $(f^{-1}(\mathcal{A}))_S \subseteq (\bigcup_{i=1}^{n}(f^{-1}(K_{\alpha i})))_S = \bigcup_{i=1}^{n}(f^{-1}(K_{\alpha i}))_S$. To prove the previous equivalence, let $\frac{m}{s}\in \bigcup_{i=1}^{n}(f^{-1}(K_{\alpha i}))_S$ where $s\in S$ and $m\in\bigcup_{i=1}^{n}(f^{-1}(K_{\alpha i}))_S$, so $\exists\in\{1, 2, 3, \ldots, n\}$ such that $m\in f^{-1}(K_{\alpha_\ell})$, thus $\frac{m}{s}\in (f^{-1}(K_{\alpha i}))_S$, hence $\frac{m}{s}\in\bigcup_{i=1}^{n}(f^{-1}(K_{\alpha i}))_S$, then $(\bigcup_{i=1}^{n}(f^{-1}(K_{\alpha i})))_S \subseteq \bigcup_{i=1}^{n}(f^{-1}(K_{\alpha i}))_S$. Now, let

$\frac{m}{s}\in\bigcup_{i=1}^{n}(f^{-1}(K_{\alpha i}))_S$, so $\frac{m}{s}\in\bigcup_{i=1}^{n}(f^{-1}(K_{\alpha_\ell}))_S$ for some $\in\{1, 2, 3, \ldots, n\}$, where $s\in S$ and $m\in f^{-1}(K_{\alpha_\ell})$, hence $m\in\bigcup_{i=1}^{n}f^{-1}(K_{\alpha i})$ thus $\frac{m}{s}\in(\bigcup_{i=1}^{n}(f^{-1}(K_{\alpha i})))_S$. Then $\bigcup_{i=1}^{n}(f^{-1}(K_{\alpha i}))_S \subseteq (\bigcup_{i=1}^{n}(f^{-1}(K_{\alpha i})))_S$. Now, as in the proof of the above proposition then $\mathcal{A}_s \subseteq \bigcup_{i=1}^{n}K_{\alpha i}$. Hence $\mathcal{W}_S$ is RNPfcp $\mathcal{R}$-modules.

V. Conclusion

- In this work, we generalize the concept of primary compactly packed modules to the concept of Restrict Nearly primary compactly packed modules and Restrict Nearly finitely primary compactly packed modules And we found important relationships between these concepts.

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