Bayes estimators with extension of Jeffery prior information for Time censored data and Failure censored data

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Abstract- *In this research, the Bayes estimator was derived based on Time censored data of the first type, and the Failure censored data of the second type. Reliance has been made on extension of Jeffery prior information. Finally, the simulation was used based on the MATLAB program and with different inputs to find the best estimator among Maximum Likelihood estimator and Bayes estimators with extension that has the least mean percentage error.*

Keywords ***: Bayes estimator; extension of Jeffery prior information; Maximum Likelihood estimator; simulation.***

I. INTRODUCTION

Al Hadithi compared standard Bayes estimations of the Pareto distribution parameter using different loss functions in 2010 functions [2]. Biswabrata Pradhan and Debasis Kundu (2014), they analyzed interval-censored data, with the Weibull distribution as the underlying lifetime distribution has been considered. Their highest posterior density the unknown parameter's credible intervals are obtained using importance sampling, technique. Small simulation experiments are conducted to investigate the finite sample performance of the proposed estimators and the analysis of two data sets; one for illustration purposes, one simulated and one real-life scenario have been provided [13]. Loaiy F. Naji and Huda A. Rasheed (2019), they presented Bayes estimators for the shape and scale parameters of Gamma" distribution under the precautionary loss function, assuming the Priors are represented "by Gamma and Exponential priors for the shape and scale parameters respectively [12]. Erma Elviana, Joko Purwadi(2020), they conducted the study to determine the estimation of Rayleigh's distribution parameters on the survival analysis of type II tuberculosis patients. The method The Bayesian method to estimate parameters used in this research is the Bayesian method. Kean University (2011), they show that considering the estimation of lifetime distribution based on missing-censoring data. Using the simple empirical approach rather than from the maximum likelihood argument, they obtain the parametric estimations of lifetime distribution under the assumption that the failure time follows exponential or gamma distribution [6]. Qesma Abadi and Iden H. AL-Kanani (2020), they proposed estimating and deriving the three parameters that include one scale parameter and one shape parameter of a new mixture distribution for the singly type one censored data, which is the branch of the right censored sample. They used to based on the maximum likelihood estimator method for singly type I censored data, the Newton-Raphson matrix procedure is used to find and estimate values of these three parameters by Utilizing the real data taken from the National Center for Research and Treatment of Hematology/University of Mustansiriyah for leukemia diseases. After that, they find and derive Then calculate the hazard function [1]. A. Ali1 and I. Al. Kanani2 (2021), they introduced some of the properties of the exponentiated Weibull distribution. The Tierney estimator method and the Lindely estimator method are proposed to estimate all the unknowns [3]. E. Mohammed, and A. Aoda Al-Tebawyb (2021), shows that the Bayes method and compares it to the standard method. The Bayes method by using a variety of loss functions to estimate the measurement parameter and the reliability [11].

The Bayes estimation method has gained significant prominence in contemporary times due to its crucial role in survival analysis. This method is applicable in scenarios involving complete data or censored data, including time-censored data of the first type and failure-censored data of the second type. The present study utilized the exponential distribution, a widely recognized failure distribution commonly employed in survival analysis. This distribution is widely regarded as one of the most significant in the field.

First, a Bayesian estimator was derived based on the extension of Jeffery prior information and also on the complete data. Then, a Bayesian estimator was derived based on extension of Jeffery prior information and Time censored data. Finally, a Bayesian estimator is derived based on extension of Jeffery prior in formation and the Failure censored data of the second type.

After completing the derivations, the MATLAB program was used to generate data by simulation to find the best estimator among Maximum Likelihood estimator and Bayes estimators with extension, based on different inputs [16] .

II. RESEARCH IMPORTANCE

The importance of the research lies in the following:

1. Knowing how to derive the Bayes estimator in the case of relying on extension of Jeffery prior information
2. Generating data by using simulation and depending on different inputs to find out the best estimator that gives us the lowest error rate

III. MATERIALS AND METHODS

1. complete data

The new extension of Jeffery prior information [4], [5], [7]

$g(\theta) \propto [I(\theta)]^{c_1}$

We find Jeffery

$I(\theta) = - nE(\frac{\partial^2 \ln f(t_i,\theta)}{\partial\theta^2})$

$$f(t_i,\theta) = \frac{1}{\theta}\exp[\frac{-t_i}{\theta}] \quad (1.1)$$

$\ln f(t_i,\theta) = -\ln\theta - \frac{t_i}{\theta}$

$$E(\frac{\partial^2 \ln f(t_i,\theta)}{\partial\theta^2}) = \frac{-1}{\theta^2} \quad (1.2)$$

$$I(\theta) = -n\left(\frac{-1}{\theta^2}\right) = \frac{n}{\theta^2} \quad (1.3)$$

$$g(\theta) \propto [I(\theta)]^{c_1}, \text{ then } g(\theta) = \frac{kn^{c_1}}{\theta^{2c_1}} \quad (1.4)$$

K is Constant, $c_1 \in R^+$

The likelihood function is

$$L(t_1,t_2,\ldots,t_n|\theta) = \prod_{i=1}^n f(t_i,\theta) = \frac{1}{\theta^n}\exp[\frac{-\sum_{i=1}^n t_i}{\theta}] \quad (1.5)$$

The joint probability density function is given by

$H(t_1,t_2,\ldots,t_n,\theta) = \prod_{i=1}^n f(t_i,\theta)\, g(\theta)$,

$$H(t_1,t_2,\ldots,t_n,\theta) = \frac{1}{\theta^n}\exp(\frac{-\sum_{i=1}^n t_i}{\theta})\frac{kn^{c_1}}{\theta^{2c_1}} = \frac{kn^{c_1}}{\theta^{n+2c_1}}\exp(\frac{-\sum_{i=1}^n t_i}{\theta}) \quad (1.6)$$

The marginal probability density function of $(t_1,t_2,\ldots,t_n)$ is given by

$$P(t_1,t_2,\ldots,t_n) = \int_0^\infty H(t_1,t_2,\ldots,t_n,\theta)d\theta = \int_0^\infty \frac{kn^{c_1}}{\theta^{n+2c_1}}\exp(\frac{-\sum_{i=1}^n t_i}{\theta})\, d\theta$$

$$= \frac{kn^{c_1}(n+2c_1-2)!}{(\sum_{i=1}^n t_i)^{n+2c_1-1}} \quad (1.7)$$

The conditional probability density function of θ given the data $(t_1,t_2,\ldots,t_n)$ is given by ,

$$\prod(\theta \mid t_1,t_2,\ldots,t_n) = \frac{H(t_1,t_2,\ldots,t_n,\theta)}{P(t_1,t_2,\ldots,t_n)}$$

$$= \frac{\theta^{-n-2c_1}\exp(\frac{-\sum_{i=1}^n t_i}{\theta})}{(\sum_{i=1}^n t_i)^{1-n-2c_1}(n+2c_1-2)!} \quad (1.8)$$

We use Risk function, by depend on loss function

$$R(\hat\theta-\theta) = \int_0^\infty c(\hat\theta-\theta)^2 \prod(\theta \mid t_1,t_2,\ldots,t_n)d\theta$$

$$= c\widehat{\theta^2} - 2c\hat\theta\int_0^\infty\left(\frac{\theta^{1-n-2c_1}\exp\left(\frac{-\sum_{i=1}^n t_i}{\theta}\right)}{(\sum_{i=1}^n t_i)^{1-n-2c_1}(n+2c_1-2)!}\right)d\theta + \zeta(\theta) \quad (1.9)$$

$$\frac{\partial R(\hat\theta,\theta)}{\partial\hat\theta} = 2c\hat\theta - 2c\int_0^\infty\left(\frac{\theta^{1-n-2c_1}\exp\left(\frac{-\sum_{i=1}^n t_i}{\theta}\right)}{(\sum_{i=1}^n t_i)^{1-n-2c_1}(n+2c_1-2)!}\right)d\theta + zero$$

, and if $\frac{\partial R(\hat\theta,\theta)}{\partial\hat\theta} = 0$, then

$$\hat\theta_B = \frac{1}{(\sum_{i=1}^n t_i)^{1-n-2c_1}(n+2c_1-2)!}\int_0^\infty \theta^{1-n-2c_1}\exp(\frac{-\sum_{i=1}^n t_i}{\theta})d\theta$$

$$\hat\theta_B = \frac{(\sum_{i=1}^n t_i)}{(n+2c_1-2)} \quad (1.10)$$

2. Time censored data (type I censored data)

[8],[9],[10]

$g(\theta) \propto [I(\theta)]^{c_1}$

$$g(\theta) = k[\frac{n}{\theta^2}]^{c_1} = \frac{kn^{c_1}}{\theta^{2c_1}} \quad (2.1)$$

k is Constant. $c_1 \in R^+$

The likelihood function is

$$L(t_1,t_2,\ldots,t_n|\theta) = \frac{n!}{(n-m)!}\prod_{i=1}^m f(t_i,\theta)[s(t_0)]^{n-m}$$

$$= \frac{n!}{(n-m)!}[\frac{1}{\theta^m}\exp\left(\frac{-\sum_{i=1}^n t_i}{\theta}\right)[\exp\left(\frac{-t_0}{\theta}\right)]^{n-m} \quad (2.2)$$

The joint probability density function is given by

$H(t_1,t_2,\ldots,t_n,\theta) = L(t_1,t_2,\ldots,t_n|\theta)g(\theta)$

$H(t_1,t_2,\ldots,t_n,\theta)$

$$= \frac{n!}{(n-m)!}[\frac{1}{\theta^m}\exp\left(\frac{-\sum_{i=1}^n t_i}{\theta}\right)\left[\exp\left(\frac{-t_0}{\theta}\right)\right]^{n-m}\frac{kn^{c_1}}{\theta^{2c_1}}$$

$$= \frac{k.n^{c_1}.n!}{\theta^{m+2c_1}(n-m)!}\exp\left(\frac{-\sum_{i=1}^n t_i}{\theta}\right)\left[\exp\left(\frac{-t_0}{\theta}\right)\right]^{n-m} \quad (2.3)$$

The marginal probability density function of $(t_1,t_2,\ldots,t_n)$ is given by

$$P(t_1,t_2,\ldots,t_n) = \int_0^\infty H(t_1,t_2,\ldots,t_n,\theta)d\theta$$

$$= \int_0^\infty \frac{k.n^{c_1}.n!}{\theta^{m+2c_1}(n-m)!}[\exp\left(\frac{-\sum_{i=1}^n t_i}{\theta}\right)\left[\exp\left(\frac{-t_0}{\theta}\right)\right]^{n-m} d\theta$$

$$= \frac{k.n^{c_1}.n!}{(n-m)!}\int_0^\infty \theta^{-m-2c_1}\left[\exp\frac{-(\sum_{i=1}^n t_i + t_0(n-m))}{\theta}\right]d\theta$$

$$= \frac{k.n^{c_1}.n!\,(m+2c_1-2)!}{(n-m)!\left(\sum_{i=1}^n t_i + t_0(n-m)\right)^{m+2c_1-1}} \quad (2.4)$$

The conditional probability density function of θ given the data $(t_1,t_2,\ldots,t_n)$ is given by,

$\prod(\theta \mid t_1,t_2,\ldots,t_n) = \frac{H(t_1,t_2,\ldots,t_n,\theta)}{P(t_1,t_2,\ldots,t_n)}$

$$= \frac{\frac{k.n^{c_1}.n!}{\theta^{m+2c_1}(n-m)!}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_0}{\theta}\right)\right]^{n-m}}{\frac{k.n^{c_1}.n!\,(m+2c_1-2)!}{(n-m)!\left(\sum_{i=1}^{n} t_i+t_0(n-m)\right)^{m+2c_1-1}}}$$

$$= \frac{\theta^{-m-2c_1}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_0}{\theta}\right)\right]^{n-m}}{\left(\sum_{i=1}^{n} t_i+t_0(n-m)\right)^{1-m-2c_1}(m+2c_1-2)!} \qquad (2.5)$$

the Risk function, using loss function

$R(\hat{\theta}-\theta) = \int_0^\infty c(\hat{\theta}-\theta)^2 \prod(\theta \mid t_1,t_2,\ldots,t_n)d\theta$

$$= \int_0^\infty \left(c\widehat{\theta^2} - 2c\hat{\theta}\theta + \zeta(\theta)\right)\frac{\theta^{-m-2c_1}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_0}{\theta}\right)\right]^{n-m}}{\left(\sum_{i=1}^{n} t_i+t_0(n-m)\right)^{1-m-2c_1}(m+2c_1-2)!}d\theta$$

$$= c\widehat{\theta^2} - 2c\hat{\theta}\int_0^\infty \frac{\theta^{-m-2c_1}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_0}{\theta}\right)\right]^{n-m}}{\left(\sum_{i=1}^{n} t_i+t_0(n-m)\right)^{1-m-2c_1}(m+2c_1-2)!}d\theta + \zeta(\theta) \qquad (2.6)$$

$$\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 2c\hat{\theta} - 2c\int_0^\infty \frac{\theta^{1-m-2c_1}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_0}{\theta}\right)\right]^{n-m}}{\left(\sum_{i=1}^{n} t_i+t_0(n-m)\right)^{1-m-2c_1}(m+2c_1-2)!}d\theta + zero \qquad (2.7)$$

, and if $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 0$, then we have

$$\hat{\theta}_B = \frac{1}{\left(\sum_{i=1}^{n} t_i + t_0(n-m)\right)^{1-m-2c_1}(m+2c_1-2)!}\int_0^\infty \theta^{1-m-2c_1}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\exp\left(\frac{-t_0}{\theta}\right)^{n-m}d\theta$$

$$\hat{\theta}_B = \frac{1}{\left(\sum_{i=1}^{n} t_i + t_0(n-m)\right)^{1-m-2c_1}(m+2c_1-2)!}\int_0^\infty \theta^{1-m-2c_1}\exp\left(\frac{-\left(\sum_{i=1}^{n} t_i + t_0(n-m)\right)}{\theta}\right)d\theta$$

$$= \frac{\sum_{i=1}^{n} t_i+t_0(n-m)}{(m+2c_1-2)} \qquad (2.8)$$

***3. Failure censored data (type II censored data)*: [14],[15]**

$g(\theta) \propto [I(\theta)]^{c_1}$

$$g(\theta) = k\left[\frac{n}{\theta^2}\right]^{c_1} = \frac{kn^{c_1}}{\theta^{2c_1}} \qquad (3.1)$$

k is Constant. $c_1 \in R^+$

The likelihood function is

$L(t_1,t_2,\ldots,t_n|\theta) = \frac{n!}{(n-r)!}\prod_{i=1}^{r} f(t_i,\theta)[s(t_r)]^{n-r}$

$$= \frac{n!}{(n-r)!}\left[\frac{1}{\theta^r}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_r}{\theta}\right)\right]^{n-r}\right. \qquad (3.2)$$

The joint probability density function is given by

$H(t_1,t_2,\ldots,t_n,\theta) = L(t_1,t_2,\ldots,t_n|\theta)g(\theta)$,

$H(t_1,t_2,\ldots,t_n,\theta)$

$$= \frac{n!}{(n-r)!}\left[\frac{1}{\theta^r}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_r}{\theta}\right)\right]^{n-r}\frac{kn^{c_1}}{\theta^{2c_1}}\right.$$

$$= \frac{k.n^{c_1}.n!}{\theta^{r+2c_1}(n-r)!}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_r}{\theta}\right)\right]^{n-r} \qquad (3.3)$$

The marginal probability density function of $(t_1,t_2,\ldots,t_n)$ is given by

$P(t_1,t_2,\ldots,t_n) = \int_0^\infty H(t_1,t_2,\ldots,t_n,\theta)d\theta$

$$= \int_0^\infty \frac{k.n^{c_1}.n!}{\theta^{r+2c_1}(n-r)!}\left[\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_r}{\theta}\right)\right]^{n-r} d\theta\right.$$

$$= \frac{k.n^{c_1}.n!}{(n-r)!}\int_0^\infty \theta^{-r-2c_1}\left[\exp\frac{-\left(\sum_{i=1}^{n} t_i+t_r(n-r)\right)}{\theta}\right]d\theta$$

$$= \frac{k.n^{c_1}.n!\left(\sum_{i=1}^{n} t_i+t_r(n-r)\right)^{1-r-2c_1}}{(n-r)!}(r+2c_1-2)!$$

$$= \frac{k.n^{c_1}.n!\,(r+2c_1-2)!}{(n-r)!\left(\sum_{i=1}^{n} t_i+t_r(n-r)\right)^{r+2c_1-1}} \qquad (3.4)$$

The conditional probability density function of θ given the data $(t_1,t_2,\ldots,t_n)$ is given by ,

$\prod(\theta \mid t_1,t_2,\ldots,t_n) = \frac{H(t_1,t_2,\ldots,t_n,\theta)}{P(t_1,t_2,\ldots,t_n)}$

$$= \frac{\frac{k.n^{c_1}.n!}{\theta^{r+2c_1}(n-r)!}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_r}{\theta}\right)\right]^{n-r}}{\frac{k.n^{c_1}.n!\,(r+2c_1-2)!}{(n-r)!\left(\sum_{i=1}^{n} t_i+t_r(n-r)\right)^{r+2c_1-1}}}$$

$$= \frac{\theta^{-r-2c_1}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_r}{\theta}\right)\right]^{n-r}}{\left(\sum_{i=1}^{n} t_i+t_r(n-r)\right)^{1-r-2c_1}(r+2c_1-2)!} \qquad (3.5)$$

the Risk function, using loss function

$R(\hat{\theta}-\theta) = \int_0^\infty c(\hat{\theta}-\theta)^2 \prod(\theta \mid t_1,t_2,\ldots,t_n)d\theta$

$$= c\widehat{\theta^2} - 2c\hat{\theta}\int_0^\infty \frac{\theta^{-r-2c_1}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_r}{\theta}\right)\right]^{n-r}}{\left(\sum_{i=1}^{n} t_i+t_r(n-r)\right)^{1-r-2c_1}(r+2c_1-2)!}d\theta + \zeta(\theta) \qquad (3.6)$$

$$\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 2c\hat{\theta} - 2c\int_0^\infty \frac{\theta^{1-r-2c_1}\exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right)\left[\exp\left(\frac{-t_r}{\theta}\right)\right]^{n-r}}{\left(\sum_{i=1}^{n} t_i+t_r(n-r)\right)^{1-r-2c_1}(r+2c_1-2)!}d\theta + zero \qquad (3.7)$$

, and if $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 0$, then

$$\hat{\theta}_B = \frac{1}{\left(\sum_{i=1}^{n} t_i + t_r(n-r)\right)^{1-r-2c_1}(r+2c_1-2)!}$$

$$\int_0^{\infty} \theta^{1-r-2c_1} \exp\left(\frac{-\sum_{i=1}^{n} t_i}{\theta}\right) \exp\left(\frac{-t_r}{\theta}\right)^{n-r} d\theta$$

$$\hat{\theta}_B = \frac{1}{\left(\sum_{i=1}^{n} t_i + t_r(n-r)\right)^{1-r-2c_1} (r+2c_1-2)!}$$

$$\int_0^{\infty} \theta^{1-r-2c_1} \exp\left(\frac{-(\sum_{i=1}^{n} t_i + t_r(n-r))}{\theta}\right) d\theta$$

$$= \frac{\sum_{i=1}^{n} t_i + t_r(n-r)}{(r+2c_1-2)!} \int_0^{\infty} y^{r+2c_1-3} \exp(-y)\, dy$$

$$= \frac{\sum_{i=1}^{n} t_i + t_r(n-r)}{(r+2c_1-2)} \qquad (3.8)$$

IV. Simulation study

In simulation study, we have chosen $n = 25{,}100$, several values of parameter θ =0.4, 1.2, value of m=20, 95 (items failed before t_0) and value of times of items failed before t_0 (10). The number of replication used was R=1000. The simulation program was written by using Matlab program. After the parameter was estimated, mean percentage error (MPE) were calculated to compare the methods of estimation, where

$$\text{MPE}\left(\theta\right) = \frac{\left[\sum_{i=1}^{1000} \frac{\left|\theta_i - \theta\right|}{\theta}\right]}{R}$$

The results of the simulation study are summarized and tabulated in Tables 1 ,2and 3, we note that the smallest values of (MPE) are in the Bayesian estimator with extension and the maximum likelihood estimator if we use complete data. In the case of using censored data, we noticed that the values of (MPE) are somewhat large compared to the values of other estimators.

Table 1: MPE for parameters with $c_1 = 0.1$ and $t_0 = 10$

n	b	M	$\widehat{\theta_M}$	$\widehat{\theta_B}$	$\widehat{\theta_M}$	$\widehat{\theta_B}$
25	0.4	20	5.4636e-05	0.0578	0.0316	0.0766
	1.2	20	2.1487e-04	0.0197	0.0315	0.0465
100	0.4	95	2.5168e-05	0.2491	0.1028	0.3403
	1.2	95	6.9769e-05	0.0860	0.1064	0.1856

Table 2: MPE for parameters with $c_1 = 0.5$ and $t_0 = 10$

n	b	M	$\widehat{\theta_M}$	$\widehat{\theta_B}$	$\widehat{\theta_M}$	$\widehat{\theta_B}$
25	0.4	20	6.9096e-05	0.0632	0.0320	0.1020
	1.2	20	6.1172e-06	0.0244	0.0259	0.0493
100	0.4	95	1.3592e-04	0.2724	0.1139	0.3763
	1.2	95	7.9430e-05	0.1037	0.0915	0.1790

Table 3: MPE for parameters with $c_1 = 1$ and $t_0 = 10$

n	b	M	$\widehat{\theta_M}$	$\widehat{\theta_B}$	$\widehat{\theta_M}$	$\widehat{\theta_B}$
25	0.4	20	1.3769e-04	0.0707	0.0337	0.1349
	1.2	20	5.3767e-05	0.0313	0.0274	0.0612
100	0.4	95	1.6909e-04	0.2855	0.0834	0.3771
	1.2	95	1.1124e-04	0.1362	0.1106	0.2085

IIV. CONCLUSION

In Table 1 When $c_1 = 0.1$we compared MPE for parameters we found When we have a sample size of 25and 100 when comparing Bayes Estimator with extension and the Maximum Likelihood Estimators, we found that the maximum likelihood estimator is the best and the value MPE is lower in the case of complete data and censored data.

In Table2 When $c_1 = 0.5$ we compared MPE for parameters we found When we have a sample size of 25and 100 when comparing Bayes Estimator with extension and the Maximum Likelihood Estimators, we found that the maximum likelihood estimator is the best and the value MPE is lower in the case of complete data and censored data.

In Table 3 When $c_1 = 1$ we compared MPE for parameters we found When we have a sample size of 25and 100 when comparing Bayes Estimator with extension and the Maximum Likelihood Estimators, we found that the maximum likelihood estimator is the best and the value MPE is lower in the case of complete data and censored data. Therefore, when taking any value of c_1, the method of the maximum likelihood estimator is preferable to achieve less MPE in small and large samples .

REFERENCES

[1] Abadi, Q. S., & AL-Kanani, I. H. (2020). Estimating of Survival Function under Type One Censoring Sample for Mixture Distribution. Ibn AL-Haitham Journal For Pure and Applied Science, 33(4), 102-109.

[2] Al-Hadithi, Ikhlas Ali Hammoudi, "Comparing the standard Bayes estimations of the Pareto distribution parameterUsing different loss functions", Master's Thesis, College of Administration and Economics-University of (Baghdad (2010).

[3] Ali, A. H., &Kanani, I. H. A. (2021, March). Bayesian Methods to Estimate the Parameters of Exponentiated Weibull Distribution. In Journal of Physics: Conference Series (Vol. 1818, No. 1, p. 012143). IOP Publishing

[4] Ellis, W. C., & Tummala, V. R. (1986). Minimum expected loss estimators of the shape & scale parameters of the Weibull distribution. IEEE transactions on reliability, 35(2), 212-215.

[5] Epstein, B., &Sobcl, M. (1984). Some Theorems to Life Testing from an Exponential Distribution. Annals of Mathematical Statistics, 25.

[6] Elviana, E., &Purwadi, J. (2020, July). Parameters Estimation Of Rayleigh Distribution In Survival Analysis On Type Ii Censored Data Using The Bayesian Method. In Journal of Physics: Conference Series (Vol. 1503, No. 1, p. 012004). IOP Publishing.

[7] Felsenstein, K. (2002). Bayesian inference for questionable data. Austrian Journal of Statistics, 31(2&3), 131-140.

[8] Fernández, A. J. (2000). Estimation and Hypothesis Testing for Exponential Lifetime Models with Double Censoring and Prior Information. Journal of Economic & Social Research, 2(2).

[9] Hahn, J. (2004). Does Jeffrey's prior alleviate the incidental parameter problem?. Economics Letters, 82(1), 135-138.

[10] Kumar, A. (2003). Bayes estimator for one parameter exponential distribution under multiply type II censoring scheme. In International Conference on Statistics, Combinatorics and Related Area.

[11] Mohammed, M. E., & Al-Tebawy, A. A. A. (2021). Bayesian estimation of the beta distribution parameter (α) when the parameter (β) is known. Turkish Journal of Computer and Mathematics Education (TURCOMAT), 12(14), 4879-4886.

[12] Naji, L. F., & Rasheed, H. A. (2019). Bayesian estimation for two parameters of gamma distribution under generalized weighted loss function. Iraqi Journal of Science, 60(5), 1161-1171.

[13] Pradhan, B., & Kundu, D. (2014). Analysis of interval-censored data with Weibull lifetime distribution. Sankhya B, 76, 120-139.

[14] Shawky, A. I., &Bakoban, R. A. (2008). Bayesian and non-Bayesian estimations on the exponentiated gamma distribution. Applied Mathematical Sciences, 2(51), 2521-2530.

[15] Sinha, S. K., & Sloan, J. A. (1988). Bayes estimation of the parameters and reliability function of the 3-parameter Weibull distribution. IEEE Transactions on Reliability, 37(4), 364-369.

[16] Haneen R. Sahib and Hadeel S. Al-Kutubi ,"A Comparison Between Parameters and Survival Function of the Exponential Distribution Model Using the Complete Data and Censored Data". accepted final of the journalMathematical Modelling of Engineering Problems .2023