Applications of Fractional-Laplace Transformation in the Field of Electrical Engineering

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Abstract: This study examines the various ways that fractional Laplace transform can be used to solve three different kinds of mathematical equations: the equation of analysis of electric circuits, simultaneous differential equations, and the heat conduction equation. This article how to use the fractional Laplace transform to calculate heat flow in semi-infinite solids in the context of heat conduction. The answers that are developed offer important information about how temperatures vary across time and space. The essay also examines how to analyse electrical circuits using the Fractional Laplace transform. This method allows researchers to measure significant electrical parameters including charge and current, which improves their comprehension of circuit dynamics. Practical examples are included throughout the essay to show how useful the Fractional Laplace transform is in various fields. As a result of the answers found using this methodology, researchers and engineers working in the fields of heat conduction, system dynamics, and circuit analysis can gain important new knowledge. In conclusion, this study explains the applicability and effectiveness of the fractional Laplace transform in resolving a variety of mathematical equations. It is a vital tool for researchers because it may be used in a wide range of scientific and engineering areas.

Keywords— ***Fractional-Laplace transform; Time derivative rule; linear differential equations; Circuit equations; Heat conduction differential model***

I. Introduction

Mathematical processes known as integral transforms reduce complex functions in one function space to simpler functions in another. Integration in the converted space enables simple analysis and manipulation of these transformed functions. The altered function is frequently returned to its native space using the inverse transform approach.

Although there is a wealth of knowledge on integral transforms, it is important to proceed with caution because the accuracy of research findings varies. The used materials and analysis techniques have a significant impact on the results. Although k-Fourier and k-Laplace transforms have not yet been compared, certain comparative studies have exhaustively examined the ideas of Fourier and Laplace transforms.

Modern integral transforms including the Laplace, Fourier, Mahgoub, Mohand, and Aboodh transforms are the subject of ongoing research since they have been essential in resolving complex technical and scientific issues.

Understanding nonlinear vibrational phenomena, which exhibit chaotic behaviour, unstable responses, and other complicated effects, is essential for comprehending vibration systems. These vibrational events are influenced by elements including electrical fields, elastic deformation, and nonlinear damping.

Second-order hyperbolic equations' numerical solutions have received a lot of attention in the past. [2] presents an algorithm for a telegraphic equation's numerical solution. The author of [3] proposed a method for solving the telegraphic hyperbolic equation with variable coefficients that combines finite differences and Haar wavelets. In [4,5], nonlinear telegraphic equation phenomena are discussed.

The Kamal integral method is used in [6] to handle delay differential equations. Differential-integral equations are resolved using modified Laplace series results and the Adomian polynomial decomposition method (LADM) in [7]. In [8], numerical approximations for systems of linear partial differential equations are obtained using the Laplace decomposition method and the Pade approximation. To solve differential equations of the Emden-Lane type, the Laplace decomposition technique is modified as described in [9].

For the purpose of resolving linear partial differential equations involving more than two independent variables, the Laplace transform substitution method is given in [10]. Using the Matlab programme, [11] investigates the effects of bioconvection and activation energy on the Maxwell equation in nano-fluid. [12] elaborates a Laplace transform-based analytical solution for the advection-diffusion problem in onedimensional semi-infinite media.

In order to solve differential equations with trigonometric coefficients in 2021, researchers developed a new transform [13,14]. [15] Is an example of the Kamal transform being used in thermal engineering to address temperature-related issues. In [16], a novel SEE transform is proposed that is effective for solving differential equations and differential equation systems. In [17], the Al-Zughair transform is used to present the solution to the differential equations for the moment Pareto distribution with logarithmic coefficients. Additionally, [18] introduces a new transform with a logarithmic kernel that facilitates the solution of both ordinary differential equations and differential equations with logarithmic coefficients.

Numerous facets of our daily lives make use of mathematical simulations and their solicitations. The populace growth equation is a crucial application that has a big impact (see [19]). Laplace transform was used by Daci [20] in a mathematical model for Albanian population projection, and its use in population growth analysis was experimentally confirmed. In relation to transfer functions of mechanical systems and nuclear physics, Sawant [21] talked about the practical uses of the Laplace transform in engineering domains.

By explaining how they relate to present discounted value and by demonstrating how they can be used to apply the time derivative property, Patil [22] provided an example of the application of Laplace transforms in finance. It should be acknowledged that Laplace transmute theory disturbs a key prerequisite of all manufacturing systems in mechanism systems.

In the partial fraction decomposition theory, the Laplace transform is commonly employed. [23] Uses the Laplace transform to solve fractional differential equations. Laplace transform theory has connections to other transforms and even to number theory, as demonstrated by [24,25] and Aleksandar [26].

This article discusses the fractional-Laplace integral transform and how it may be used to solve differential linear models, analyse electric circuits, and solve the heat conduction equation. The following is a definition of the fractional Laplace transform.

II. Material And Methods

A. Fractional Laplace Transform [27]

Integral Fractional Laplace transform of the function $f(t)$ for all $t \geq 0$ is defined as:

$$L_\alpha\{f(t)\} = \int_0^\infty e^{-v^{\frac{1}{\alpha}}t} f(t)dt = F\left(v^{\frac{1}{\alpha}}\right) \qquad (1)$$

Where α is real numbers and v is transformed variable. By choosing the values of parameter $\alpha = 1$ as arbitrary real number, we can get the Laplace transform.

B. Inverse Fractional Laplace Transform [27]

Inverse of integral Fractional Laplace transform of the function $F\left(v^{\frac{1}{\alpha}}\right)$ for all $0 < \alpha < 1$ is defined as:

$$L_\alpha^{-1}\left\{F\left(v^{\frac{1}{\alpha}}\right)\right\} = \frac{1}{2\pi i\alpha}\int_{a^\alpha - i\infty}^{a^\alpha - i\infty} e^{v^\alpha t} F\left(v^{\frac{1}{\alpha}}\right) v^{\left(\frac{1-\alpha}{\alpha}\right)} dv \qquad (2)$$

C. Time Derivative Rule [27]

If $L_\alpha\{f(t)\} = F(v^{\frac{1}{\alpha}})$ and all of its higher derivative are defined, then

$$L_\alpha\{f'(t)\} = \frac{1}{v^{\frac{1}{\alpha}}} F\left(v^{\frac{1}{\alpha}}\right) - f(0) \qquad (3)$$

$$L_\alpha\{f''(t)\} = \frac{1}{v^{\frac{2}{\alpha}}} F\left(v^{\frac{1}{\alpha}}\right) - \frac{1}{v^{\frac{1}{\alpha}}} f(0) - f'(0) \qquad (4)$$

Similarly,

$$L_\alpha\{f^n(t)\} = \frac{1}{v^{\frac{n}{\alpha}}} F\left(v^{\frac{1}{\alpha}}\right) - \frac{1}{v^{\frac{n-1}{\alpha}}} f(0) - \frac{1}{v^{\frac{n-2}{\alpha}}} f'(0) - \cdots - f^{n-1}(0) \qquad (5)$$

III. Results And Discussions

A. Fractional-Laplace Technique in Solving Simultaneous Differential Equations

Numerous mathematical models and challenges in engineering use simultaneous linear differential equations in addition to single differential equations. Fractional Laplace transforms can be used to solve these simultaneous equations.

Problem 1: Consider a particle that is travelling in a straight line. The following system of differential equations provide the coordinates of the particle at any moment t:

$$\frac{dy}{dx} + 2x = sin(2t) \quad (6)$$

$$\frac{dx}{dt} - 2y = cos(2t) \quad (7)$$

Given the initial conditions at $t = 0$, where $x = 1$ and $y = 0$, we can use Fractional Laplace transforms to demonstrate that the particle moves along the path described by the equation $4x^2 + 4xy + 5y^2 = 4$.

To begin, we take the Fractional Laplace transform of the given simultaneous system of equations:

$$L_\alpha\{\frac{dy}{dx}\} + 2L_\alpha\{x\} = L_\alpha\{sin(2t)\} \quad (8)$$

$$L_\alpha\{\frac{dx}{dt}\} - 2L_\alpha\{y\} = L_\alpha\{cos(2t)\} \quad (9)$$

We may solve equations (8) and (9) to get the Fractional Laplace-transformed expressions of the variables x and y by using the properties of Fractional Laplace transform and taking into account the beginning circumstances.

$$\left[v^{\frac{1}{\alpha}}Y\left(v^{\frac{1}{\alpha}}\right) - y(0)\right] + 2X\left(v^{\frac{1}{\alpha}}\right) = \frac{2}{v^{\frac{2}{\alpha}}+4}$$

or $$2X\left(v^{\frac{1}{\alpha}}\right) + v^{\frac{1}{\alpha}}X\left(v^{\frac{1}{\alpha}}\right) = \frac{2}{v^{\frac{2}{\alpha}}+4} \quad (10)$$

and $$\left[v^{\frac{1}{\alpha}}X\left(v^{\frac{1}{\alpha}}\right) - x(0)\right] - 2Y\left(v^{\frac{1}{\alpha}}\right) = \frac{v^{\frac{1}{\alpha}}}{v^{\frac{2}{\alpha}}+4}$$

or $$\left[v^{\frac{1}{\alpha}}X\left(v^{\frac{1}{\alpha}}\right)\right] - 2Y\left(v^{\frac{1}{\alpha}}\right) = \frac{v^{\frac{1}{\alpha}}}{v^{\frac{2}{\alpha}}+4} + 1 \quad (11)$$

To solve for $X(v^{\frac{1}{\alpha}})$ and $Y(v^{\frac{1}{\alpha}})$, we multiply equation (10) by v, equation (6) by 2, and take the difference.

$$Y\left(v^{\frac{1}{\alpha}}\right) = -\frac{2}{v^{\frac{2}{\alpha}}+4} \ ;$$

$$X\left(v^{\frac{1}{\alpha}}\right) = \frac{1}{2}\left[\frac{2}{v^{\frac{2}{\alpha}}+4} + \frac{2v^{\frac{1}{\alpha}}}{v^{\frac{2}{\alpha}}+4}\right].$$

and hence, by applying the inverse Fractional Laplace transform, we can obtain the solutions for $x(t)$ and $y(t)$.

$$y(t) = -sin2t \ ; \ x(t) = \frac{1}{2}(sin2t + cos2t)$$

Hence

$$2x = sin2t + cos2t \Rightarrow 2x + y = 2cos2t$$

$$\Rightarrow (2x + y)^2 = 4(1 - \sin^2 2t = 4 - 4y^2$$

$$\Rightarrow 4x^2 + 4xy + 5y^2 = 4$$

Hence, it can be concluded that the desired particle moves along the path described by the equation $4x^2 + 4xy + 5y^2 = 4$.

B. Fractional-Laplace in Equation of Heat Conduction [28]

The application of Fractional Laplace transform in solving the heat conduction equation can be demonstrated for a infinite solid ($x > 0$). Primarily, the solid is at $u = 0$, and at t=0, the initial condition $x = 0$ is upraised to a heat u_0 and preserved at u_0. The one-dimensional conduction equation is given by

$$c^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, x > 0, t > 0, \quad (12)$$

with initial conditions $u = 0$ when $t = 0$ $(x \geq 0)$ and $u = u_0$ when $x = 0$, $(t > 0)$.

Solution:

By multiplying the given partial equation by $e^{-v^{\frac{1}{\alpha}}t}$ and integrating with respect to t from 0 ∞, and utilizing the initial conditions, the equation can be transformed to;

$$c^2\frac{\partial^2 u}{\partial x^2} = v^{\frac{1}{\alpha}}\bar{u}(x, v^{\frac{1}{\alpha}}),$$

Where $\bar{u}(x, v^{\frac{1}{\alpha}})$ represents the Fractional Laplace transformed variable.

The second condition leads to $\bar{u} = \frac{u}{v^{\frac{1}{\alpha}}}$ $when\ x = 0$. The general solution of $c^2\frac{\partial^2 u}{\partial x^2} = v^{\frac{1}{\alpha}}\bar{u}\ (x, v^{\frac{1}{\alpha}})$ is given by;

$$\bar{u} = Ae^{\frac{x\sqrt{v^{\frac{1}{\alpha}}}}{c}} + Be^{-\frac{x\sqrt{v^{\frac{1}{\alpha}}}}{c}} \quad (13)$$

To ensure the solution remains finite as x approaches infinity, we set $A = 0$, resulting in $B = e^{-\frac{x\sqrt{v^{\frac{1}{\alpha}}}}{c}}$.

Further, $\bar{u} = \frac{u}{v^{\frac{1}{\alpha}}} \Rightarrow B = \frac{u_0}{v^{\frac{1}{\alpha}}}$.

Thus, the transformed variable $\bar{u}$ becomes $\frac{u_0}{v^{\frac{1}{\alpha}}}e^{-\frac{x\sqrt{v^{\frac{1}{\alpha}}}}{c}}$.

By taking the inverse Fractional Laplace transform, we obtain the solution as;

$$u = u_0\,(1 - erf(\frac{x}{2c\sqrt{t}})) \quad (14)$$

Where $erf(x)$ represents the error function defined as $erf(x) = \left(\frac{2}{\sqrt{\pi}}\right)\int_0^x e^{-t^2}dt$ [30].

Problem 2: Consider an extremely long string where one side at $x = 0$ is motionless, and the other end at $x = 0$ is given a translation (t) for $t > 0$. To determine the move mentation at a specific point of the rope at any time t, assuming the displacement $y(x,t)$ remains bounded, we have the partial differential equation:

$$\frac{c^2(\partial^2 u)}{\partial x^2} = \frac{\partial u}{\partial t} \quad \text{for } x > 0 \text{ and } t > 0 \qquad (15)$$

with boundary conditions $y(x,0) = 0$ and $\left(\frac{\partial}{\partial t}\right) y(x,0) = 0$, and the condition that $y(x,t)$ is bounded. Additionally, we have $u(0,t) = f(t)$.

Solution:

By multiplying the given partial differential equation by $e^{-v^{\frac{1}{\alpha}}t}$ and integrating with respect to t from 0 to ∞, we obtain:

$$v^{\frac{2}{\alpha}}\bar{y} - v^{\frac{1}{\alpha}}y(x,0) = c^2 \frac{\partial^2 \bar{y}}{\partial x^2} \qquad (16)$$

Considering the condition $y(x,0) = 0$, we get:

$$v^{\frac{2}{\alpha}}\bar{y} = c^2 \frac{\partial^2 \bar{y}}{\partial x^2}.$$

We have, $\bar{y} = F(v^{\frac{1}{\alpha}})$ at $x = 0$.

Since $\bar{y}(x, v^{\frac{1}{\alpha}})$ is bounded, the constant A must be zero and $B = F(v^{\frac{1}{\alpha}})$ based on $\bar{y} = F(v^{\frac{1}{\alpha}})$ at $x = 0$.

Hence,

$$\bar{y}(x, v^{\frac{1}{\alpha}}) = F\left(v^{\frac{1}{\alpha}}\right) e^{-\frac{x v^{\frac{1}{\alpha}}}{c}} \qquad (17)$$

Using the inversion formula, we find:

$$y = \left(\frac{1}{2\pi i}\right) \int_{a-i\infty}^{a+i\infty} F\left(v^{\frac{1}{\alpha}}\right) e^{\left(t-\frac{x}{c}\right)v^{\frac{1}{\alpha}}} \, dv$$

$$= f\left(t - \frac{x}{c}\right) \qquad (18)$$

C. Analysis of Electric Circuit via Fractional Laplace transform [29]

Let suppose a series circuit consisting of a resistor (R), inductor (L), and capacitor (C), connected with an electromotive power of voltage E with a switch. Applying Kirchhoff's rule, we obtain;

$$L\frac{di}{dt} + Ri + \frac{q}{c} = E \qquad (19)$$

Problem 3: If $R = 16\,\Omega$, $I = 3\,H$, and $C = 0.02\,F$ are connected in sequence with a source of 300 Volt (refer to figure). Initially, at $t = 0$, the capacitor is uncharged and the current is zero. To determine the charge and current at any time $t > 0$, we can use the equations and principles of circuit analysis.

$$L\frac{di}{dt} + Ri + \frac{q}{c} = E.$$

$i.e.$

$$\frac{d^2q}{dt^2} + 8\frac{dq}{dt} + 25q = 150. \qquad (20)$$

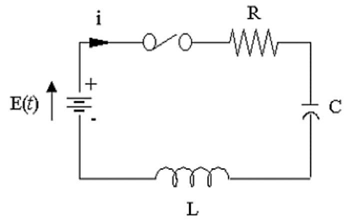

***Figure 1**:* R-L-C Series Circuit

Given the initial conditions $q = 0$ and $i = 0$ at time $t = 0$, we can use these conditions to solve for the charge and current at any subsequent time $t > 0$.

Applying Fractional Laplace transform yields the following equations:

$$L_\alpha\left[\frac{d^2q}{dt^2}\right] + 8L_\alpha\left[\frac{dq}{dt}\right] + 25L_\alpha[q] = 150L_\alpha[1]$$

$$v^{\frac{2}{\alpha}}Q\left(v^{\frac{1}{\alpha}}\right) - v^{\frac{1}{\alpha}}q(0) - q'(0) + 8\left(v^{\frac{1}{\alpha}}Q\left(v^{\frac{1}{\alpha}}\right) - q(0)\right) + 25Q\left(v^{\frac{1}{\alpha}}\right) = 150\frac{1}{v^{\frac{1}{\alpha}}}$$

Applying the initial conditions, we obtain the following results:

$$v^{\frac{1}{\alpha}}Q\left(v^{\frac{1}{\alpha}}\right) + 8\left(v^{\frac{1}{\alpha}}Q\left(v^{\frac{1}{\alpha}}\right)\right) + 25Q\left(v^{\frac{1}{\alpha}}\right) = 150\frac{1}{v^{\frac{1}{\alpha}}}$$

$$Q\left(v^{\frac{1}{\alpha}}\right) = \frac{150}{v^{\frac{1}{\alpha}}\left(v^{\frac{2}{\alpha}} + 8v^{\frac{1}{\alpha}} + 25\right)} \qquad (21)$$

Taking the inverse transform on both sides yields:

$$q = 6 - 6e^{-4t}cos3t - 8e^{-4t}sin3t$$

and $\quad i = te^{-4t}sin3t \qquad (22)$

These expressions represent the charge q and current i at any time $t > 0$, respectively.

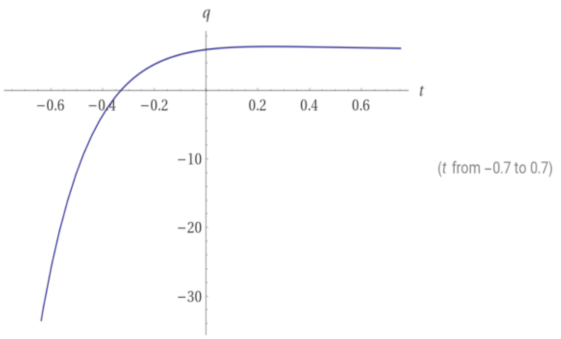

Figure 2: Graph presenting the charge q when t is from $-0.7\ to\ 0.7$.

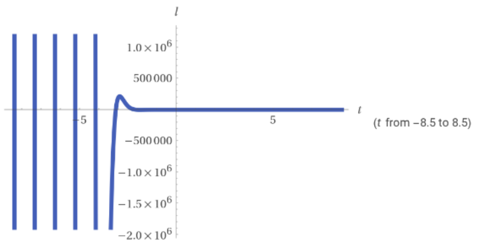

Figure 3: Graph presenting the current i when time t from $-8.5\ to\ 8.5$.

IV. CONCLUSION

In conclusion, the Fractional Laplace transform has proven useful in a variety of scientific and engineering fields by being used to the solution of heat conduction equations, simultaneous differential equations, and electric circuit analysis. It is clear from the examples given that the fractional Laplace transform offers strong mathematical instruments for deriving solutions to challenging issues. Researchers have used the fractional Laplace transform to examine time distribution in materials and create precise predictions regarding heat flow in the setting of heat conduction.

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