Sensitivity Analysis of the $\mathbb{PSITPS}$ Epidemic Model's Parameters for COVID-19

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Abstract— *Given the prevailing circumstances surrounding the transmission of COVID-19, which has been classified as a worldwide epidemic by the World Health Organization, a sensitivity analysis was performed on the Protected-Susceptible-Infected-Treated- Protected- Susceptible ($\mathbb{PSITPS}$) epidemic model. The objective of this analysis was to identify the specific parameters associated with the basic reproduction number that contribute to the propagation of the COVID-19 epidemic in Najaf. The findings indicate that the sensitivity index of parameters exhibiting a positive sign is positively correlated with the extent of the epidemic's spread in Najaf. Conversely, parameters displaying a negative sign are inversely correlated with the spread of the epidemic. The analysis was conducted numerically using MATLAB to confirm the results of the study.*

Keywords—: $\mathbb{PSITPS}$ ***Model; COVID-19 Epidemic; The Basic Reproduction Number; Sensitivity Analysis; Experimental Result;***

1- INTRODUCTION

The emergence of COVID-19 was first reported in December 2019 in Wuhan, located in the Hubei Province of China [1]. From December 18 to December 29, 2019, five people were hospitalized with symptoms of fever, dyspnea, and cough, which were further complicated by acute respiratory distress syndrome. Then one of these people passed away [2]. After short period, on January 30, 2020, the World Health Organization (WHO) proclaimed the COVID-19 epidemic in China to be a public health emergency of worldwide concern [3]. 765,222,932 COVID-19 cases have been confirmed by the WHO, together with 6,921,614 fatalities [4]. Among these countries that have been severely affected by the virus is Iraq. The initial infection with COVID-19 was verified on February 25, 2020, in Najaf Governorate by an Iranian student, followed by a family in Kirkuk Governorate on February 26, 2020. These cases had all previously visited Iran [5]. In Najaf Governorate, the cumulative total of COVID-19 cases from March 31, 2020, to May 9, 2022, is 95,147 confirmed cases with 728 deaths [6]. COVID-19 cases in total from January 3, 2020, to May 10, 2023, are 2,465,545 confirmed cases, with 25,375 deaths in Iraq [7].

Human beings commonly encounter infectious diseases such as COVID-19, measles, and tuberculosis on a regular basis. Given the significant risks that these viruses pose to both human populations and national economies, humanity has developed a variety of strategies, tools, and cutting-edge technological devices to mitigate the associated hazards. The containment of infectious diseases cannot be achieved solely through biological and medical interventions by the human population. The mathematical modeling of infectious diseases is considered a crucial tool for effectively managing such endeavors [8].

According to works that have been published, the idea of sensitivity analysis has been around to find the parameter sensitivity of the studied model.

Gurmu et al. (2020) established a dynamic model to analyze the transmission dynamics of COVID-19. The existence of the formulated model equations is shown. The model equation was analyzed to determine the stability of both the disease-free equilibrium point and the endemic equilibrium point at the local as well as global levels. This analysis was conducted by considering the basic reproduction number. The results of a numerical simulation study show that increasing the level of contact rate among people has an impact on reducing the occurrence of COVID-19 and COVID-19 disease. In addition, the model equation was subjected to sensitivity analysis to determine the importance and potential effects of the main parameters on the transmission dynamics of COVID-19 [9].

A segmented mathematical model was proposed by Samui et al. (2020) to analyze and forecast the transmission and control dynamics of the COVID-19 pandemic in India. The model utilized epidemic data up to April 30, 2020. The sensitivity indices of the reproduction number R0, which is a key determinant of disease transmission in the early stages, were computed from the estimated parameter values. The estimated model parameters yielded a R_0 value of 1.6632, indicating a substantial outbreak of COVID-19 in India. The results of the model simulation indicate that the disease transmission rate (βs) has a greater impact on mitigating the basic reproduction number R_0. Depending on the estimated data, the model projects that the peak of COVID-19 cases in India will occur in approximately 60 days, with a higher magnitude. Subsequently, the curve is expected to plateau, indicating a stabilization in the number of cases. However, it is important to note that the persistence of the coronavirus disease is anticipated to endure for an extended period [10].

Rida et al. (2021) introduced a mathematical Susceptible-Exposed-Infectious-Recovered (SEIAR) Model to analyze the ongoing COVID-19 pandemic outbreak. To perform a sensitivity analysis and determine the key parameters for estimating the volume of COVID-19 transmission, the researchers utilized the reproduction number R_0. A numerical simulation was conducted utilizing data in order to estimate the global dissemination of COVID-19. Furthermore, a comparison was made between the results obtained from our simulation and the empirical data on recorded instances of infectious cases collected by the World Health Organization [11].

Salam et al. (2022) enhanced the Susceptible-Infected Asymptomatic-Unreported with Symptoms-Reported with Symptoms-Recovered (SIUWR) model by incorporating additional transmission parameters that were not included in their previous work. Consequently, the calculation of the basic reproduction number and elasticity coefficients has been conducted at the points of equilibrium. Subsequently, several crucial model parameters are determined in accordance with the local sensitivities. The model's validity was evaluated by comparing it to actual data collected in Iraq and France from January 1st to December 25th, 2021. The findings presented in this study provide potential biological explanations that can be leveraged to effectively manage and mitigate the impact of this epidemic [12].

Khoshnaw et al. (2022) examine the Susceptible–Exposed–Infected–Recovered (SEIR) model as a means to forecast the transmission dynamics of the disease. The authors emphasize the significance of modeling critical transmissions and sensitivities in order to comprehensively investigate the scope of this pandemic. The equilibrium points were analyzed in terms of the basic reproduction number and its coefficient of elasticity. In addition, the estimation of model parameters and validation of the model were conducted using authentic data on confirmed cases in the Kurdistan Region of Iraq. The outcomes of the computational model yielded significant enhancements to the existing model and proposed potential control strategies. The cases within the model population exhibit distinct dynamics, characterized by estimated parameters and initial values. Another consequence of the model is that the majority of states within it exhibit sensitivity to model parameters to varying degrees. The researchers discovered that the contact rate, transition rate from the exposed class to the infected class, and natural recovery rate are the key parameters that can be manipulated to decrease the basic reproduction number. These parameters are also identified as critical factors within the model. Moreover, the arithmetic findings derived from the actual data suggest that the basic reproduction number in the Kurdistan Region was approximately 1.28, surpassing the threshold of unity. This implies that the novel coronavirus retains a significant capacity for interpersonal transmission, necessitating additional interventions and novel strategies to effectively manage the spread of this disease [13].

Al-Yahyai et al. (2023) put forth a deterministic mathematical model to examine the impact of various forms of isolation and quarantine on the transmission dynamics of COVID-19. The quarantine compartment is divided into two categories: short and long quarantine classes. Similarly, the isolation compartment is further divided into two groups: home-isolated people who have been tested and those who have not been tested, as well as institutionally isolated people. The studied model has undergone a comprehensive analysis. The estimation of the model parameters has been conducted using a dataset specific to Oman. The control reproduction number and the influence of all transmission routes on the reproduction number have been computed based on the fitted parameters. Additionally, a sensitivity analysis has been conducted to examine the impact of model parameters on the control reproduction number. In the end, the study conducted numerical simulations to illustrate the impact of various model parameters pertaining to distinct forms of isolation and quarantine on the dynamics of disease transmission. The findings have been visually presented through graphical representations [14].

The objective of this study is to perform a sensitivity analysis on the $\mathbb{PSITPS}$ Epidemic Model, which was employed to demonstrate the effects of protection (vaccination) after treatment for COVID-19 infection in Najaf [15]. The analysis will yield numerical findings that will enable the identification of the parameters that made a significant contribution to the propagation of the epidemic in Najaf. The results will be presented using tables and figures to enhance the overall understanding of the findings.

The present paper is organized as follows: Section 2 outlines the structure of the $\mathbb{PSITPS}$ Epidemic Model. Section 3 provides an opportunity to conduct a comprehensive review of the model analysis. In Section 4, a sensitivity analysis of the model was performed. Section 5 presented the experimental result, followed by the conclusion in Section 6.

2- THE STRUCTURE OF THE MODEL

According to [15], the people living in Najaf were divided into four groups at time t: the protected ($\mathbb{P}(t)$) are people who have been vaccinated, the Susceptible ($\mathbb{S}(t)$) are people who have not yet contracted the disease, the Infected ($\mathbb{I}(t)$) refer to individuals who are infected, and the Treated ($\mathbb{T}(t)$) are people who successfully receive treatment.

The following mathematical model was provided, which consists of four nonlinear first-order differential equations, and Table 1 contains the descriptions of each parameter and their values.

$$\left.\begin{aligned} \frac{d\mathbb{P}}{dt} &= \varphi\Lambda + \alpha\mathbb{T} - (\delta+\beta)\mathbb{P} \\ \frac{d\mathbb{S}}{dt} &= (1-\varphi)\Lambda + \delta\mathbb{P} + \eta\mathbb{T} - (\varepsilon+\beta)\mathbb{S} \\ \frac{d\mathbb{I}}{dt} &= \varepsilon\mathbb{S} - (\gamma+\beta+\omega)\mathbb{I} \\ \frac{d\mathbb{T}}{dt} &= \omega\mathbb{I} - (\eta+\alpha+\beta)\mathbb{T} \end{aligned}\right\} \quad (1)$$

where $\varepsilon = \frac{(1-\vartheta)\pi\theta}{\mathbb{N}}\mathbb{I}$ is an effective force of infection, ϑ is the rate of infection protection efficacy, π is the COVID-19 transmission probability rate, θ is the infection rate per contact, and total population size is $\mathbb{N} = \mathbb{P} + \mathbb{S} + \mathbb{I} + \mathbb{T}$ with initial conditions $\mathbb{P}(0) = \mathbb{P}_0$, $\mathbb{S}(0) = \mathbb{S}_0$, $\mathbb{I}(0) = \mathbb{I}_0$, and $\mathbb{T}(0) = \mathbb{T}_0$.

TABLE 1: DESCRIPTION OF COVID-19 MODEL PARAMETERS AND THEIR VALUES [15].

Parameter	*Description*	*Value*
Λ	Birth rate	2.92
φ	Already protected rate	0.206
δ	Rate of loss of protection	0.0037
ε	Effective force of infection	0.0002
ω	Rate of treatment	0.095
η	Transmission rate from $\mathbb{T}$ to $\mathbb{S}$	0.0067
α	Transmission Rate from $\mathbb{T}$ to $\mathbb{P}$	0.0074
γ	Death rate induced COVID-19	0.58
β	Natural death rate	0.556
π	Transmission probability rate of COVID-19	0.074
θ	Contact rate of infection	0.3
ϑ	Rate of protection against infected	0.787

3- *Review The Model Analysis [15]*

The positive solutions of the system of model equations (1) are bounded. The Basic Reproduction Number of System (1) is $\mathcal{R}_0^D = \frac{\pi\theta(1-\vartheta)(\delta+\beta-\varphi\beta)}{(\delta+\beta)(\gamma+\beta+\omega)}$. Also, two equilibria exist in System (1), the Disease-Free Equilibrium $\mathbb{E}_0 = \left(\frac{\varphi\Lambda}{(\delta+\beta)}, \frac{(\delta+\beta-\varphi\beta)\Lambda}{\beta(\delta+\beta)}, 0, 0\right)$, and the Endemic Equilibrium $\mathbb{E}^* = (\mathbb{P}^*, \mathbb{S}^*, \mathbb{I}^*, \mathbb{T}^*)$, where

$$\mathbb{P}^* = \frac{\varphi\Lambda + \alpha\left[\frac{\omega}{(\eta+\alpha+\beta)}\left(\frac{(\eta+\alpha+\beta)(\pi\theta(1-\vartheta)(\delta+\beta)(1-\varphi)\Lambda+\pi\theta(1-\vartheta)\delta\varphi\Lambda-\beta\mathbb{N}(\delta+\beta)(\gamma+\beta+\omega))}{((\delta+\beta)(\eta+\alpha+\beta)(\gamma+\beta+\omega)-\delta\alpha\omega-\eta\omega(\delta+\beta))(\pi\theta(1-\vartheta))}\right)\right]}{(\delta+\beta)},$$

$$\mathbb{S}^* = \frac{\mathbb{N}(\beta+\gamma+\omega)}{\pi\theta(1-\vartheta)},$$

$$\mathbb{I}^* = \frac{(\eta+\alpha+\beta)(\pi\theta(1-\vartheta)(\delta+\beta)(1-\varphi)\Lambda+\pi\theta(1-\vartheta)\delta\varphi\Lambda-\beta\mathbb{N}(\delta+\beta)(\gamma+\beta+\omega))}{((\delta+\beta)(\eta+\alpha+\beta)(\gamma+\beta+\omega)-\delta\alpha\omega-\eta\omega(\delta+\beta))(\pi\theta(1-\vartheta))}, \text{ and}$$

$$\mathbb{T}^* = \frac{\omega}{(\eta+\alpha+\beta)}\left[\frac{(\eta+\alpha+\beta)(\pi\theta(1-\vartheta)(\delta+\beta)(1-\varphi)\Lambda+\pi\theta(1-\vartheta)\delta\varphi\Lambda-\beta\mathbb{N}(\delta+\beta)(\gamma+\beta+\omega))}{((\delta+\beta)(\eta+\alpha+\beta)(\gamma+\beta+\omega)-\delta\alpha\omega-\eta\omega(\delta+\beta))(\pi\theta(1-\vartheta))}\right].$$

Proposition 1. if $\mathcal{R}_0^D < 1$ Then the Model's (1) disease-free equilibrium is locally asymptotically stable, and unstable if $\mathcal{R}_0^D > 1$.

Theorem 1. The endemic equilibrium points of the Model (1) are globally asymptotically stable in Υ if the basic reproduction number $\mathcal{R}_0^D > 1$.

4- *SENSITIVITY ANALYSIS*

A sensitivity analysis was conducted in order to assess the robustness of the model with respect to variations in parameter values. This methodology is expected to facilitate the identification and validation of model parameters that have the most significant impact on the pathogen fitness threshold. Moreover, the sensitivity indexes offer valuable insights into the parameters that warrant prioritization for the purpose of intervention [16]. The present study conducts a sensitivity analysis of $\mathcal{R}_0^D$ in relation to each individual parameter. The methodology for computing the parameters of the sensitivity analysis is outlined as follows [8]:

$$\varphi^{\mathcal{R}_0^D} = \frac{\partial\mathcal{R}_0^D}{\partial\varphi} \times \frac{\varphi}{\mathcal{R}_0^D} = +0.998,$$

$$\delta^{\mathcal{R}_0^D} = \frac{\partial\mathcal{R}_0^D}{\partial\delta} \times \frac{\delta}{\mathcal{R}_0^D} = +0.002.$$

Similarly, the sensitivity index for each parameter was calculated, which is presented in Table 2.

TABLE 2: SENSITIVITY INDEX TABLE.

Parameter	*Sensitivity Index*
π	+0.998
θ	+0.998
ϑ	-3.689
ε	+0.2009
δ	+0.002
ω	-0.077
β	-0.215
φ	-0.257
γ	-0.470
α	0

5- *EXPERIMENTAL RESULT*

Table 2 shows the $\mathcal{R}_0^{\mathrm{D}}$ sensitivity index to the parameters for the ℙ𝕊𝕀𝕋ℙ𝕊 Epidemic Model for the COVID-19 epidemic in Najaf, determined by the values in Table 1.

The following result has been reached by numerically solving the model equations (1) using MATLAB. The numerical method fourth-order Runge-Kutta (RK4) was used, along with the parameter values shown in Table 1.

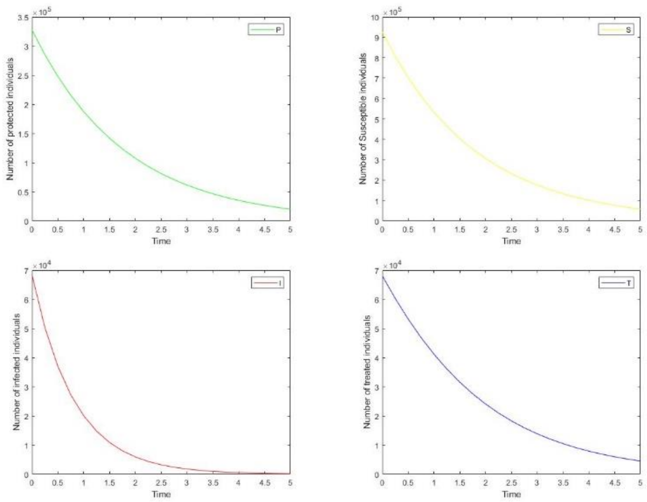

FIGURE 1: COVID-19 DYNAMICS IN CLASSES ℙ, 𝕊, 𝕀, AND 𝕋.

Tables 3–8 with Figure 2 represent the simulated interpretation for the sensitivity of the parameters in infected cases, as shown in Table 2. According to the sensitivity indices in Table 2, the sensitivity value of the effective force of infection (ε) is positive. As a result, there is a noticeable increase in infected cases when the effective force of infection in Table 3 with (A) of Figure 2 is raised. We conclude from Table 5 and Table 7 with (C) and (E), respectively, of Figure 2 that the improvement in the treatment rate (ω) and the rate of those who are already protected (φ) will contribute to controlling the spread of the disease. In Table 5, Table 7, Table 6, and Table 8 with (C), (E), (D), and (F), respectively, of Figure 2, an increase in the treatment rate (ω), the rate of those who are already protected (φ), and the death rate (β and γ) leads to a decrease in COVID-19 infections in Najaf. However, the increase in the loss of protection (δ) contributes to the spread of the disease in Table 4 with (B) of Figure 2.

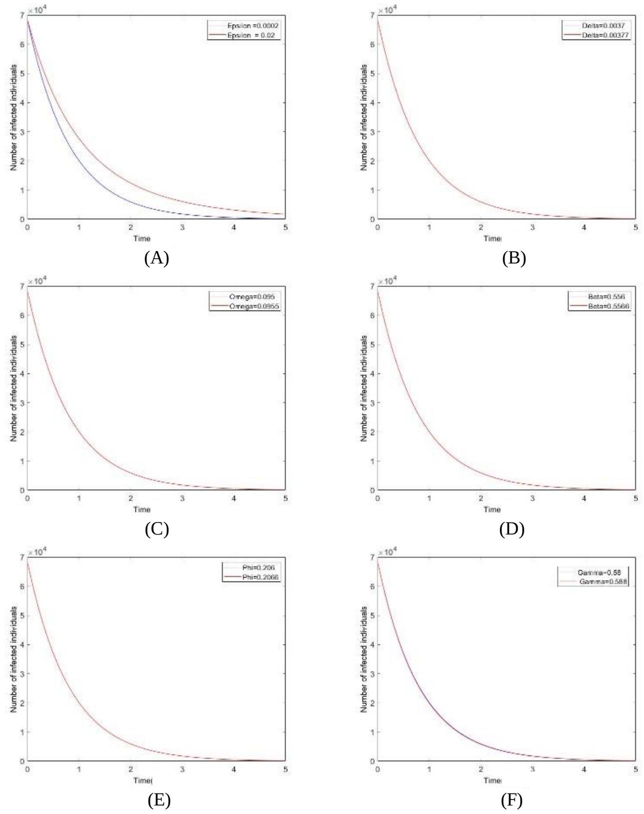

FIGURE 2: (A) THE EFFECT OF THE EFFECTIVE FORCE OF INFECTION ON COVID-19 TRANSMISSION WHEN $\varepsilon = 0.02$. (B) THE EFFECT OF THE RATE OF LOSS OF PROTECTION ON COVID-19 TRANSMISSION WHEN $\delta = 0.00377$. (C) THE EFFECT OF THE RATE OF TREATMENT ON COVID-19 TRANSMISSION WHEN $\omega = 0.0955$. (D) THE EFFECT OF THE NATURAL DEATH RATE ON COVID-19 TRANSMISSION WHEN $\beta = 0.5566$. (E) THE EFFECT OF THE ALREADY PROTECTED RATE ON COVID-19 TRANSMISSION WHEN $\varphi = 0.2066$. (F) THE EFFECT OF THE DEATH RATE INDUCED COVID-19 ON COVID-19 TRANSMISSION WHEN $\gamma = 0.588$.

TABLE 3: NUMERICAL RESULTS FOR THE EFFECT OF THE EFFECTIVE FORCE OF INFECTION ON COVID-19 TRANSMISSION WHEN $\varepsilon = 0.02$.

$\varepsilon = 0.0002$ 1.0e+04 *	$\varepsilon = 0.02$ 1.0e+04 *	The difference 1.0e+03 *
6.839400000000000	6.839400000000000	0
6.048924317780305	6.216520016091240	-1.675956983109347
5.349913718136892	5.656338145471814	-3.064244273349221
4.731779230224717	5.152119651989639	-4.203404217649222
4.185158079828752	4.697880412431973	-5.127223326032203
3.701771680071964	4.288301621098795	-5.865299410268308
3.274300069506756	3.918654281792723	-6.443542122859665
2.896270892621483	3.584732358745624	-6.884614661241405
2.561961238433434	3.282793588419109	-7.208323499856761
2.266310847924056	3.009507069443809	-7.431962215197535
2.004845373561186	2.761906849947707	-7.570614763865204
1.773608526663646	2.537350821711256	-7.637422950476102
1.569102083209836	2.333484310342165	-7.643822271323288
1.388232837919821	2.148207821193210	-7.599749832733893
1.228265701858810	1.979648463119902	-7.513827612610916
1.086782232018464	1.826134627335371	-7.393523953169075
0.961643963745149	1.686173547402259	-7.245295836571096
0.850959989751735	1.558431409543833	-7.074714197920974
0.753058293877098	1.441715720611627	-6.886574267345284
0.666460404722980	1.334959674791801	-6.684992700688208
0.58985898465305	1.237208289977374	-6.473493053120695
0.522098014270824	1.147606111129751	-6.255080968589267
0.462155271525700	1.065386301299537	-6.032310297738372
0.409126840097714	0.989860961626282	-5.807341215285681
0.362213411636533	0.920412539901513	-5.581991282649800
0.320708174333808	0.856486203435032	-5.357780291012246
0.283986104029879	0.797583066254914	-5.135969622250350
0.251494495431568	0.743254173312606	-4.917596778810385
0.222744589818807	0.693095155546785	-4.703505657279789
0.197304172252130	0.646741479551732	-4.494373072996021
0.174791026000715	0.603864224347200	-4.290731983464856
0.154867144914827	0.564166325488851	-4.092991805740243
0.137233615964570	0.527379233608004	-3.901456176434343
0.121626094333102	0.493259940529780	-3.716338461966777
0.107810802441318	0.461588331480924	-3.537775290396059
0.095580992228725	0.432164826643358	-3.365838344146326
0.084753817042431	0.404808279508035	-3.200544624656044
0.075167565699487	0.379354103199195	-3.041865374997085
0.066679216781509	0.355652599227235	-2.889733824457264
0.059162276077895	0.333567466038607	-2.744051899607117
0.052504864388787	0.312974467306922	-2.604696029181356
0.046608026696299	0.293760242189449	-2.471522154931492
0.041384237070142	0.275821241791490	-2.344370047213481
0.036756076642452	0.259062777867976	-2.223067012255240
0.032655064611533	0.243398171373546	-2.107431067620130
0.029020624555091	0.228747989873251	-1.997273653181600
0.025799170385580	0.215039364066374	-1.892401936807943
0.022943298094726	0.202205374774508	-1.792620766797811
0.020411071038540	0.190184502718012	-1.697734316794714
0.018165387932583	0.178920134266929	-1.607547463343463
0.016173423981472	0.168360117116050	-1.521866931345785

The Table 3 illustrates the impact of the parameter (ε), whose influence is not obvious in (A) of Figure 2.

TABLE 4: NUMERICAL RESULTS FOR THE EFFECT OF THE RATE OF LOSS OF PROTECTION ON COVID-19 TRANSMISSION WHEN $\delta = 0.00377$.

δ=0.0037 1.0e+04 *	δ=0.00377 1.0e+04 *	The difference
6.839400000000000	6.839400000000000	0
6.048924317780305	6.048924319901806	-0.000021215011657
5.349913718136892	5.349913725986295	-0.000078494034824
4.731779230224717	4.731779246565548	-0.000163408316439
4.185158079828752	4.185158106714139	-0.000268853873422
3.701771680071964	3.701771718959233	-0.000388872686017
3.274300069506756	3.274300121356395	-0.000518496392033
2.896270892621483	2.896270957982447	-0.000653609633446
2.561961238433434	2.561961317516512	-0.000790830785263
2.266310847924056	2.266310940664844	-0.000927407880226
2.004845373561186	2.004845479673978	-0.001061127921275
1.773608526663646	1.773608645687438	-0.001190237919218
1.569102083209836	1.569102214547458	-0.001313376216785
1.388232837919821	1.388232980871105	-0.001429512847608
1.228265701858810	1.228265855648591	-0.001537897807793
1.086782232018464	1.086782395820087	-0.001638016237848
0.961643963745149	0.961644136700118	-0.001729549681841
0.850959989751735	0.850960170986000	-0.001812342641642
0.753058293877098	0.753058482514474	-0.001886373760499
0.666460404722980	0.666460599896086	-0.001951731060217
0.589858984665305	0.589859185524375	-0.002008590701735
0.522098014270824	0.522098219990706	-0.002057198819784
0.462155271525700	0.462155481311304	-0.002097856044202
0.409126840097714	0.409127053188148	-0.002130904341357
0.362213411636533	0.362213627308121	-0.002156715879664
0.320708174333808	0.320708391902173	-0.002175683654059
0.283986104029879	0.283986322851241	-0.002188213626596
0.251494495431568	0.251494714903386	-0.002194718179453
0.222744589818807	0.222744809379877	-0.002195610704803
0.197304172252130	0.197304391382247	-0.002191301167159
0.174791026000715	0.174791244219966	-0.002182192509963
0.154867144914827	0.154867361782604	-0.002168677777490
0.137233615964570	0.137233831078355	-0.002151137852252
0.121626094333102	0.121626307327074	-0.002129939718770
0.107810802441318	0.107811012984835	-0.002105435168687
0.095580992228725	0.095581200024713	-0.002077959884900
0.084753817042431	0.084754021825715	-0.002047832840276
0.075167565699487	0.075167767235083	-0.002015355962726
0.066679216781509	0.066679414862911	-0.001980814019589
0.059162276077895	0.059162470525364	-0.001944474684137
0.052504864388787	0.052505055047662	-0.001906588750671
0.046608026696299	0.046608213435346	-0.001867390469499
0.041384237070142	0.041384419779940	-0.001827097977525
0.036756076642452	0.036756255233832	-0.001785913803644
0.032655064611533	0.032655239014076	-0.001744025431037
0.029020624555091	0.029020794715681	-0.001701605901133
0.025799170385580	0.025799336267024	-0.001658814446500
0.022943298094726	0.022943459674441	-0.001615797142279
0.020411071038540	0.020411228307297	-0.001572687566465
0.018165387932583	0.018165540893329	-0.001529607462174
0.016173423981472	0.016173572648211	-0.001486667395596

The Table 4 illustrates the impact of the parameter (δ), whose influence is not obvious in (B) of Figure 2.

TABLE 5: NUMERICAL RESULTS FOR THE EFFECT OF THE RATE OF TREATMENT ON COVID-19 TRANSMISSION WHEN $\omega = 0.0955$.

ω = 0.095 1.0e+04 *	ω = 0.0955 1.0e+04 *	The difference
6.83940000000000	6.839400000000000	0
6.04892431778035	6.048621925233567	3.023925467379740
5.34991371813682	5.349378917857057	5.348002798353264
4.73177923022477	4.731069854440362	7.093757843547792
4.18515807982872	4.184321684402533	8.363954262196785
3.70177168007194	3.700847149117672	9.245309542930045
3.27430006950676	3.273318986152324	9.810833544321213
2.89627089262143	2.895258708565055	10.121840564279410
2.56196123843344	2.560938270513948	10.229679194853816
2.26631084792406	2.265293126089402	10.177218346536392
2.00484537356116	2.003845361289170	10.000122720164654
1.77360852666366	1.772635732006980	9.727946566657920
1.56910208320986	1.568163576137998	9.385070718382849
1.38823283791981	1.387333687467213	8.991504526076824
1.22826570185880	1.227409344716194	8.563571426166163
1.08678223201844	1.085970782584667	8.114494337965880
0.96164396374519	0.960878474255282	7.654894898676503
0.85095998975175	0.850240667887448	7.193218642876673
0.75305829387708	0.752384684218645	6.736096584534607
0.66646040472290	0.665831539499945	6.288652230348816
0.58985898466535	0.589273508483817	5.854761814874109
0.52209801427084	0.521554286823305	5.437274475194499
0.46215527152570	0.461651451710274	5.038198154256861
0.40912684009774	0.408660954475882	4.658856218319215
0.36221341163653	0.361783409728651	4.300019078814785
0.32070817433388	0.320311972883015	3.962014507927961
0.283986104029879	0.283621622048093	3.644819817861389
0.251494495431568	0.251159681569042	3.348138625256070
0.222744589818807	0.222437443365299	3.071464535079031
0.197304172252130	0.197022758877697	2.814133744336232
0.174791026000715	0.174533489172910	2.575368278044152
0.154867144914827	0.154631713782642	2.354311321846581
0.137233615964570	0.137018610374361	2.150055902094891
0.121626094333102	0.121429927535054	1.961667980476250
0.107810802441318	0.107631981954086	1.788204872322922
0.095580992228725	0.095418119252519	1.628729762059152
0.084753817042431	0.084605584745151	1.482322972799693
0.075167565699487	0.075032756644760	1.348090547269862
0.066679216781509	0.066556699720340	1.225170611689350
0.059162276077895	0.059051002285788	1.112737921072835
0.052504864388787	0.052403863696603	1.010006921836748
0.046608026696299	0.046516403334880	0.916233614190105
0.041384237070142	0.041301165425019	0.830716451237208
0.036756076642452	0.036680796995158	0.752796472938712
0.032655064611533	0.032586878927523	0.681856840096486
0.029020624555091	0.028958892364538	0.617321905532094
0.025799170385580	0.025743304791989	0.558655935901356
0.022943298094726	0.022892761936975	0.505361577516823
0.020411071038540	0.020365373224280	0.456978142606971
0.018165387932583	0.018124079954766	0.413079778171067
0.016173423981472	0.016136096624712	0.373273567597266

The Table 5 illustrates the impact of the parameter (ω), whose influence is not obvious in (C) of Figure 2.

TABLE 6: NUMERICAL RESULTS FOR THE EFFECT OF THE NATURAL DEATH RATE ON COVID-19 TRANSMISSION WHEN $\beta = 0.5566$.

β = 0.556 1.0e+04 *	β = 0.5566 1.0e+04 *	The difference
6.839400000000000	6.839400000000000	0
6.048924317780305	6.048561397124076	3.629206562291074
5.349913718136892	5.349271773931323	6.419442055681429
4.731779230224717	4.730927595794545	8.516344301722711
4.185158079828752	4.184153773235656	10.043065930956800
3.701771680071964	3.700661327092613	11.103529793515918
3.274300069506756	3.273121546322014	11.785231847421528
2.896270892621483	2.895054727353070	12.161652684131695
2.561961238433434	2.560731805358069	12.294330753644317
2.266310847924056	2.265087383593134	12.234643309217063
2.004845373561186	2.003642840064254	12.025334969326650
1.773608526663646	1.772438343816366	11.701828472803754
1.569102083209836	1.567972748451934	11.293347579021429
1.388232837919821	1.387150450114985	10.823878048358893
1.228265701858810	1.227234402943806	10.312989150039357
1.086782232018464	1.085804578506801	9.776535116629020
0.961643963745149	0.960721238411472	9.227253336775902
0.850959989751735	0.850092462371894	8.675273798413400
0.753058293877098	0.752245438645303	8.128552317946742
0.666460404722980	0.665701080885221	7.593238377590751
0.589858984665305	0.589151585974473	7.073986908319967
0.522098014270824	0.521440592063824	6.574222070005817
0.462155271525700	0.461545635528996	6.096359967039462
0.409126840097714	0.408562640470280	5.641996274338453
0.362213411636533	0.361692205244996	5.212063915365434
0.320708174333808	0.320227477812472	4.806965213357216
0.283986104029879	0.283543435798430	4.426682314491245
0.251494495431568	0.251087408517156	4.070869144120934
0.222744589818807	0.222370697049488	3.738927693180813
0.197304172252130	0.196961165148969	3.430071031613579
0.174791026000715	0.174476688490656	3.143375100585445
0.154867144914827	0.154579362811027	2.877821037995773
0.137233615964570	0.136970383010987	2.632329535833378
0.121626094333102	0.121385515482326	2.405788507762509
0.107810802441318	0.107591094925714	2.197075156037499
0.095580992228725	0.095380484892298	2.005073364273130
0.084753817042431	0.084570948322124	1.828687203069080
0.075167565699487	0.075000880577901	1.666851215861243
0.066679216781509	0.066527362976519	1.518538049896392
0.059162276077895	0.059023999686941	1.382763909542291
0.052504864388787	0.052379005165367	1.258592234195135
0.046608026696299	0.046493513102394	1.145135939057980
0.041384237070142	0.041280081219892	1.041558502504302
0.036756076642452	0.036661369228723	0.947074137283835
0.032655064611533	0.032568969887201	0.860947243325768
0.029020624555091	0.028942375424455	0.782491306364022
0.025799170385580	0.025728063647760	0.711067378193547
0.022943298094726	0.022878689869694	0.646082250328845
0.020411071038540	0.020352372397285	0.586986412550033
0.018165387932583	0.018112060745509	0.533271870732563
0.016173423981472	0.016124976993072	0.484469884001328

The Table 6 illustrates the impact of the parameter (β), whose influence is not obvious in (D) of Figure 2.

TABLE 7: NUMERICAL RESULTS FOR THE EFFECT OF THE ALREADY PROTECTED RATE ON COVID-19 TRANSMISSION WHEN $\varphi = 0.2066$.

$\varphi = 0.206$ 1.0e+04 *	$\varphi = 0.2066$ 1.0e+04 *	The difference 1.0e-06 *
6.839400000000000	6.839400000000000	0
6.048924317780305	6.048924317780140	0.001651642378420
5.349913718136892	5.349913718136269	0.006228219717741
4.731779230224717	4.731779230223395	0.013220414984971
4.185158079828752	4.185158079826533	0.022184394765645
3.701771680071964	3.701771680068690	0.032749085221440
3.274300069506756	3.274300069502298	0.044579792302102
2.896270892621483	2.896270892615743	0.057396391639486
2.561961238433434	2.561961238426338	0.070955138653517
2.266310847924056	2.266310847915551	0.085052306530997
2.004845373561186	2.004845373551235	0.099509634310380
1.773608526663646	1.773608526652229	0.114174326881766
1.569102083209836	1.569102083196944	0.128919054986909
1.388232837919821	1.388232837905457	0.143640136229806
1.228265701858810	1.228265701842986	0.158246621140279
1.086782232018464	1.086782232001197	0.172667569131590
0.961643963745149	0.961643963726465	0.186841134564020
0.850959989751735	0.850959989731664	0.200718204723671
0.753058293877098	0.753058293855672	0.214260580833070
0.666460404722980	0.666460404700236	0.227437340072356
0.589858984665305	0.589858984641282	0.240224835579284
0.522098014270824	0.522098014245563	0.252607605943922
0.462155271525700	0.462155271499242	0.264572918240447
0.409126840097714	0.409126840070102	0.276113496511243
0.362213411636533	0.362213411607810	0.287227067019558
0.320708174333808	0.320708174304016	0.297913175018039
0.283986104029879	0.283986103999061	0.308174548990792
0.251494495431568	0.251494495399766	0.318016191158677
0.222744589818807	0.222744589786062	0.327444467984606
0.197304172252130	0.197304172218484	0.336468019668246
0.174791026000715	0.174791025966205	0.345096395903965
0.154867144914827	0.154867144879493	0.353339601133484
0.137233615964570	0.137233615928449	0.361209231414250
0.121626094333102	0.121626094296230	0.368716655430035
0.107810802441318	0.107810802403730	0.375873923985637
0.095580992228725	0.095580992190456	0.382693315259530
0.084753817042431	0.084753817003512	0.389187334803864
0.075167565699487	0.075167565659950	0.395368260797113
0.066679216781509	0.066679216741384	0.401248598791426
0.059162276077895	0.059162276037211	0.406840399591601
0.052504864388787	0.052504864347571	0.412155714002438
0.046608026696299	0.046608026654579	0.417206422298477
0.041384237070142	0.041384237027942	0.422004006850329
0.036756076642452	0.036756076599796	0.426559722654929
0.032655064611533	0.032655064568445	0.430884540492116
0.029020624555091	0.029020624511592	0.434989033237798
0.025799170385580	0.025799170341691	0.438883432707371
0.022943298094726	0.022943298050469	0.442577629655716
0.020411071038540	0.020411070993932	0.446081116933783
0.018165387932583	0.018165387887642	0.449403103175428
0.016173423981472	0.016173423936216	0.452552399110573

The Table 7 illustrates the impact of the parameter (φ), whose influence is not obvious in (E) of Figure 2.

TABLE 8: NUMERICAL RESULTS FOR THE EFFECT OF THE DEATH RATE INDUCED COVID-19 ON COVID-19 TRANSMISSION WHEN $\gamma = 0.588$.

γ = 0.58 1.0e+04 *	γ = 0.588 1.0e+04 *	The difference 1.0e+02 *
6.839400000000000	6.839400000000000	0
6.048924317780305	6.044087851464437	0.483646631586817
5.349913718136892	5.341363328345292	0.855038979160017
4.731779230224717	4.720441975793403	1.133725443131407
4.185158079828752	4.171795802683390	1.336227714536217
3.701771680071964	3.687006878261840	1.476480181012448
3.274300069506756	3.258637988633586	1.566208087317063
2.896270892621483	2.880118365118267	1.615252750321670
2.561961238433434	2.545642728168080	1.631851026535333
2.266310847924056	2.250082095200937	1.622875272311831
2.004845373561186	1.988904981516932	1.594039204425426
1.773608526663646	1.758107783209316	1.550074345432950
1.569102083209836	1.554153272109443	1.494881110039278
1.388232837919821	1.373916257487600	1.431658043222087
1.228265701858810	1.214635579384767	1.363012247404295
1.086782232018464	1.073871695767236	1.291053625122786
0.961643963745149	0.949469211672472	1.217475207267762
0.850959989751735	0.839523774472241	1.143621527949436
0.753058293877098	0.742352826485172	1.070546739192578
0.666460404722980	0.656469765457246	0.999063926573399
0.589858984665305	0.580561115806312	0.929786885899257
0.522098014270824	0.513466359800782	0.863165447004258
0.462155271525700	0.454160118724359	0.799515280134065
0.409126840097714	0.401736410196433	0.739042990128032
0.362213411636533	0.355394739726709	0.681867190982371
0.320708174333808	0.314427812773259	0.628036156054850
0.283986104029879	0.278210678478738	0.577542555114114
0.251494495431568	0.246191138262811	0.530335716875657
0.222744589818807	0.217881271888147	0.486331793066001
0.197304172252130	0.192849950791320	0.445422146081025
0.174791026000715	0.170716223642767	0.407480235794744
0.154867144914827	0.151143472504575	0.372367241025211
0.137233615964570	0.133834249797563	0.339936616700732
0.121626094333102	0.118525716751748	0.310037758135425
0.107810802441318	0.104985613257708	0.282518918361004
0.095580992228725	0.093008697202703	0.257229502602244
0.084753817042431	0.082413598590097	0.234021845233403
0.075167565699487	0.073040040114641	0.212752558484574
0.066679216781509	0.064746381497405	0.193283528410366
0.059162276077895	0.057407449859212	0.175482621868306
0.052504864388787	0.050912622806750	0.159224158203733
0.046608026696299	0.045164134788677	0.144389190762242
0.041384237070142	0.040075580709714	0.130865636042845
0.036756076642452	0.035570593821590	0.118548282086165
0.032655064611533	0.031581677587457	0.107338702407624
0.029020624555091	0.028049173582063	0.097145097302765
0.025799170385580	0.024920349580026	0.087882080555374
0.022943298094726	0.022148593831025	0.079470426370167
0.020411071038540	0.019692703152089	0.071836788645154
0.018165387932583	0.017516253908374	0.064913402420898
0.016173423981472	0.015587046227154	0.058637775431815

The Table 8 illustrates the impact of the parameter (γ), whose influence is not obvious in (F) of Figure 2.

6- Conclusion

The findings indicate that augmenting the parameter values of ε (the effective force of infection), and δ (rate of loss of protection), while holding the remaining parameters steady elevates the $\mathcal{R}_0^D$ value, thereby enhancing the disease's endemicity. The reason for this phenomenon is that individuals who are susceptible to the disease become infected as a result of either direct or indirect contact with individuals who are already infected. Subsequently, certain individuals who have contracted the infection may commence recuperation or mortality, thereby instigating a gradual reduction in the infected populace. Elevating the values of parameters ω (rate of treatment), β (natural death rate), φ (already protected rate), and γ (death rate induced COVID-19), while holding the remaining parameters constant results in a reduction of the value of $\mathcal{R}_0^D$. This decrease in $\mathcal{R}_0^D$ indicates a decrease in the endemicity of the disease.

The results of the analysis indicate a discernible correlation between vaccination and the mitigation of the spread of COVID-19. Consequently, precautionary measures are implemented to mitigate the risk of exposure to the virus. To provide a sustainable solution, it is recommended to offer vaccination. This plays a vital role in mitigating the number of cases of infection in Najaf.

In order to further enhance the scope of this study, it could be to employ sensitivity analysis technique to examine the correlation between the parameters of the mathematical model and their influence on the propagation of the disease, hence mitigating infection rates and augmenting immunity.

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