Proximally Syndetical Point in Topological Transference Group

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Abstract—*In this research will study the syndetic set in topological transformation group, that represents one of the important sets in topological dynamics that Gottschak referred to in a book topological dynamics. Relationship between syndetic set and translation (left-right- bilateral) provided, In right topological transformation group have presented semi replete set and extensive set and referred to the relationship of semi replete set and extensive set on the one hand, and their relationship with syndetic set on the other hand. Introduced a new idea syndetic proximal point that depended on syndetic set and indicated to a relationship syndetic proximal point with proximal point and extensive proximal point, and concluded a number of relationships and theories.**Keywords— syndetic set, topological transformation group, proximal point*

I. INTRODUCTION

Let (Z, T, ϖ) be a right topological transference group, whose phase space Z is compact hausdorff and phase group T is syndetical transformaton[1].Topological transference group (Z, T, ϖ) has algebraic structure and topological structure , depended on structure to study properties dynamical properties for (sets - points) [2]. Proximally is one of the important subject in topological dynamics, where the proximal between points in different orbits and their dynamic properties are studied [3] . The idea of Proximally depends on the study of continuity and compaction between groups[4].In this paper been studied Proximally in the periodic points and the almost periodic point depende on syndetical set [4]. Relationship syndetical proximally point with some characteristic dynamical system (proximally point -extensively proximally point-almost periodic point-invariant set-minimal set- periodic point) presented ,also we studied syndetical proximally and extensively proximally independed on syndetical set and extensively set , and it will be give a necessary and sufficient condition syndetical proximally and extensively proximally to be proximally and we give some theorem a bout syndetical proximally point .We use symbol Δ to indication the end

II.Preliminaries

In this section we given basic definitions and important concepts and necessary relations that need in this work

Definition (1) [5]

Let (T, $*$) be a group. Suppose that G is a topological space and there is a continues function f: T × T → T which defined by $f(x, y) = x * y^{-1}$. Then (T,$*$) is a topological group.

Definition (2) [6]

Let (T,$*$) be a topological group and let A be a semi group of T. Then A is said to be semi repletly group if for each compact set K there subsist $g \epsilon$T that $gK \subset$ A or Kg $\subset$ A.

Definition (3) [7]
Let (T,$*$) be a topological group and let A be a subset of G said to be extensively set provided that A intersects every semi repletly groups in T is non-empty set.

Definition (4) [7]

Let (T,*) is a topological group, a subset of T is said to be a right syndetical and a left syndetical if there is a subsist compact subset K of T ($K \subset T$) that ($\{G = AK\}, G = KA$).

Definition (5) [8]

Triple(Z, T, ϖ) is said to be a right topological transference group is a where Z is a topological space called the phase space, T is a topological group called the phase group and mapping $\pi: X \times G \to X, \pi(x,t) \to xt$ is a continuous condition right topological transference group as follows:

(1) $xe = x$ ($x \in X$) where is the identity element of T

(2) $(xt)s = x(ts)$, ($x \in X, t, s \in T$)

Definition (6) [8]

Let (Z, T, ϖ)be a topological transference group

(1) A subset $\Xi \subseteq T$ is said to be invariant set if $\Xi T = \Xi$

(2)A non-empty closed invariant set $\Xi \subseteq T$ is said to be minimal set if it contains no non-empty closed invariant set.

(3) $x \in Z$, is said to be fixed point if $xT = x$.

(4) $x \in Z$ is said to be invariant almost periodic point under T if for each invariant open neighborhood U of Z there subsist syndetical subset Ξ of T that $x\Xi \subseteq U$.

(5) $x \in Z$ is said to be almost periodic point under T if for each open neighborhood U of Z there subsist syndetical subset Ξ of T that $x\Xi \subseteq U$.

Theorem (7)

Let (Z, T, ϖ) be a topological group and T be abelin statements are equivalent

1- A extensively set of T
2- Ag extensively set of T

Proof Suppose(1) we prove (2) Let A be a extensively set of T there subsist semi repletly P of T that $A \cap P \neq \emptyset$ So $Ag \cap Pg \neq \emptyset$ for some g ∈ T .It is enough to prove that Pg semi repletly of T by hypothesis we have that compact M of T that $g_1 M \subset P$ for some $g_1 \in T$ because T group there subsist $g^{-1} \in T$ that $g_1 M g g^{-1} \subset P$,and $g_1 M g \subset P$ g because M compact set of T and right transference to be continuous then Mg compact set therefore Pg semi repletly of T and Ag extensively set of T. Suppose (2) We prove (1) .Let Ag extensively set of ,then there semi-repletly P of T that $Ag \cap P \neq \phi$ for each $a \in A$ there subsist $g \in T, p \in P$ that $ag = p$ so $a = pg^{-1}$ thus $Pg^{-1} \cap A \neq \phi$.It is enough to prove that Pg^{-1}semi repletly of T by hypothesis we have that compact M of T that $g_1 M \subset P$ for some for some $g_1 \in T$ because T group there subsist $g^{-1} \in T$ that $g_1 M g^{-1} \subset Pg^{-1}$because M compact set by hypothesis we obtain Mg^{-1}compact set therefore Pg^{-1} semi repletly of G and A extensively set of T. Δ

Theorem (8)

Let (Z, T, ϖ)be a topological group and T be abelin statements are equivalent

1- gA extensively set of T
2- gAg_1 extensively set of T
3- Ag extensively set of T

Proof : Suppose(1) we prove (2) Let gA be a extensively set of T there subsist semi repletly gP of T that $gA \cap gP \neq \emptyset$ So $gAg_1 \cap gPg_1 \neq \emptyset$ for some g,g_1∈ T .It is enough to prove that gPg_1 semi repletly of T by hypothesis we have that compact M of T that $g_2 M \subset gP$ for some $g_2 \in T$ because T group there subsist $g_1^{-1} \in T$ that $g_2 M g_1 g_1^{-1} \subset gP$,and $g_2 M g_1 \subset gP g_1$ because M compact set of T and right transference to be continuous then Mg_1 compact set therefore $gP g_1$ semi repletly of T and gAg_1 extensively set of T .Suppose (2) we prove (1) let gAg_1

extensively set of ,then there semi- repletly P of T that $gAg_1 \cap P \neq \phi$ for $a \in A$ there subsist $g, g_1 \in T, p \in P$ that $gag_1 = p$ so $ga = pg_1^{-1}$ thus $Pg_1^{-1} \cap gA \neq \phi$.It is enough to prove that Pg_1^{-1}semi repletly of T by hypothesis we have that compact M of T that $g_1 M \subset P$ for some for some $g_1 \in T$ because T group there subsist$g_1^{-1} \in T$ that $g_1 M g_1^{-1} \subset P g_1^{-1}$because M compact set by hypothesis we obtain Mg_1^{-1}compact set therefore Pg_1^{-1} semi repletly of T and gA extensively set of T. Δ

Prove the rest of the cases in the same way.

Theorem (9)

Let (Z, T, ϖ) be a topological group and G be abelin statements are equivalent

1. g𝔸 syndetical subset of T.
2. 𝔸 syndetical subset of T.
3. 𝔸g syndetical subset of T

proof : Suppose(1) we prove (2) g𝔸 syndetical subset of T then there subsist compact set K of T that g𝔸 k = T , because T group then there subsist $g^{-1} \in T$ that g^{-1}g$\mathbb{A}K = g^{-1}T = T\ thus\ e\mathbb{A}K = T\ so\ \mathbb{A}K = T$ there for 𝔸 syndetical subset of T . Suppose(2) we prove (1) let 𝔸 syndetical subset of T then there subsist compact subset K of T that 𝔸K= T , because T group there subsist $g \in T$ that $g\mathbb{A}K = gT = G$ then g𝔸 syndetical subset of T . Suppose(3) we prove (2) let 𝔸g syndetical subset of T then there subsist compact set K of T that 𝔸gK =T by hypothesis 𝔸 syndetical subset of T .Suppose(2) we prove (3) let 𝔸 syndetical subset of T then there subsist compact subset K of T that 𝔸K=T, because T group there subsist ($e \in T$) that $\mathbb{A}ek = T, \mathbb{A}gg^{-1}k = T$ by theorem by hypothesis can obtain $g^{-1}k$ compact subset of T then 𝔸g syndetical subset of T . Δ

Theorem (10)

Let (Z, T, ϖ)be a topological group and T be abelin statements are equivalent

1. 𝔸 syndetical set
2. 𝔸 semi repletly set.

Proof**:**Suppose(1) we prove (2) Let 𝔸 syndetical set there subsist compact subset K of T that $\mathbb{A}K = T$,all g ∈ T there subsist a ∈ A , k ∈ K that t = ak , t k^{-1} = a thus tk^{-1} ⊂ 𝔸 because k compact set and (T,∗) topological group then k^{-1} compact set therefore 𝔸 semi repletly . Suppose(2) we prove (1) Let 𝔸 repletly semi group there subsist compact subset K of T that gk⊂ 𝔸 all g ∈ T there a ∈ 𝔸 , k ∈ K that gk =a , g = a k^{-1} ,T⊂ 𝔸 k^{-1} because K compact and (T,∗) topological group then k^{-1} compact sub set of T because k^{-1} ⊂T , 𝔸 ⊂T and (*) binary operation on T , 𝔸 k^{-1} ⊂ T , T= 𝔸 k^{-1} therefore 𝔸 syndetical **.** Δ

Theorem(11)

Let (Z, T, ϖ) be a toplogical transference group. and T be an abelian Then the following statement are equivalent.

1) 𝔸 extensively set of T
2) 𝔸 syndetical set of T

Proof**:**Suppose(1) we prove (2) Let 𝔸 be a extensively subset of T there subsist semi repletly P of T that $\mathbb{A} \cap P \neq \emptyset$ Because intersects be abelin then P $\subset A$ there subsist a compact subset K of T $PK \subset \mathbb{A}K$ form theorem (10) P K = T so some $T \subset$ 𝔸 K it follows that 𝔸 syndetical set of T. Suppose(2) we prove (1) Let 𝔸 be a syndetical subset of T there subsist a compact subset K of T that 𝔸 K =T Because K compact set and (T,∗) topological group then k^{-1} compact sub set of T, 𝔸 =T K^{-1} ,for each t∈ T there subsist k∈K^{-1} , a∈ 𝔸 that tk^{-1}= a and tK^{-1} ⊂ 𝔸 Hence 𝔸 semi repletly of T and 𝔸 ∩ 𝔸 ≠ ∅ there fore 𝔸 extensively set . Δ

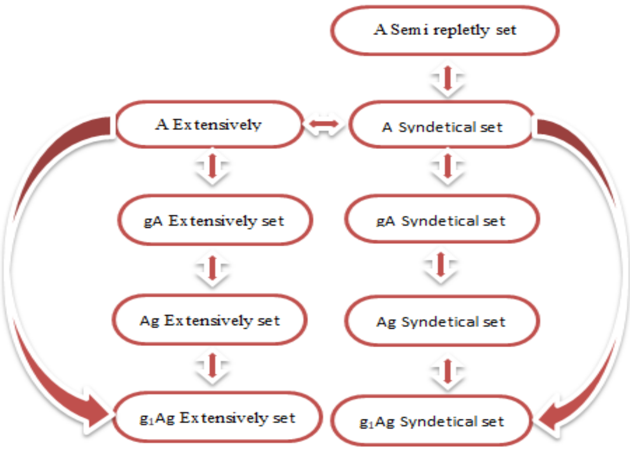

Fig. (1) Show relationship Syndetic point with dynamical properties

II. BASE RESULTS

In this part, we introduce syndetical proximally point in transference group and show that relationship syndetical proximally with dynamical properties.

Definition (1) [9]

Let (Z,T,ϖ) be a transference group , a two points ξ and ς of Z are called proximally proved that for each neighborhood index λ in Z there subsist $t\in T$ that $(\xi,\varsigma)t\in\lambda$ .The set of all proximally pairs are called the proximally relation and denoted by $P(Z,T)$.

Definition (2)

Let (Z,T,ϖ) be a transference group , a two points ξ and ς of Z are called syndetical proximally proved that for each neighborhood index λ in Z there subsist syndetical subset $\hbar$ of Z that $(\xi,\varsigma)\hbar\subset\lambda$ .The set of all syndetical proximally pairs are called the syndetical proximally relation and denoted by $SP(Z,T)$.

Definition (3)

Let (Z,T,ϖ) be a transference group , a two points ξ and ς of Z are called extensively proximally proved that for each neighborhood index λ in Z there subsist semi repletly subset $\wp$ of Z that $(\xi,\varsigma)\wp\in\lambda$ .The set of all extensively proximally pairs are called the extensively proximally relation and denoted by $EP(Z,T)$.

Theorem (4)

Let (Z,T,ϖ) be a transference group abelian group (ξ,ς) syndetical proximally point then λ is invariant .

Proof: Let (ξ,ς) are syndetical proximally points then for each neighborhood index λ in Z there subsist syndetical subset $\hbar$ of T that $(\xi,\varsigma)\hbar\subset\lambda$,because $\hbar$ syndetical set there subsist compact set $\wp$ that $\hbar\wp=T$,so $(\xi,\varsigma)\hbar\wp\subset\lambda\wp$ by hypothesis $(\xi,\varsigma)T\subset\lambda\wp$, $(\xi,\varsigma)TT\subset\lambda\wp T$ because T syndetical set then $(\xi,\varsigma)T\subset\lambda T$, *since* $(\xi,\varsigma)T$ orbit of (ξ,ς)then $(\xi,\varsigma)T\cap(\xi,\varsigma)\hbar\neq\phi$ .So $\lambda T\subset(\xi,\varsigma)\hbar\subset\lambda$,because $\lambda e\subset\lambda T$ so $\lambda\subset\lambda T$ Thus λ invariant . Δ

Theorem (5)

Let (Z,T,ϖ) be a transference group ,then the folllowing statements are equivalent .

1. (ξ,ς) proximally point.
2. (ξ,ς) syndetical proximally point.

Proof: Let (ξ,ς) are proximally points then for each neighborhood index λ in T there subsist $\hbar\in T$ such that $(\xi,\varsigma)\hbar\in\lambda$ so $(\xi,\varsigma)T\subset\lambda$ because T topological group , then we obtain T syndetical set .Then (ξ,ς) are syndetical proximally points. Conversely, let (ξ,ς) are syndetical proximally points then for each neighborhood index λ in Z there subsist syndetical subset $\hbar$ of T such that $(\xi,\varsigma)\hbar\subset\lambda$,because $\hbar$ syndetical set there subsist compact set $\wp$ of T such that $\hbar\wp=T$, $(\xi,\varsigma)\hbar\wp\subset\lambda\wp$ and $\lambda\wp\sqsubset\lambda T$ by theorem (4) we obtain $(\xi,\varsigma)\hbar\wp\subset\lambda$ so $(\xi,\varsigma)T\subset\lambda$,there subsist $t\in T$ that $(\xi,\varsigma)t\in\lambda$ .Therefore (ξ,ς) are proximally points. Δ

Theorem (6)

Let (Z,T,ϖ) be a transference group ,then the folllowing statements are equivalent

1. (ξ,ς) proximally point
2. (ξ,ς) extensively proximally point

Proof:
Suppose that υ be compact subset of T ,let (ξ,ς) are proximally points then for each neighborhood index λ in T there subsist $\hbar \in T$ that $(\xi,\varsigma)\hbar \in \lambda$ so $(\xi,\varsigma)T \subset \lambda$ it is enough to prove T extensively set ,by hypothesis $\upsilon t \subset T\ t \subset T$ Then T be semi repletly set and $T \cap T \neq \phi$ then T extensively set so (ξ,ς) extensively proximally points. Conversely, let (ξ,ς) are extensively proximally points then for each neighborhood index λ in Z there subsist extensively subset E of T that $(\xi,\varsigma)E \subset \lambda$,because E extensively set then there subsist semi repletly set $\Im$ in T that $E \cap \Im \neq \phi$ so $(\xi,\varsigma)E \cap (\xi,\varsigma)\Im \neq \phi$ because $(\xi,\varsigma)E \subset \lambda$ then $(\xi,\varsigma)\Im \subset \lambda$ because $\Im$ subset of T there subsist $t \in \Im$ that $(\xi,\varsigma)t \in \lambda$. Δ

Theorem (7)

Let (Z,T,ϖ) be a transference group ,then folllowing statements are equivalent

1. (ξ,ς) syndetical proximally point
2. (ξ,ς) extensively proximally point

Proof: let (ξ,ς) are syndetical proximally points then for each neighborhood neighborhood neighborhood index λ in Z there subsist syndetical subset $\hbar$ of T that $(\xi,\varsigma)\hbar \subset \lambda$,because $\hbar$ syndetical set there subsist compact set $\wp$ of T that $\hbar\wp = T$,for each $t \in T$ there subsist $a \in \hbar, b \in \wp$ that $ab = t$ because T topological group there subsist $\wp^{-1} \in T$ that $t\wp^{-1} \subset \hbar$ because $\wp$ compact set and (T,∗) topological group then $\wp^{-1}$ compact subset of T thus $\hbar$ semi repletly set and $\hbar \cap \hbar \neq \phi$ then $\hbar$ extensively set and (ξ,ς) extensively proximally point. Conversely, let (ξ,ς) are extensively proximally points then for each neighborhood index λ in Z there subsist extensively subset E of T that $(\xi,\varsigma)E \subset \lambda$,because E extensively set then there subsist semi repletly set $\Im$ in T that $E \cap \Im \neq \phi$ and $(\xi,\varsigma)E \subset (\xi,\varsigma)\Im \neq \phi$ by hypothesis we obtain $(\xi,\varsigma)\Im \subset \lambda$ because $\Im$ semi repletly subset of T by theorem (10) $\Im$ syndetical subset of T therefore (ξ,ς) syndetical proximally point. Δ

Theorem (8)

1.Let (Z,T,ϖ) be a transference group , and β open neighborhood of (ξ,ς) then β invariant open neighborhood of (ξ,ς).

2. Let (Z,T,ϖ) be a transference group , and P period of (ξ,ς) then P syndetical subset of T.

Theorem (9)

Let (Z,T,ϖ) be a transference group , and (ξ,ς) syndetical proximally point then $\lambda = (\xi,\varsigma)T$

Proof: Suppose $(\xi,\varsigma)T$ orbit of (ξ,ς) because (ξ,ς) are syndetical proximally points then for each neighborhood index λ in Z there subsist syndetical subset $\hbar$ of T that $(\xi,\varsigma)\hbar \subset \lambda$ because $(\xi,\varsigma)T$ orbit of (ξ,ς) then $(\xi,\varsigma) \in (\xi,\varsigma)T$,by hypothesis $(\xi,\varsigma) \in \lambda\hbar^{-1} \subset \lambda T$ by theorem (4) we obtain $(\xi,\varsigma) \in \lambda$ thus $(\xi,\varsigma)T \subset \lambda$ because $(\xi,\varsigma)\hbar \subset \lambda$,$(\xi,\varsigma)\eta \cap (\xi,\varsigma)T \neq 0$,so $\lambda \subset (\xi,\varsigma)T$ therefore $\lambda = (\xi,\varsigma)T$. Δ

Lemma (10)

Let (Z,T,ϖ) be a transference group , and (ξ,ς) syndetical proximally point then $\lambda \sqsubset \overline{(\xi,\varsigma)T}$

Theorem (11)

Let (Z,T,ϖ) be a transference group , and (ξ,ς) syndetical proximally point then (ξ,ς) almost invariant periodic point

Proof : Let (ξ,ς) are syndetical proximally points then for each neighborhood index λ in Z there subsist syndetical subset $\hbar$ of T that $(\xi,\varsigma)\hbar\subset\lambda$ .Because λ subset in topological spase then λ open set by throrem (4) λ invariant set .thus (ξ,ς) almost invariant periodic point .

Theorem (12)

Let (Z,T,ϖ) be a transference group , and (ξ,ς) syndetical proximally point then (ξ,ς) almost periodic point

Theorem (13)

Let (Z,T,ϖ) be a transference group , and (ξ,ς) periodic point under T then:

1. (ξ,ς) syndetical proximally point .
2. $(\xi,\varsigma)P=(\xi,\varsigma)T$
3. λ minimal neighborhood index

Proof:

(1) Suppose λ neighborhood of (ξ,ς) because periodic point under T there subsist period P that $(\xi,\varsigma)P=(\xi,\varsigma)$, so $(\xi,\varsigma)P\sqsubset\lambda$, by theorem (8-2) P syndetical set and λ Open neighborhood subset of Z therefore (ξ,ς) syndetical proximally point . Δ

(3) Let (ξ,ς) periodic point under T by throrem (13-1) there subsist syndetical subset $\hbar$ of T that $(\xi,\varsigma)\hbar\subset\lambda$ by throrem (9) we obtain $\lambda=(\xi,\varsigma)T\sqsubset\overline{(\xi,\varsigma)T}$ because (ξ,ς) periodic point then λ be minimal index. Δ

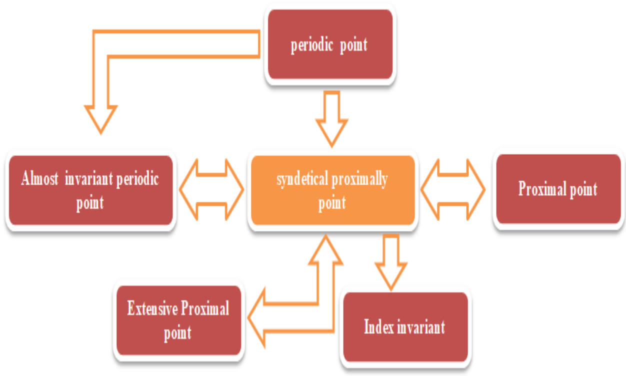

Fig.(2) Show relationship Syndetic Proximally point with dynamical properties

III.Conclusion

In this research, study the proximally between the points (almost periodic –periodic) in topological transference group depended on syndetical set , and reached the following results

1. (ξ,ς) syndetical proximally point neighborhood index λ is invariant

2.Relationship syndetical proximally point with proximally point

3..Relationship syndetical proximally point with extensively proximally point

4..Relationship extensively proximally point with proximally point

5. Relationship syndetical proximally point with almost periodic point

6. Relationship periodic point with almost periodic point. And λ minimal index

V. Reference

[1] W.Gottschalk, and ,G .Hedlund.. "Topological Dynamics", American Mathematical Society, (1955).

[2] J. Mai and W. Sun," Almost periodic points and minimal sets in regular spaces", Topology Appl. 154 2873–2879, 2007

[3] E.Yahya , Ameera N.Alkiffai and Rana H.Hilal "Relation proximal point with some dynamical properties"Mathematical Theor and Modeling, Vol.6, No.8,p.100-107,2016

[4] E.Glasner ,W.Huang,S.Shao"Regionally proximal relation Of order D along arithmetic progressipons

and nilsystems"Science Ching Mathematics ,Vol.63,p1757- 1776,2020

[5] A. Fedeli and A. Le Donne, "On almost periodic orbits and minimal sets", Topology Appl.Vol 156 , p.473– 475. 2008

[6] J .Jezierski, and W. Marzantowicz, ," Homotopy Methods in Topological Fixed and Periodic Point Theory Springer Published,2006

[7] E.Yahya, " Distal set in topological transference group " journal of kerbala university, , Volume 10, Issue 1, PP 270-277, 2012

[8] M.l Megrelishvili "Topological Transformation Groups: Selected Topics"In book: Open Problems in Topology II,April 2007

[9] Y.Smirnov. "Some topological aspects of the theory of topological transformation groups"In book: General Topology and Its Relations to Modern Analysis and Algebra IV,November 2006