Characteristic Zero Complex in Event of a Partitioning (4,4,3)

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Abstract*— *A project's aim is to investigate use the complex of characteristic '0' in the event of a partitioning (4, 4, 3) using the idea of mapping Cone and the concepts divided power the place polarization, illustrations, and Capelli (1)in this partitioning.

Keywords*— *Mapping Cone, Divided Power Algebra, Place Polarization, Resolution of Weyl Module.

I. INTRODUCTION

Let R be abelian, F be a free R-module, $D_b F$ be the divided power degree b . Think about layouts

$$\partial_{21}^{(\kappa)} : D_{\rho+\kappa} \otimes D_{q-\kappa} \otimes D_r \rightarrow D_\rho \otimes D_q \otimes D_r$$

The layouts is a place polarization from $D_{\rho+\kappa}$ to $D_{q-\kappa}$ and

$$\partial_{32}^{(\kappa)} : D_\rho \otimes D_{q+\kappa} \otimes D_{r-\kappa} \rightarrow D_\rho \otimes D_q \otimes D_r$$

The layouts is a place polarization from $D_{q+\kappa}$

The characteristic zero complex in the event of a partitioning (2, 2, 2) were also conducted a survey either by reviewers through [1], [2], [3], [5], [6] and [7]. Feature the characteristic '0' complex as shown in illustration event of some partitioning in [8]. [9] features the concepts and the precision of either the Weyl resolution in the case of partition (8,7) and (8,7,3).

We discuss during this case paper the characteristic '0' resolution in the event of a partitioning (4, 4, 3) utilizing the concept mapping con last but not least part, after we have shown the phrases of the characteristic '0' complex in the following portion of the same division. The layouts $\partial_{b_i}^{(\kappa)}$ that is to say the divided power the polarization ∂_{b_i} ; i is really lower b capelli (ONE) [10].

II. CONDITIONS CHARACTERISTIC ZERO COMPLEX IN EVENT A PARTITIONING (4,4,3)

The stances of the conditions of the complex are dictate by the duration of the permutation they match to [11].

In the event of a partitioning (4, 4, 3) we control the pursuit matrix;

$$\begin{pmatrix} D_4 & D_3 & D_1 \\ D_5 & D_4 & D_2 \\ D_6 & D_6 & D_3 \end{pmatrix}$$

Then the characteristic zero complex having the following correspondences between its The Conditions;

$$D_4 \otimes D_4 \otimes D_3 \leftrightarrow (id)$$
$$D_4 \otimes D_5 \otimes D_2 \leftrightarrow (23)$$
$$D_5 \otimes D_3 \otimes D_3 \leftrightarrow (12)$$
$$D_6 \otimes D_3 \otimes D_2 \leftrightarrow (123)$$
$$D_5 \otimes D_5 \otimes D_1 \leftrightarrow (132)$$
$$D_6 \otimes D_4 \otimes D_1 \leftrightarrow (13)$$

Consequently the characteristic zero resolution in the event of a partitioning (4, 4, 3) possesses the expression;

$$D_6 \otimes D_4 \otimes D_1 \longrightarrow \begin{matrix} D_6 \otimes D_3 \otimes D_2 \\ \oplus \\ D_5 \otimes D_5 \otimes D_1 \end{matrix} \longrightarrow \begin{matrix} D_4 \otimes D_5 \otimes D_2 \\ \oplus \\ D_5 \otimes D_3 \otimes D_3 \end{matrix} \longrightarrow D_4 \otimes D_4 \otimes D_3$$

III. THE SCHEMATIC FOR THE CHARACTERISTIC ZERO COMPLEX IN THE EVENT OF A PARTITIONING (4, 4, 3)

Look at the approach illustration ;

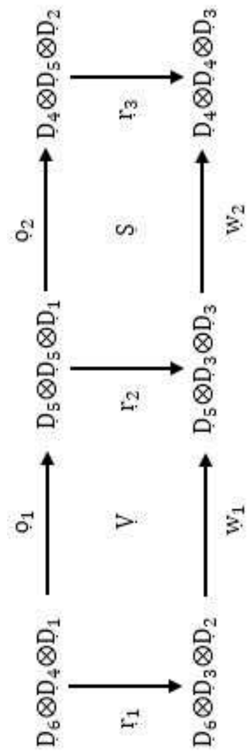

Figure (1)

We get acquainted
$ọ_1: Ḍ_6 \otimes Ḍ_4 \otimes Ḍ_1 \rightarrow Ḍ_5 \otimes Ḍ_5 \otimes Ḍ_1$ *as*
$ọ_1(\mathscr{v})ọ_1 = \dot{\partial}_{21}(\mathscr{v}); where\ \mathscr{v} \in Ḍ_6 \otimes Ḍ_4 \otimes Ḍ_1$

$ṛ_1: Ḍ_6 \otimes Ḍ_4 \otimes Ḍ_1 \rightarrow Ḍ_6 \otimes Ḍ_3 \otimes Ḍ_2$ *as*
$ṛ_1(\mathscr{v}) = \dot{\partial}_{32}(\mathscr{v}); where\ \mathscr{v} \in Ḍ_6 \otimes Ḍ_4 \otimes Ḍ_1$
And
$ṛ_2: Ḍ_5 \otimes Ḍ_5 \otimes Ḍ_1 \rightarrow Ḍ_5 \otimes Ḍ_3 \otimes Ḍ_3$ *as*
$ṛ_2(\mathscr{v}) = \dot{\partial}_{32}^{(2)}(\mathscr{v}); where\ \mathscr{v} \in Ḍ_5 \otimes Ḍ_5 \otimes Ḍ_1$

Now, we have the authority to familiarize the layout
$ẉ_1: Ḍ_6 \otimes Ḍ_3 \otimes Ḍ_2 \rightarrow Ḍ_5 \otimes Ḍ_3 \otimes Ḍ_3$
Which of the following constructs the layout Ṿ commutative
$ẉ_{1\circ}ṛ_1 = ṛ_{2\circ}ọ_1$
It implies
$ẉ_{1\circ}\dot{\partial}_{32} = \dot{\partial}_{32}^{(2)}{}_\circ\dot{\partial}_{21}$
By utilizing Capelli (1)
$\dot{\partial}_{32}^{(2)}{}_\circ\dot{\partial}_{21} = \dot{\partial}_{21\circ}\dot{\partial}_{32}^{(2)} + \dot{\partial}_{32\circ}\dot{\partial}_{31}$
$= {}^1\!/_2\, \dot{\partial}_{21\circ}\dot{\partial}_{32}\dot{\partial}_{32} + \dot{\partial}_{32\circ}\dot{\partial}_{31}$
$= [{}^1\!/_2\, \dot{\partial}_{21\circ}\dot{\partial}_{32} + \dot{\partial}_{31}]_\circ\dot{\partial}_{32}$
$\Rightarrow\ ẉ_1 = [{}^1\!/_2\, \dot{\partial}_{21\circ}\dot{\partial}_{32} + \dot{\partial}_{31}]$
We get acquainted the layout
$ẉ_2: Ḍ_5 \otimes Ḍ_3 \otimes Ḍ_3 \rightarrow Ḍ_4 \otimes Ḍ_4 \otimes Ḍ_3$ *as*
$ẉ_2(\mathscr{v}) = \dot{\partial}_{12}(\mathscr{v}); where\ \mathscr{v} \in Ḍ_5 \otimes Ḍ_3 \otimes Ḍ_3$
And
$ṛ_3: Ḍ_4 \otimes Ḍ_5 \otimes Ḍ_2 \rightarrow Ḍ_4 \otimes Ḍ_4 \otimes Ḍ_3$ *as*
$ṛ_3(\mathscr{v}) = \dot{\partial}_{32}(\mathscr{v}); where\ \mathscr{v} \in Ḍ_4 \otimes Ḍ_5 \otimes Ḍ_2$

We need to get to know UU in order to create the Ṣ commute layout.
$ọ_2: Ḍ_5 \otimes Ḍ_5 \otimes Ḍ_1 \rightarrow Ḍ_4 \otimes Ḍ_5 \otimes Ḍ_2$
$ṛ_{3\circ}ọ_2 = ẉ_{2\circ}ṛ_2$
$\dot{\partial}_{32\circ}ọ_2 = \dot{\partial}_{21\circ}\dot{\partial}_{32}^{(2)}$
By utilizing Capelli (1)
$\dot{\partial}_{12\circ}\dot{\partial}_{32}^{(2)} = \dot{\partial}_{32}^{(2)}{}_\circ\dot{\partial}_{21} - \dot{\partial}_{32\circ}\dot{\partial}_{31}$

$= {}^1\!/_2\, \dot{\partial}_{32}\dot{\partial}_{32\circ}\dot{\partial}_{21} - \dot{\partial}_{32\circ}\dot{\partial}_{31}$

$= \dot{\partial}_{32\circ}[{}^1\!/_2\, \dot{\partial}_{32\circ}\dot{\partial}_{21} - \dot{\partial}_{31}]$

$\Rightarrow ọ_2 = [{}^1\!/_2\, \dot{\partial}_{32\circ}\dot{\partial}_{21} - \dot{\partial}_{31}]$

Think about the following schematic layout:

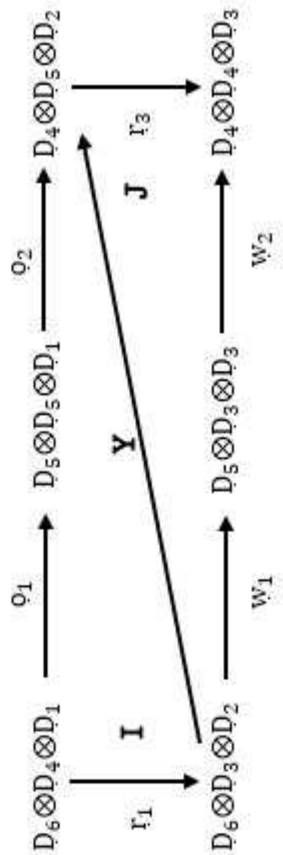

Figure (2)

Familiarize
$Y: Ḍ_6 \otimes Ḍ_3 \otimes Ḍ_2 \rightarrow Ḍ_4 \otimes Ḍ_5 \otimes Ḍ_2$ *as*
$Y(\mathscr{v}) = \dot{\partial}_{21}^{(2)}(\mathscr{v}); where\ \mathscr{v} \in Ḍ_6 \otimes Ḍ_3 \otimes Ḍ_2$

Proposition (3.1):
The schematic I in figure (2) is commute.
Proof:
To illustrate the schematic I is commute, We must illustrate
$ọ_{2\circ}ọ_1 = Y_\circ ṛ_1$

$$ọ_{2\circ}ọ_1 = [{}^1\!/_2\, \dot\partial_{32\circ}\dot\partial_{21} - \dot\partial_{31}]_\circ\dot\partial_{21}$$
$$= \dot\partial_{32\circ}\dot\partial_{21}^{(2)} - \dot\partial_{31\circ}\dot\partial_{21}$$
$$= \dot\partial_{21}^{(2)}{}_\circ\dot\partial_{32}$$
$$= \Upsilon_\circ ṛ_1$$

Proposition (3.2):

The schematic J in figure (2) is commute.

Proof:

To illustrate the schematic J is commute, we must illustrate

$$ẉ_{2\circ}ẉ_1 = ṛ_{3\circ}\Upsilon$$
$$ẉ_{2\circ}ẉ_1 = \dot\partial_{12\circ}[{}^1\!/_2\, \dot\partial_{21\circ}\dot\partial_{32} + \dot\partial_{31}]$$
$$= \dot\partial_{32\circ}\dot\partial_{21}^{(2)} + \dot\partial_{21\circ}\dot\partial_{31}$$
$$= \dot\partial_{32\circ}\dot\partial_{21}^{(2)}$$
$$= ṛ_{3\circ}\Upsilon$$

Ultimately, we familiarize ourselves with the layouts $\sigma_{\mathbb{1}}, \sigma_{\mathbb{2}}$ *and* $\sigma_{\mathbb{3}}$:

$$\sigma_{\mathbb{3}}(\dot x) = \big(ọ_1(\dot x), ṛ_1(\dot x)\big)\ ;\ \forall \dot x \in Ḍ_6 \otimes Ḍ_4 \otimes Ḍ_1$$

$$\sigma_{\mathbb{2}}(\dot x_1, \dot x_2) = \big(ọ_2(\dot x_1) - \Upsilon(\dot x_2), ẉ_1(\dot x_2) - ṛ_2(\dot x_1)\big)\ ;\ \forall \dot x \in Ḍ_6 \otimes Ḍ_3 \otimes Ḍ_2 \oplus Ḍ_5 \otimes Ḍ_5 \otimes Ḍ_1$$

$$\sigma_{\mathbb{1}}(\dot x_1, \dot x_2) = \big(ṛ_3(\dot x_1) + ẉ_2(\dot x_2)\big)\ ;\ \forall \dot x \in Ḍ_4 \otimes Ḍ_5 \otimes Ḍ_2 \oplus Ḍ_5 \otimes Ḍ_3 \otimes Ḍ_3$$

Where

$$\sigma_{\mathbb{3}}: Ḍ_6\otimes Ḍ_4\otimes Ḍ_1 \longrightarrow \begin{matrix} Ḍ_6\otimes Ḍ_3\otimes Ḍ_2 \\ \oplus \\ Ḍ_5\otimes Ḍ_5\otimes Ḍ_1 \end{matrix}$$

$$\sigma_{\mathbb{2}}: \begin{matrix} Ḍ_6\otimes Ḍ_3\otimes Ḍ_2 \\ \oplus \\ Ḍ_5\otimes Ḍ_5\otimes Ḍ_1 \end{matrix} \longrightarrow \begin{matrix} Ḍ_4\otimes Ḍ_5\otimes Ḍ_2 \\ \oplus \\ Ḍ_5\otimes Ḍ_3\otimes Ḍ_3 \end{matrix}$$

$$\sigma_{\mathbb{1}}: \begin{matrix} Ḍ_4\otimes Ḍ_5\otimes Ḍ_2 \\ \oplus \\ Ḍ_5\otimes Ḍ_3\otimes Ḍ_3 \end{matrix} \longrightarrow Ḍ_4\otimes Ḍ_4\otimes Ḍ_3$$

Proposition (3.3): The sequence

$$0 \longrightarrow Ḍ_6\otimes Ḍ_4\otimes Ḍ_1 \xrightarrow{\sigma_{\mathbb{3}}} \begin{matrix} Ḍ_6\otimes Ḍ_3\otimes Ḍ_2 \\ \oplus \\ Ḍ_5\otimes Ḍ_5\otimes Ḍ_1 \end{matrix} \xrightarrow{\sigma_{\mathbb{2}}} \begin{matrix} Ḍ_4\otimes Ḍ_5\otimes Ḍ_2 \\ \oplus \\ Ḍ_5\otimes Ḍ_3\otimes Ḍ_3 \end{matrix} \xrightarrow{\sigma_{\mathbb{1}}} Ḍ_4\otimes Ḍ_4\otimes Ḍ_3$$

Is complex.

Proof:

It is well recognized among the familiar that $\dot\partial_{21}$ *and* $\dot\partial_{32}$ are injective [12], yet we have $\sigma_{\mathbb{3}}$ injective utilizing 'Capelli (1)'. Now

$$(\sigma_{\mathbb{2}\circ}\sigma_{\mathbb{3}})(\dot x) = \sigma_{\mathbb{2}}(ọ_1(\dot x), ṛ_1(\dot x))$$
$$= \sigma_{\mathbb{2}}(\dot\partial_{21}(\dot x), \dot\partial_{32}(\dot x))$$
$$= \big(ọ_2(\dot x_1) - \Upsilon(\dot x_2), ẉ_1(\dot x_2) - ṛ_2(\dot x_2)\big)\ (\dot\partial_{21}(\dot x), \dot\partial_{32}(\dot x))$$
$$= \Big(ọ_2\big(\dot\partial_{21}(\dot x)\big) - \Upsilon\big(\dot\partial_{32}(\dot x)\big), ẉ_1\big(\dot\partial_{32}(\dot x)\big) - ṛ_2\big(\dot\partial_{21}(\dot x)\big)\Big)$$

$$ọ_2\big(\dot\partial_{21}(\dot x)\big) - \Upsilon\big(\dot\partial_{32}(\dot x)\big)$$
$$= [{}^1\!/_2\, \dot\partial_{32\circ}\dot\partial_{21} - \dot\partial_{31}]_\circ\dot\partial_{21}(\dot x) - \dot\partial_{21}^{(2)}{}_\circ\dot\partial_{32}(\dot x)$$
$$= [\dot\partial_{21}^{(2)}{}_\circ\dot\partial_{32} - \dot\partial_{21\circ}\dot\partial_{31} - \dot\partial_{21}^{(2)}{}_\circ\dot\partial_{32}](\dot x)$$
$$= [\dot\partial_{21}^{(2)}{}_\circ\dot\partial_{32} + \dot\partial_{21\circ}\dot\partial_{31} - \dot\partial_{21\circ}\dot\partial_{31} - \dot\partial_{21}^{(2)}{}_\circ\dot\partial_{32}](\dot x)$$
$$= 0$$

$$ẉ_1\big(\dot\partial_{32}(\dot x)\big) - ṛ_2\big(\dot\partial_{21}(\dot x)\big)$$
$$= [{}^1\!/_2\, \dot\partial_{21\circ}\dot\partial_{32} + \dot\partial_{31}]_\circ\dot\partial_{32}(\dot x) - \dot\partial_{32}^{(2)}{}_\circ\dot\partial_{21}(\dot x)$$
$$= [\dot\partial_{32}^{(2)}{}_\circ\dot\partial_{21} + \dot\partial_{32\circ}\dot\partial_{31} - \dot\partial_{32}^{(2)}{}_\circ\dot\partial_{21}](\dot x)$$
$$= [\dot\partial_{32}^{(2)}{}_\circ\dot\partial_{21} - \dot\partial_{32\circ}\dot\partial_{31} + \dot\partial_{32\circ}\dot\partial_{31} - \dot\partial_{32}^{(2)}{}_\circ\dot\partial_{21}](\dot x)$$
$$= 0$$

$$\Rightarrow (\sigma_{\mathbb{2}\circ}\sigma_{\mathbb{3}})(\dot x) = 0$$

And

$$(\sigma_{\mathbb{1}} \circ \sigma_{\mathbb{2}})(\dot{x}_1, \dot{x}_2) = \sigma_{\mathbb{1}}(\underset{\cdot}{o}_2(\dot{x}_1) - \Upsilon(\dot{x}_2), \underset{\cdot}{w}_1(\dot{x}_2) - \underset{\cdot}{r}_2(\dot{x}_1))$$

$$= \sigma_{\mathbb{1}}([{}^1/_2\, \dot{\partial}_{32} \circ \dot{\partial}_{21} - \dot{\partial}_{31}](\dot{x}_1) - \dot{\partial}_{21}^{(2)}(\dot{x}_2), [{}^1/_2\, \dot{\partial}_{32} \circ \dot{\partial}_{21} + \dot{\partial}_{31}](\dot{x}_2) - \dot{\partial}_{32}^{(2)}(\dot{x}_1))$$

$$= \dot{\partial}_{32} \circ \left([{}^1/_2\, \dot{\partial}_{32} \circ \dot{\partial}_{21} - \dot{\partial}_{31}](\dot{x}_1) - \dot{\partial}_{21}^{(2)}(\dot{x}_2)\right) + \dot{\partial}_{21} \circ ([{}^1/_2\, \dot{\partial}_{32} \circ \dot{\partial}_{21} + \dot{\partial}_{31}](\dot{x}_2) - \dot{\partial}_{32}^{(2)}(\dot{x}_1))$$

$$= ([\dot{\partial}_{32}^{(2)} \circ \dot{\partial}_{21} - \dot{\partial}_{32} \circ \dot{\partial}_{31}](\dot{x}_1) - \dot{\partial}_{32} \circ \dot{\partial}_{21}^{(2)}(\dot{x}_2) + [\dot{\partial}_{21}^{(2)} \circ \dot{\partial}_{32} + \dot{\partial}_{21} \circ \dot{\partial}_{31}](\dot{x}_2) - \dot{\partial}_{21} \circ \dot{\partial}_{32}^{(2)}(\dot{x}_1))$$

$$= ([\dot{\partial}_{32}^{(2)} \circ \dot{\partial}_{21} - \dot{\partial}_{32} \circ \dot{\partial}_{31} - \dot{\partial}_{21} \circ \dot{\partial}_{32}^{(2)}](\dot{x}_1) + [\dot{\partial}_{21}^{(2)} \circ \dot{\partial}_{32} + \dot{\partial}_{21} \circ \dot{\partial}_{31} - \dot{\partial}_{32} \circ \dot{\partial}_{21}^{(2)}](\dot{x}_2)$$

$$= ([\dot{\partial}_{21} \circ \dot{\partial}_{32}^{(2)} + \dot{\partial}_{32} \circ \dot{\partial}_{31} - \dot{\partial}_{32} \circ \dot{\partial}_{31} - \dot{\partial}_{21} \circ \dot{\partial}_{32}^{(2)}](\dot{x}_1) + [\dot{\partial}_{32} \circ \dot{\partial}_{21}^{(2)} - \dot{\partial}_{21} \circ \dot{\partial}_{31} + \dot{\partial}_{21} \circ \dot{\partial}_{31} - \dot{\partial}_{32} \circ \dot{\partial}_{21}^{(2)}](\dot{x}_2))$$

$$= 0$$
$$\Rightarrow (\sigma_{\mathbb{1}} \circ \sigma_{\mathbb{2}})(\dot{x}_1, \dot{x}_2) = 0$$

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