ON QUASI- CONFLUENT MAPPINGS

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ABSTRACT

In this work we study quasi-confluent mappings, where we the relation between these mappings and confluent mappings. Also, we study some operations on quasi-confluent mappings, such as: the pullback, factrization, composition and product of quasi-confluent mappings .

1 . Introduction :

In this work a space ***X*** is a topological space (X,T), a "***mapping***" is a continuous function and, we use "→→" to denote the mapping is " ***onto***" .

Let X be a space and let p be any point in X, then :

i)The ***component*** of p in X is the largest connected subset of X containing p and it is denoted by C(X,p),[5] .If m(X) = number of components in the space X, then the space X is connected if and only if, m(X)=1, which means X will be disconnected if and only if, m(X)>1.

ii)The ***quasi-component*** of p in X is the set containing of p together with all points x of X such that there exist no disjoint two open sets W and V with $X = W\cup V$, where $x\in W$ and $p\in V$, and it is denoted by Q(X, p).Also, we can say that it is the intersection of all non- empty clopen sets in X containing p.Q(X,p) may be different from C(X, p), and each component of a space X is contained in some quasi-component of X. In a compact Hausdorff space X, the quasi-components are connected and coincide with components of X, [3].

We will use the symbol 2 to indicate the end of the proof.

2 . Basic definitions and Examples :

In this section we recall the basic definitions needed in this work.

2.1 Definition[1]: A subset K of a space X is said to be ***continuum*** if, K is connected and compact.

2.2 Definition[6]: A subset U of a space X is said to be ***strongly connected*** if, for any open subsets W and V of X, if $U\subseteq W\cup V$, then either $U\subseteq W$ or $U\subseteq V$**.**

2.3 **Definition**[2] :A mapping f: X→→Y is said to be ***confluent*** if, given a continuum $K\subseteq Y$, every component C of $f^{-1}(K)$ we have f (C) = K .

2.4 Example : Let X = [2,5] and Y = [4,25] with the usual topology defined on X and Y. Let f : X→→Y be a function defined by: $f(x) = x^2$, $\forall x\in X$.

f is a confluent mapping.

2.5 Definition[4] : A mapping f: X→→Y is said to be ***quasi-confluent*** if, for each continuum K in Y, and for each quasi-component C of $f^{-1}(K)$, we have f(C) = K.

2.6 Remark: Every confluent mapping is a quasi-confluent mapping, but the converse is not true in general since Q(X,p) may be different from C(X, p), [3].

2.7 Theorem [4]: Every quasi-confluent mapping f: X→→Y of a compact Hausdorff space X onto a Hausdorff space Y is confluent.

3. Pullback of Quasi-Confluent Mappings :

In this section , we study the pullback of quasi-confluent mappings . So, we recall the following definitions :

3.1 Definition [5]: A ***fiber structure*** is a triple (X, f, Y) consisting of two spaces X and Y, and a mapping f: X→→Y.

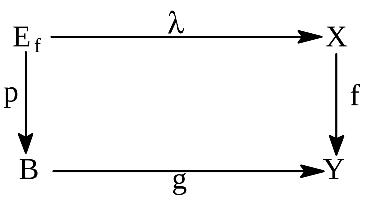
The space X is said to be the ***fibered*** (or, ***total***) space, f is termed the ***projection***, and Y is the ***base*** space.

Next, we recall the definition of the pullback.

3.2 Definition [5]: Let (X, f, Y) be a fiber structure. Let B be any space and let g: B ⟶Y be any mapping into the base Y. Let E_f be a subspace of the Cartesian product X×B where $E_f = \{(x,b): f(x) = g(b)\}$, and let p:E_f →→B be the projection of E_f in B such that $p(x,b) = b$, $\forall(x,b)\in E_f$.

The fiber structure (E_f , p, B) is said to be the ***fiber structure over B*** induced by the mapping g and the projection p is said to be the ***pullback*** of f by g.

Now, let λ: E_f⟶X be the projection such that $\lambda(x,b) = x$, $\forall\ (x,b)\in E_f$.



We observe that the following diagram is commutative.

Before we prove the main result in this section, we state and prove the following lemma.

3.3 Lemma [4]: Let f: X→→Y be a mapping, let B be any space, and let g: B⟶Y be any mapping, if K⊆B,then $p^{-1}(K)=f^{-1}(g(K))\times K$,where p is the pullback of f by g.

3.4 Theorem: The pullback of a quasi-confluent mapping is a quasi-confluent mapping.

Proof: Let f: X→→Y be any quasi-confluent mapping, let B be any space, and let g: B ⟶Y be any mapping.Let K be any continuum in B, and C be any quasi-component of $p^{-1}(K)$, where p is the pullback of f by g.Then, C is a quasi-component of $f^{-1}(g(K))\times K$ by (3.3).So, there exists a quasi-component C_1 in $f^{-1}(g(K))$ such that $C = C_1\times K$.Since f is a quasi-confluent mapping, and since g(K) is a continuum in Y.So, $f(C_1) = g(K)$.Therefore, $p(C) = p(C_1\times K) = K$.Therefore, p is a quasi-confluent mapping. 2

4. Factorization of Quasi- Confluent Mappings :

In the present section, we study a direct consequence of Whyburn's factorization theorem for quasi-confluent mappings.So, we recall few definitions that will be needed in this section.First, we recall the definition of a factorable mapping.

4.1 Definition [1]: If h: X→→W is a mapping, any representation of h in the form h = g∘f where f: X→→Y and g: Y→→W are two mappings and Y is a certain space, will said to be ***factorization*** of h, and h is said to be a ***factorable*** mapping and Y is a ***middle*** space.

4.2 Example : Let h: R→→R^+∪{0} be a mapping defined by: $h(x) = 9x^2 + 6x + 1$, ∀ x∈R. The mapping h is factorable and has a factorization in the form h = g∘f, where f : R→→R is a mapping defined by: f(x) = 3x+1, ∀ x∈ R , and g: R→→R^+ ∪{0}is a mapping defined by: $g(x) = x^2$, ∀x∈R.

4.3 Theorem[4]: If h: X→→W is a confluent mapping of a strongly connected compact space X onto a Hausdorff space W, then there exists a unique factorization for h into two confluent mappings in the form h = g∘f.

Now, we can get the factorization of a quasi- confluent mapping in the following theorem :

4.3 Theorem: If h:X→→W is a quasi-confluent mapping of a strongly connected compact Hausdorff space X onto a Hausdorff space W, then there exists a unique factorization for h into two quasi-confluent mappings in the form h = g∘f.

5. Composition of Quasi - Confluent Mappings :

In the present section, we study the composition of two quasi-confluent mappings. So , we recall the following theorem :

5.1 Theorem [4]: If f: X→→Y and g: Y→→W are two confluent mappings, then h = g∘f is a confluent mapping.

5.2 Theorem [4]: If X , Y and W are Hausdorff spaces,X is a compact space , and if f: X→→Y and g: Y→→W are two quasi-confluent mappings, then h = g∘f is a quasi-confluent mapping.

Proof : Since f and g are two quasi-confluent mappings, and since X and Y are two compact Hausdorff spaces. Then , from (2.7) f and g are confluent mappings .Thus, h = g∘f is a confluent mapping , by (5.1) . From (2.5), h= g∘f: X→→W is a quasi-confluent mapping. 2

From (2.6) , we canclude that the following result:

5.3 Corllary: If X , Y and W are three spaces, and if f: X→→Y and g: Y→→W are two confluent mappings, then h = g∘f is a quasi-confluent mapping.

From (2.7) , we can get the following results:

5.4 Corllary : If X, Y and W are Hausdorff spaces, X is a compact space, and if f: X→→Y and g: Y→→W are two quasi-confluent mappings, then h = g∘f is a confluent mapping.

5.5 Corllary : If X, Y and W are Hausdorff spaces, X is a compact space, and if f: X→→Y is a confluent mapping and g: Y→→W is a quasi-confluent mapping (or,if f: X→→Y is a quasi- confluent mapping and g: Y→→W is a confluent mapping), then h = g∘f is a confluent mapping (or, h = g∘f is a quasi-confluent mapping).

6. Product of Quasi -C onfluent Mappings*:*

In this section, we study the product of quasi-confluent mappings.So, we recall the following theorem:

6.1 Theorem [4]: If X and Y are two strongly connected compact spaces, W and V are two Hausdorff spaces, and if f: X→→W and g: Y→→V are two confluent mappings, then f×g: X×Y→→W×V is a confluent mapping.

6.2 Theorem: If X and Y are two strongly connected compact Hausdorff spaces, W and V are two Hausdorff spaces, and if f: X→→W and g: Y→→V are two quasi-confluent mappings, then f×g: X×Y→→W×V is a quasi-confluent mapping.

Proof: Since f and g are quasi-confluent mappings of a compact Hausdorff space onto a Hausdorff space . Thus, f and g are confluent mappings , by (2.2). From (6.1) we get that f×g is a confluent mapping . Therefore, f×g is a quasi-confluent mapping , by(2.4) . 2

From (2.6) , we canclude that the following result:

6.3 Corllary: If X and Y are two strongly connected compact spaces, W and V are two Hausdorff spaces, and if f: X→→W and g: Y→→V are two confluent mappings, then f×g: X×Y→→W×V is a quasi-confluent mapping.

From (2.7) , we can get the following results:

6.4 Corllary: If X and Y are two strongly connected compact Hausdorff spaces, W and V are two Hausdorff spaces, and if f: X→→W and g: Y→→V are two quasi-confluent mappings, then f×g: X×Y→→W×V is a confluent mapping.

6.5 Corllary: If X and Y are two strongly connected compact Hausdorff spaces, W and V are two Hausdorff spaces, and if f: X→→W is a confluent mapping and g: Y→→V is a quasi-confluent mapping(or,if f: X→→Y is a quasi-confluent mapping and g: Y→→W is a confluent mapping), then is a confluent mapping. (or, f×g: X×Y→→W×V is a quasi-confluent mapping).

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حول التطبيقات شبه – المندمجة

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الخلاصة

في هذا العمل درسنا التطبيقات شبه - المندمجة ، حيث درسنا العلاقة بين هذه التطبيقات والتطبيقات المندمجة . وكـذلك درسـنا بعـض العمليات على التطبيقات شبه- المندمجة ، مثل السحب الخلفي ، التحليل ، التركيب و الضرب للتطبيقات شبه- المندمجة.