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Application of the Two Rowed Weyl Module in the Case of Partitions (7,5) and Skew-Partition (7, 5) / (1, 0)

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Abstract— *The main purpose of this paper is to study the application of Weyl module and it's resolution in the case of two rows which way be specified in the partitions (7, 5) and skew-partition (7, 5)/(1, 0), using the homological way (i.e. the contracting homotopy, place polarization.*

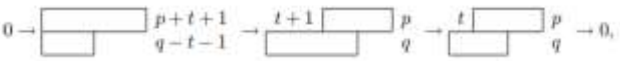
Keywords— Weyl module, skew- shape, graded contracting, homotopy,divided power algebra.

I. INTRODUCTION

Let Ɍ be a commutative ring with identity, F be a free Ɍ-module where D_n F is the divided power of dgree n.[1]

The issue of resolving Schur modules in terms of direct sums of tensor products of exterior powers was first studied by Akin and Buchsbaum in the early 1980 (hence referred to as A&B) (or fundamental representations).

Using the "basic precise sequence" for two-row Schur modules (more on this will be covered in a later section)



An induction argument on the amount of overlaps between the two rows led to a description of the desired result, and a Koszul-like complex (which we named a "arithmetic Koszul Complex" and which will be discussed in section 4.1) what kind of resolution [2] .

As stated in the introduction, we'll briefly explain how letter-place methods can be used to perform "intrinsic" module theory In particular, one can define the equivariant filtration on, say, a two-rowed skew shape using the letter-place algebra, leading to the Pieri decomposition of the appropriate Weyl module [4], [5]. For assuming we have the skew-shaped, two-rowed:

This is the image of $D_p \otimes D_q$ under the Weyl map, as previously stated [6], and the letter-place algebra for $D_p \otimes D_q$ is the set of all double standard tableaux $\left\{\begin{pmatrix} w & 1^{(p)}2^{(l)} \\ w' & 2^{(q-l)} \end{pmatrix}\right\}$ with $q \le p+l$, and where w and w' are words in the letter alphabet (In this situation, only the numbers 1 and 2 in their customary order make up the place alphabet).

Weyl module $K_{\lambda/\mu}\mathcal{F} = Im\,(\,d'_{\lambda/\mu})$

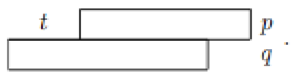
Where $d'_{\lambda/\mu}$ and $d'_{\lambda/\mu}: Đ_{\lambda/\mu}\mathcal{F} \longrightarrow \Lambda_{\lambda/\mu}\mathcal{F}$ is the Weyl map whose images will be Called "Weyl module "

$\sum Đ_{p+k} \otimes Đ_{q-k} \xrightarrow{\square} Đ_p \otimes Đ_q$ [7].

And by using letter place, the maps will be explained as follows :

$$\begin{pmatrix} W \\ W' \end{pmatrix}\begin{vmatrix} 1^{(p+k)} \\ 2^{(q-k)} \end{vmatrix} \xrightarrow{\partial_{21}^{(k)}} \begin{pmatrix} W \\ W' \end{pmatrix}\begin{vmatrix} 1^{(p)}2^{(k)} \\ 2^{(q-k)} \end{vmatrix} \longmapsto \sum_W \begin{pmatrix} W\ W_{(1)} \\ W'W_{(2)} \end{pmatrix}\begin{vmatrix} (t+1)'(t+2)' \ldots\ldots (p+t)' \\ 1'2'3' \ldots q' \end{vmatrix} \quad \ldots(2)$$

Where $w \otimes w' \in D_{p+k} \otimes D_{q-k}$, $\square = \sum_{k=t+1}^{q} \partial_{21}^{(k)}$ is the box map ,

(A) 

And $d'_{\lambda/\mu} = \partial_{1'2}\,\partial_{(p+t)'1'}$, is the composition of place polarization, from positive places $\{1,2\}$ to negative places $\{1', 2', \ldots, (p+t)'\}$.

also ,as shown in (2) , $\square$ is deliver a component $x \otimes y$ of $Đ_{p+k} \otimes Đ_{q-k}$ to

$\sum x_p \otimes x'_k y$, where is $\sum x_p \otimes x'_k$ is the element of the diagonal of x in $Đ_p \otimes Đ_q$.

divided power element $z_{21}^{(k)}$ of degree k of the free generator $(\mathcal{Z}_{21})$ acts on $Đ_{p+k} \otimes Đ_{q-k}$ by place polarization of degree k from place (1) to place (2)

The ''graded'' algebra with identity. A = $Đ(\mathcal{Z}_{21})$ acts on the graded module $\mathcal{M}$= $Đ_{p+k} \otimes Đ_{q-k} = \sum \mathcal{M}_{q-k}$.

Hence, $\mathcal{M}$ is a graded -module, where $w = z_{21}^{(k)} \in A$ and $\mathbb{v} \in Đ_{\beta 1} \otimes Đ_{\beta 2}$, So we have:

$w\,(\mathbb{v}) = z_{21}^{(k)}\,(\mathbb{v}) = \partial_{21}^{(k)}(\mathbb{v})$

If we take (t^{+}) graded strand of degree q.

$$\mathcal{M}_{\bullet} : 0 \to \mathcal{M}_{q-t} \xrightarrow{\partial_{21}} \ldots \to \mathcal{M}_{e} \xrightarrow{\partial_{s}} \mathcal{M}_{1} \xrightarrow{\partial_{s}} \mathcal{M}_{0},$$

Of the normalized *B*ar complex, Bar ($\mathcal{M}$, A, ;•),where $S = \{x\}$. Some important standard concepts which are needed in our work are illustrated.

The maps $\{S_i\}$ are defined as follows: [2]

$$S_0 : Đ_p \otimes Đ_q \longrightarrow \sum_{k>0} z^{(t+k)} \text{ x } Đ_{p+t+k} \otimes Đ_{q-t-k}$$

$$\begin{pmatrix} W \\ W' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(p)} 2^{(k)} \\ 2^{(q-k)} \end{matrix}\right. \longrightarrow \left\{ \begin{matrix} z_{21}^{(k)} \text{ x} \left| \begin{pmatrix} W & 1^{(p+k)} \\ W' & 2^{(q-k)} \end{pmatrix} \right. & ;\text{if } k \leq t \\ 0 & ;\text{if } k > t \end{matrix} \right\}$$

And for the higher dimensionses as:

$$s_{t-1} : \sum k_i > 0\ z_{21}^{(t+k_1)} \text{x } z_{21}^{(k_2)} \text{x} \ldots z_{21}^{(k_{t-1})} \text{x } Đ_{p+t+|K|} \otimes Đ_{q-t-|K|}$$

$$z_{21}^{(t+k_1)} \text{x } z_{21}^{(k_2)} \ldots z_{21}^{(k_{t-1})} \quad x \quad Đ_{p+t+|K|} \otimes \quad Đ_{q-t-|K|}$$

$$\to z_{21}^{(t+k_1)} \text{x } z_{21}^{(k_2)} \text{x} \ldots z_{21}^{(k_{t-1})} \text{x} \begin{pmatrix} W & 1^{(p+t+k)} 2^{(v)} \\ W' & 2^{(q-t-k)} \end{pmatrix} \to$$

$$\begin{cases} z_{21}^{(t+k_1)} \text{x } z_{21}^{(k_2)} \text{x} \ldots z_{21}^{(k_{t-1})} \text{x} z_{21}^{(v)} \text{x} \begin{pmatrix} W & 1^{(p+t+k)} 2^{(v)} \\ W' & 2^{(q-t-k)} \end{pmatrix} & ;\text{if } v > 0 \\ 0 & ;\text{if } v = 0 \end{cases}$$

Where is the modules of the resolution were we define the terms as:-

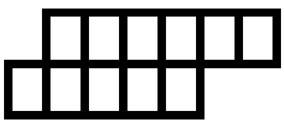
($\mathcal{M}_i$) for ($i = 0,1,\ldots\ldots q-t$) ,with $\mathcal{M}_0 = Đ_p \otimes Đ_q$,

$\mathcal{M}_i = z_{21}^{(t+k_1)} \text{x } z_{21}^{(k_2)} \text{x} \ldots z_{21}^{(k_i)} \text{x } Đ_{p+t+|K|} \otimes Đ_{q-t-|K|}$; $for\ i \geq 1$ [2].

The resolution of Weyl module in the subject of two_rowed skew_shape (p +t,q)/(t,0) studied in [8], while another study [3] and [9] exhibited the terms and the exactness of the Weyl resolution in the case of skew_shape (8,8)/(1,0) and two rows.

II. THE RESULTS OF THE CASE (7,5)/(1,0)

In this section, we find the term Weyl module resolution in the cases of a skew partition (7,5) / (1,0) and give the proof of its exactness .



2.1 Resolution of Weyl module in the case of partition (7,5)

In this section we study exhibited the terms and the exactness of the Weyl resolution.

2.2 The terms of Weyl module resolution in the case of partition (7,5)

These terms illustrated as follows:

$M_{0=}\ D_7 \otimes D_5$

$M_{1=}\ z_{21}^{(1)} X \quad D_8 \otimes D_4 \oplus Z_{21}^{(2)} D_9 \otimes D_3 \oplus Z_{21}^{(3)} X D_{10} \otimes D_2 \oplus Z_{21}^{(4)} X$
$D_{11} \otimes D_1 \oplus Z_{21}^{(5)} X\ D_{12} \otimes D_0$

$M_{2=}\ z_{21}^{(1)} X\ z_{21}^{(1)} X D_9 \otimes D_3 \oplus Z_{21}^{(2)} X\ z_{21}^{(1)} X D_{10} \otimes D_2 \oplus Z_{21}^{(1)} X z_{21}^{(2)} X D_{10}$
$\otimes D_2 \oplus z_{21}^{(3)} X\ z_{21}^{(1)} X D_{11} \otimes D_1 \oplus z_{21}^{(1)} X\ z_{21}^{(3)} X\ D_{11} \otimes D_1 \oplus z_{21}^{(2)} X\ z_{21}^{(2)} D_{11}$
$\otimes D_1 \oplus z_{21}^{(4)} X\ z_{21}^{(1)} D_{12} \otimes D_0 \oplus z_{21}^{(1)} X\ z_{21}^{(4)} D_{12} \otimes D_0$
$\oplus z_{21}^{(2)} X\ z_{21}^{(3)} D_{12} \otimes D_0 \oplus z_{21}^{(3)} X\ z_{21}^{(2)} D_{12} \otimes D_0$

$M_{3=}\ z_{21}^{(1)} X\ z_{21}^{(1)} X\ z_{21}^{(1)} X\ D_{10} \otimes D_2 \oplus z_{21}^{(2)} X\ z_{21}^{(1)} X\ z_{21}^{(1)} X D_{11} \otimes D_1$
$z_{21}^{(1)} X\ z_{21}^{(2)} X\ z_{21}^{(1)} X D_{11} \otimes D_1 \oplus z_{21}^{(1)} X\ z_{21}^{(1)} X\ z_{21}^{(2)} X D_{11} \otimes D_1 \oplus$
$z_{21}^{(3)} X\ z_{21}^{(1)} X\ z_{21}^{(1)} X D_{12} \otimes D_0 \oplus z_{21}^{(1)} X\ z_{21}^{(3)} X\ z_{21}^{(1)} X D_{12} \otimes D_0 \oplus$
$z_{21}^{(1)} X\ z_{21}^{(1)} X\ z_{21}^{(3)} X D_{12} \otimes D_0 \oplus z_{21}^{(2)} X\ z_{21}^{(2)} X\ z_{21}^{(1)} X D_{12} \otimes D_0 \oplus$
$z_{21}^{(2)} X\ z_{21}^{(1)} X\ z_{21}^{(2)} X D_{12} \otimes D_0 \oplus z_{21}^{(1)} X\ z_{21}^{(2)} X\ z_{21}^{(2)} X D_{12} \otimes D_0 \oplus$

$M_{4=} z_{21}^{(1)} X\ z_{21}^{(1)} X z_{21}^{(1)} X\ z_{21}^{(1)} X D_{11} \otimes D_1 \oplus z_{21}^{(2)} X\ z_{21}^{(1)} X$
$z_{21}^{(1)} X\ z_{21}^{(1)} X D_{12} \otimes D_0 \oplus\ z_{21}^{(1)} X\ z_{21}^{(2)} X\ z_{21}^{(1)} X\ z_{21}^{(1)} X\ D_{12} \otimes D_0 \oplus$
$z_{21}^{(1)} X\ z_{21}^{(1)} X\ z_{21}^{(2)} X\ z_{21}^{(1)} X\ D_{12} \otimes D_0 \oplus\ z_{21}^{(1)} X\ z_{21}^{(1)} X\ z_{21}^{(1)} X\ z_{21}^{(2)} X$
$D_{12} \otimes D_0$

$M_{5=}\ z_{21}^{(1)} X\ z_{21}^{(1)} X\ z_{21}^{(1)} X\ z_{21}^{(1)} X\ z_{21}^{(1)} X\ D_{12} \otimes D_0$

2.2 The Exactness of Weyl module Resolution in the Case of Partition (7,5)

Define the map by:

$$s_0 \left(\begin{pmatrix} W & 1^{(7)}\, 2^{(k)} \\ W' & 2^{(5-k)} \end{pmatrix} \right) =$$

$$\begin{cases} 0 & ;if\ \ k \leq 0 \\ z_{21}^{(k)}\ \chi \begin{pmatrix} W & 1^{(7+k)} \\ W' & 2^{(5-k)} \end{pmatrix} & ;if\ \ k = 1,2 \ldots..5 \end{cases} ,$$

$$s_1 : \sum_{k>0} z_{21}^{(k)} X\ D_{7+k} \otimes D_{5-k} \longrightarrow \sum_{k>0} z_{21}^{(K_1)} X\ z_{21}^{(K_2)} X\ D_{7+k} \otimes D_{5-k} \text{ by}$$

$$s_1 \left(\left(z_{21}^{(k)} X \begin{pmatrix} w & 1^{(7+k)}\ \ 2^{(m)} \\ w' & 2^{(5-k-m)} \end{pmatrix} \right) \right) =$$

$$\begin{cases} 0 & ;if\ m{=}0 \\ z_{21}^{(k)} X \begin{pmatrix} w & 1^{(7+k)}\ \ 2^{(m)} \\ w' & 2^{(5-k-m)} \end{pmatrix} & ;if\ \ m{=}1,2\ldots\ldots4 \end{cases}$$

Also define the map

$$s_2 : \sum_{k_i>0} z_{21}^{(K_1)} X z_{21}^{(K_2)} X\ D_{7+|k|} \otimes D_{5-|k|} \to \sum_{k_i>0} z_{21}^{(K_1)} X\ z_{21}^{(K_2)} X z_{21}^{(K_3)} X\ D_{7+|k|} \otimes D_{5-|k|}$$ Such that

$s_2\left(\left(z_{21}^{(K_1)}Xz_{21}^{(K_2)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ & 2^{(5-|k|-m)} \end{matrix}\right)\right)=$

$\begin{cases} 0 & ;if\ m=0 \\ z_{21}^{(K_1)}Xz_{21}^{(K_2)}X\,z_{21}^{(m)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix} & ;if\ m=1,2,3 \end{cases}$

$Where\ |k| = K_1 + K_2 .$

And the map

$s_3:\ \sum_{k_i>0} z_{21}^{(K_1)}Xz_{21}^{(K_2)}X\,z_{21}^{(K_3)}XD_{7+|k|}\otimes D_{5-|k|} \longrightarrow \sum_{k_i>0} z_{21}^{(K_1)}X\,z_{21}^{(K_2)}Xz_{21}^{(K_3)}Xz_{21}^{(K_4)}X \quad D_{7+|k|}\otimes\ D_{5-|k|}$

such that

$s_3\left(\left(z_{21}^{(K_1)}Xz_{21}^{(K_2)}Xz_{21}^{(K_3)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ & 2^{(5-|k|-m)} \end{matrix}\right)\right)=$

$\begin{cases} 0 & ;if\ m=0 \\ z_{21}^{(K_1)}Xz_{21}^{(K_2)}Xz_{21}^{(K_3)}X\,z_{21}^{(m)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix} & ;if\ m=1,2 \end{cases}$

; where$|k| = K_1 + K_2 + K_3$

And finaly we define the map

$s_4:\sum_{k_i>0} z_{21}^{(K_1)}Xz_{21}^{(K_2)}X\,z_{21}^{(K_3)}X\,z_{21}^{(K_4)}XD_{7+|k|}\otimes D_{5-|k|} \longrightarrow \sum_{k_i>0} z_{21}^{(K_1)}X\,z_{21}^{(K_2)}Xz_{21}^{(K_3)}Xz_{21}^{(K_4)}Xz_{21}^{(K_5)}X \quad D_{7+|k|}\otimes D_{5-|k|}$ such that

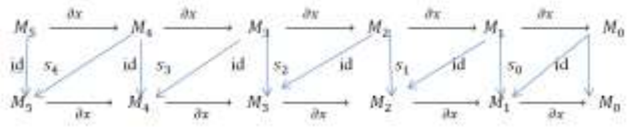
$s_4\left(\left(z_{21}^{(K_1)}Xz_{21}^{(K_2)}Xz_{21}^{(K_3)}Xz_{21}^{(K_4)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ & 2^{(5-|k|-m)} \end{matrix}\right)\right)=$

$\begin{cases} 0 & ;if\ m=0 \\ z_{21}^{(K_1)}Xz_{21}^{(K_2)}Xz_{21}^{(K_3)}X\,z_{21}^{(K_4)}Xz_{21}^{(m)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix} & ;if\ m=1,2 \end{cases}$

$Where|k| = K_1 + K_2 + K_3 + K_4 .$

2.3 The exactness of the resolution of Wely module

Consider the following diagram



Now

$s_1\ \partial_x\left(\left(z_{21}^{(K)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+k)} & 2^{(m)} \\ & 2^{(5-k-m)} \end{matrix}\right)\right) = s_0\ \partial_{21}^{(k)}\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+k)} & 2^{(m)} \\ & 2^{(5-k-m)} \end{matrix} = \binom{k+m}{m} z_{21}^{(k+m)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+k)+m} \\ 2^{(5-k-m)} \end{matrix}$

And

$\partial_x s_1\left(z_{21}^{(K)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+k)} & 2^{(m)} \\ & 2^{(5-k-m)} \end{matrix}\right)=$

$\partial_x\left(z_{21}^{(K)}Xz_{21}^{(m)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+k)+m} \\ 2^{(5-k-m)} \end{matrix}\right)$

$= -\binom{k+m}{m} z_{21}^{(k+m)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+k)+m} \\ 2^{(5-k-m)} \end{matrix} + z_{21}^{(k)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+k)} & 2^{(m)} \\ & 2^{(5-k-m)} \end{matrix}$

It is clear $s_0\partial_x+\partial_x s_1=id_{M_1}$

$s_1\partial_x\left(z_{21}^{(k_1)}Xz_{21}^{(k_2)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ & 2^{(5-|k|-m)} \end{matrix}\right)=$

$\left(s_1 - \binom{|k|}{k_2} z_{21}^{(|k|)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ & 2^{(5-|k|-m)} \end{matrix} + z_{21}^{(k_1)}Xz_{21}^{(k_2)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ & 2^{(5-|k|-m)} \end{matrix}\right)$

$= -\binom{|k|}{k_2} z_{21}^{(|k|)}Xz_{21}^{(m)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix} + \binom{k_2+m}{m} z_{21}^{(k_1)}Xz_{21}^{(k_2+m)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix},$

And

$\partial_x s_2\left(z_{21}^{(k_1)}Xz_{21}^{(k_2)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ & 2^{(5-|k|-m)} \end{matrix}\right)=$

$=\partial_x\left(z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(m)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix}\right)=$

$\binom{|k|}{k_2} z_{21}^{(|k|)}Xz_{21}^{(m)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix} - \binom{k_2+m}{m} z_{21}^{(k_1)}Xz_{21}^{(k_2+m)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix} +$

$z_{21}^{(k_1)}Xz_{21}^{(k_2)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ & 2^{(5-|k|-m)} \end{matrix}$; where $|k|=k_1 + k_2$

It is clear that $s_1\partial_x+\partial_x s_2=id_{M_2}$.

$s_2\partial x\left(z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ & 2^{(5-|k|-m)} \end{matrix}\right)$

$=s_2\left(\binom{k_1+k_2}{k_2} z_{21}^{(k_1+k_2)}Xz_{21}^{(k_3)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ & 2^{(5-|k|-m)} \end{matrix} - \binom{k_2+k_3}{k_3} z_{21}^{(k_1)}Xz_{21}^{(k_2+k_3)}X\begin{pmatrix} w \\ w' \end{pmatrix}\begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ & 2^{(5-|k|-m)} \end{matrix} + \right.$

$z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)\Big)$

$=\binom{k_1+k_2}{k_2} z_{21}^{(k_1+k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$-

$\binom{k_2+k_3}{k_3} z_{21}^{(k_1)}Xz_{21}^{(k_2+k_3)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$+

$\binom{k_3+m}{m} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3+m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$

And

$\partial x s_3\left(z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)\right)=$

$\partial x\left(z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)\right)=$

$-\binom{k_1+k_2}{k_2} z_{21}^{(k_1+k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$+

$\binom{k_2+k_3}{k_3} z_{21}^{(k_1)}Xz_{21}^{(k_2+k_3)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$-

$\binom{k_3+m}{m} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3+m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$+

$z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$=-

$\binom{k_1+k_2}{k_2} z_{21}^{(k_1+k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$+

$\binom{k_2+k_3}{k_3} z_{21}^{(k_1)}Xz_{21}^{(k_2+k_3)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$-

$\binom{k_3+m}{m} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3+m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$+

$z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$; where $|k| = k_1 + k_2 + k_3$

It is clear that $s_2\,\partial_x + \partial_x s_3 = id_{M_3}$.

$s_3\partial x\left(z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)\right)$

=

$s_3\Bigg(-\binom{k_1+k_2}{k_2} z_{21}^{(k_1+k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$\binom{k_2+k_3}{k_3} z_{21}^{(k_1)}Xz_{21}^{(k_2+k_3)}Xz_{21}^{(k_4)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$\binom{k_3+k_4}{k_4} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3+k_4)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)\Bigg)$

=

$-\binom{k_1+k_2}{k_2} z_{21}^{(k_1+k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$\binom{k_2+k_3}{k_3} z_{21}^{(k_1)}Xz_{21}^{(k_2+k_3)}Xz_{21}^{(k_4)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$\binom{k_3+k_4}{k_4} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3+k_4)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$\binom{k_4+m}{m} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4+m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right),$

And

$\partial x s_4\left(z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)\right)$

=

$\partial x\left(z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)\right)$

=

$\binom{k_1+k_2}{k_2} z_{21}^{(k_1+k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)-$

$\binom{k_2+k_3}{k_3} z_{21}^{(k_1)}Xz_{21}^{(k_2+k_3)}Xz_{21}^{(k_4)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$\binom{k_3+k_4}{k_4} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3+k_4)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)-$

$\binom{k_4+m}{m} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4+m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$

$+\, z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$; where

$|k| = k_1 + k_2 + k_3 + k_4$

It is clear that $s_3\,\partial x + \partial x\, s_4 = id_{M_4}$.

$s_4\partial x\left(z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}Xz_{21}^{(k_5)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)\right)$

=

$s_4\Bigg(\Bigg(\binom{k_1+k_2}{k_2} z_{21}^{(k_1+k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}Xz_{21}^{(k_5)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$\binom{k_2+k_3}{k_3} z_{21}^{(k_1)}Xz_{21}^{(k_2+k_3)}Xz_{21}^{(k_4)}Xz_{21}^{(k_5)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$\binom{k_3+k_4}{k_4} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3+k_4)}Xz_{21}^{(k_5)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$\binom{k_4+k_5}{k_5} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4+k_5)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}X\partial_{21}^{(k_5)}\left(\begin{array}{c|c} w & 1^{(7+|k|)} \; 2^{(m)} \\ w' & 2^{(5-|k|-m)} \end{array}\right)\Bigg)$

=

$\binom{k_1+k_2}{k_2} z_{21}^{(k_1+k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}Xz_{21}^{(k_5)}X_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)-$

$\binom{k_2+k_3}{k_3} z_{21}^{(k_1)}Xz_{21}^{(k_2+k_3)}Xz_{21}^{(k_4)}Xz_{21}^{(k_5)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$\binom{k_3+k_4}{k_4} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3+k_4)}Xz_{21}^{(k_5)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)-$

$\binom{k_4+k_5}{k_5} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4+k_5)}Xz_{21}^{(m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)+$

$\binom{k_4+m}{m} z_{21}^{(k_1)}Xz_{21}^{(k_2)}Xz_{21}^{(k_3)}Xz_{21}^{(k_4)}Xz_{21}^{(k_5+m)}X\left(\begin{array}{c|c} w & 1^{(7+|k|)+m} \\ w' & 2^{(5-|k|-m)} \end{array}\right)$

,

And

$$\partial x s_5\left(z_{21}^{(k_1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X z_{21}^{(k_5)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ 2^{(5-|k|-m)} \end{matrix}\right)\right)$$
$=$
$$\partial x\left(z_{21}^{(k_1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X z_{21}^{(k_5)} X z_{21}^{(m)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix}\right)\right)$$
$=$
$$-\binom{k_1+k_2}{k_2} z_{21}^{(k_1+k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X z_{21}^{(k_5)} X z_{21}^{(m)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix}\right) +$$
$$\binom{k_2+k_3}{k_3} z_{21}^{(k_1)} X z_{21}^{(k_2+k_3)} X z_{21}^{(k_4)} X z_{21}^{(k_5)} X z_{21}^{(m)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix}\right) -$$
$$\binom{k_3+k_4}{k_4} z_{21}^{(k_1)} X z_{21}^{(k_2)} X z_{21}^{(k_3+k_4)} X z_{21}^{(k_5)} X z_{21}^{(m)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix}\right) +$$
$$\binom{k_4+k_5}{k_5} z_{21}^{(k_1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4+k_5)} X z_{21}^{(m)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)+m} \\ 2^{(5-|k|-m)} \end{matrix}\right) -$$
$$\binom{k_5+m}{m} z_{21}^{(k_1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X z_{21}^{(k_5+m)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ 2^{(5-|k|-m)} \end{matrix}\right)$$
$$+z_{21}^{(k_1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X z_{21}^{(k_5)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)} & 2^{(m)} \\ 2^{(5-|k|-m)} \end{matrix}\right);$$ Where

$|k| = k_1 + k_2 + k_3 + k_4 + k_5$.

It is clear that $s_4 \partial x + \partial x\, s_5 = id_{M_5}$

Form the above, we get that { $s_0, s_1, s_2, s_3, s_4,$}be contracting homotopy [6] which means that our complex is exacted.

III. Resolution of Weyl module in the case of Skew –Shap (7,5)/(1,0)

In this section we study exhibited the terms and the exactness of the Weyl resolution.

3.2 The terms of Weyl module resolution in the case of Skew –Shap (7,5)/(1,0)

These terms are:

$M_0 = D_6 \otimes D_5$

$M_1 = z_{21}^{(2)} X D_8 \otimes D_3 \oplus Z_{21}^{(3)} X D_9 \otimes D_2 \oplus Z_{21}^{(4)} X D_{10} \otimes D_1 \oplus Z_{21}^{(5)} X D_{11} \otimes D_0$

$M_2 = z_{21}^{(2)} X z_{21}^{(1)} X D_9 \otimes D_2 \oplus Z_{21}^{(3)} X z_{21}^{(1)} X D_{10} \otimes D_1 \oplus Z_{21}^{(2)} X z_{21}^{(2)} X D_{10} \otimes D_1 \oplus z_{21}^{(4)} X z_{21}^{(1)} X D_{11} \otimes D_0$
$\oplus z_{21}^{(2)} X z_{21}^{(3)} X D_{11} \otimes D_0 \oplus z_{21}^{(3)} X z_{21}^{(2)} X D_{11} \otimes D_0$

$M_3 = z_{21}^{(2)} X z_{21}^{(1)} X z_{21}^{(1)} X D_{10} \otimes D_1 \oplus z_{21}^{(3)} X z_{21}^{(1)} X z_{21}^{(1)} X D_{11} \otimes D_0$
$z_{21}^{(2)} X z_{21}^{(2)} X z_{21}^{(1)} X D_{11} \otimes D_0 \oplus z_{21}^{(2)} X z_{21}^{(1)} X z_{21}^{(2)} X D_{11} \otimes D_0 \oplus z_{21}^{(1)} X z_{21}^{(1)} X z_{21}^{(2)} X D_{11}$

$M_4 = z_{21}^{(2)} X z_{21}^{(1)} X z_{21}^{(1)} X z_{21}^{(1)} X D_{11} \otimes D_0$

3.3 The results of the case (7,5)/(1,0).,

In this section we find the terms of the characteristic-free resolution are:

As for homologies we have:

$s_0 = M_0 \rightarrow M_1$

$$s_0\left(\left(\begin{matrix} W \\ W' \end{matrix}\middle| \begin{matrix} 1^{(6)}\ 2^{(k)} \\ 2^{(5-k)} \end{matrix}\right)\right) = \begin{cases} 0 & ; if\ k \le 2 \\ z_{21}^{(k)} \chi \left(\begin{matrix} W \\ W' \end{matrix}\middle| \begin{matrix} 1^{(7+k+m)} \\ 2^{(5-k-m)} \end{matrix}\right) & ; if\ k = 2,3,4,5 \end{cases}$$

And

$\backslash s_1 = M_1 \rightarrow M_2$

$$s_1\left(\left(z_{21}^{(k+1)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k)} & 2^{(v)} \\ 2^{(4-k-v)} \end{matrix}\right)\right)\right) = \begin{cases} 0 & ; if v=0 \\ z_{21}^{(k+1)} X z_{21}^{(v)}\ \chi \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k+v)} \\ 2^{(4-k-v)} \end{matrix}\right) & ; if\ v=1,2,3 \end{cases}$$

And

$s_2 = M_2 \rightarrow M_3$

$$s_2\left(\left(z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ 2^{(4-|k|-v)} \end{matrix}\right)\right)\right) = \begin{cases} 0 & if\ v=0 \\ z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix}\right) & if\ \ v=1,2, \end{cases};$$

where $|k| = k_1 + k_2$

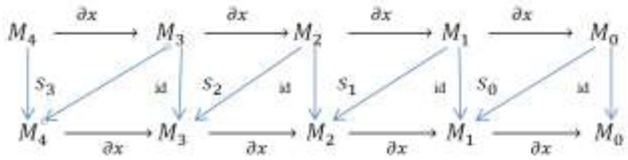
And

$s_3 = M_3 \rightarrow M_4$

$$s_3\left(z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ 2^{(4-|k|-v)} \end{matrix}\right)\right) = \begin{cases} 0 & if\ v=0 \\ z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix}\right) & if\ \ v=1, \end{cases}$$

; where $|k| = k_1 + k_2 + k_3$

So we have the following diagram:



Now we have

$$s_0\, \partial x\left(z_{21}^{(k+1)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k)} & 2^{(v)} \\ 2^{(4-k-v)} \end{matrix}\right)\right) =$$

$$s_0\, \partial_{21}^{(k+1)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k)} & 2^{(v)} \\ 2^{(4-k-v)} \end{matrix}\right) = \binom{k+1+v}{v} z_{21}^{(k+1+v)} X \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(7+k+v)} \\ 2^{(4-k-v)} \end{matrix}\right)$$

And

$\partial x\, s_1 = \left(z_{21}^{(k+1)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+k)} & 2^{(v)} \\ & 2^{(4-k-v)} \end{pmatrix} \right)$

$\partial x \left(z_{21}^{(k+1)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+k+v)} \\ 2^{(4-k-v)} \end{pmatrix} = -\binom{k+1+v}{v}$

$z_{21}^{(k+1+v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+k+v)} \\ 2^{(4-k-v)} \end{pmatrix} + z_{21}^{(k+1)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+k)} & 2^{(v)} \\ & 2^{(4-k-v)} \end{pmatrix}$

clearly $s_0 \partial x + \partial x s_1 = id_{M_1}$

$s_1\, \partial x \left(z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{pmatrix} \right) =$

$s_1 \left(-\binom{|k|+1}{k_2} z_{21}^{(|k|+1)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{pmatrix} \right.$

$+ z_{21}^{(k+1)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{pmatrix} +$

$\left. z_{21}^{(k_1+1)} X\, \partial_{21}^{(k_2)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{pmatrix} \right)$

$= -\binom{|k|+1}{k_2} z_{21}^{(|k|+1)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix} +$

$\binom{k_2+v}{v} z_{21}^{(k_1+1)} X z_{21}^{(k_2+v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix}$

And

$\partial x S_1 (z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{pmatrix} =$

$\partial x \left(z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix} \right) =$

$\binom{|k|+1}{k_2} z_{21}^{(|k|+1)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix} -$

$\binom{k_2+v}{v} z_{21}^{(k_1+1)} X z_{21}^{(k_2+v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix}$

$+(z_{21}^{(k_1+1)} X z_{21}^{(k_2+v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{pmatrix}$; *where* $|k| = k_1 + k_2$.

$clearly s_1 \partial x + \partial x s_2 = id_{M_2}$

$s_2\, \partial x \left(z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{pmatrix} \right) =$

$s_2 \left(\binom{k_1+1+k_2}{k_2} z_{21}^{(k_1+1+k_2)} X \right.$

$z_{21}^{(k_3)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{pmatrix} -$

$\binom{k_2+k_3}{k_3} z_{21}^{(k_1+1)} X z_{21}^{(k_2+k_3)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{pmatrix} +$

$\left. z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{pmatrix} \right)$

$= \binom{k_1+1+k_2}{k_2} z_{21}^{(k_1+1+k_2)} X z_{21}^{(k_3)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix} -$

$\binom{k_2+k_3}{k_3} z_{21}^{(k_1+1)} z_{21}^{(k_2+k_3)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix} +$

$\binom{k_3+v}{v} z_{21}^{(k_1+1)} z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix}$

And

$\partial x s_3 \left(z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{pmatrix} \right) =$

$\partial x \left(z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix} \right) =$

$-\binom{k_1+1+k_2}{k_2}$

$z_{21}^{(k_1+1+k_2)} X z_{21}^{(k_3)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix} +$

$\binom{k_2+k_3}{k_3} z_{21}^{(k_1+1)} X z_{21}^{(k_2+k_3)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix} -$

$\binom{k_3+v}{v} z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix} +$

$z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X\, \partial_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix} =$

$-\binom{k_1+1+k_2}{k_2} z_{21}^{(k_1+1+k_2)} X z_{21}^{(k_3)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix}$

$+\binom{k_2+k_3}{k_3} z_{21}^{(k_1+1)} X z_{21}^{(k_2+k_3)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix} -$

$\binom{k_3+v}{v} z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(v)} X \begin{pmatrix} w \\ w' \end{pmatrix} \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{pmatrix}$

$+ z_{21}^{(k_1+1)} X\, z_{21}^{(k_2)} X z_{21}^{(k_3)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{matrix} \right)$; where $|k| = k_1 + k_2 + k_3$.

$Clearly\ \ s_2 \partial x + \partial x s_3 = id_{M_3}$

$$s_3\, \partial x \left(z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{matrix} \right) \right) =$$

$$s_3 \Bigg(-\binom{k_1+1+k_2}{k_2} z_{21}^{(k_1+1+k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{matrix} \right) +$$

$$\binom{k_2+k_3}{k_3} z_{21}^{(k_1+1)} X z_{21}^{(k_2+k_3)} X z_{21}^{(k_4)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{matrix} \right) - \binom{k_3+k_4}{k_4} z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3+k_4)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{matrix} \right)$$

$$+ z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X \partial_{21}^{(k_4)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{matrix} \right) \Bigg) =$$

$$-\binom{k_1+1+k_2}{k_2} z_{21}^{(k_1+1+k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X z_{21}^{(v)} \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right) +$$

$$\binom{k_2+k_3}{k_3} z_{21}^{(k_1+1)} z_{21}^{(k_2+k_3)} X z_{21}^{(k_4)} X z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right)$$

$$-\binom{k_3+k_4}{k_4} z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3+k_4)} X z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right)$$

$$+ \binom{k_4+v}{v} z_{21}^{(k_1+1)} z_{21}^{(k2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right)$$

And

$$\partial x s_4 (z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{matrix} \right) =$$

$$\partial x \left(z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X\, z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right) \right)$$

$$= \binom{k_1+1+k_2}{k_2} z_{21}^{(k_1+1+k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right)$$

$$-\binom{k_2+k_3}{k_3} z_{21}^{(k_1+1)} X z_{21}^{(k_2+k_3)} X z_{21}^{(k_4)} X\, z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right) +$$

$$\binom{k_3+k_4}{k_4} z_{21}^{(k_1+1)} X\, z_{21}^{(k_2)} X z_{21}^{(k_3+k_4)}\, z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right) -$$

$$\binom{k_4+v}{v} z_{21}^{(k_1+1)} X\, z_{21}^{(k_2)} X z_{21}^{(k_3)} X\, z_{21}^{(k_4)} X\, z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right)$$

$$+ z_{21}^{(k_1+1)} X z_{21}^{(k_2)} X z_{21}^{(k_3)} X\, z_{21}^{(k_4)} X \partial_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right) =$$

$$-\binom{k_1+1+k_2}{k_2} z_{21}^{(k_1+1+k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X\, z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right)$$

$$-\binom{k_2+k_3}{k_3} z_{21}^{(k_1+1)} X z_{21}^{(k_2+k_3)} X z_{21}^{(k_4)} X\, z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right)$$

$$+\binom{k_3+k_4}{k_4} z_{21}^{(k_1+1)} X\, z_{21}^{(k_2)} X z_{21}^{(k_3+k_4)} X\, z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right) -$$

$$\binom{k_4+v}{v} z_{21}^{(k_1+1)} X\, z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X\, z_{21}^{(v)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|+v)} \\ 2^{(4-|k|-v)} \end{matrix} \right)$$

$$+ z_{21}^{(k_1+1)} X\, z_{21}^{(k_2)} X z_{21}^{(k_3)} X z_{21}^{(k_4)} X \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(7+|k|)} & 2^{(v)} \\ & 2^{(4-|k|-v)} \end{matrix} \right);$$

Where $|k| = k_1 + k_2 + k_3 + k_4$.

$clearly\ S_3 \partial x + \partial x S_4 = id_{M_4}.$

From the above, we get that $\{s_0, s_1, s_2, s_3, s_4,\}$ be contracting homotopy ,which mean that our complex is exact.

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