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THE CYCLIC DECOMPOSITION OF THE FACTOR GROUP CF(D_{nh},Z)/R($\overline{D_{nh}}$) WHEN N IS AN ODD NUMBER

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ABSTRACT

For fixed positive integer $n \geq 3$,let $\mathbf{D_n}$ be **the dihedral group**, $\mathbf{D_{nh}} = \mathbf{D_n} \times \mathbf{C_2}$ and cf(D_{nh},Z) be the abelian group of Z-valued class functions of the group $\mathbf{D_{nh}}$ .The intersection of cf(D_{nh},Z) with the group of all generalized characters of D_{nh} , R(D_{nh}) is a normal subgroup of cf(D_{nh},Z) denoted by $\overline{R}$ (D_{nh}),then cf(D_{nh},Z)/ $\overline{R}$ (D_{nh}) is a finite abelian factor group which is denoted by K(D_{nh}).

In this paper ,we determine the cyclic decomposition of the finite abelian factor group **cf(D_{nh},Z)/R(D_{nh})** when n is an odd number, we find that the cyclic decomposition of K(D_{nh}) depends on the elementary divisor of n ,if n= $p_1^{\alpha_1}.p_2^{\alpha_2}..p_m^{\alpha_m}$ where $p_1, p_2, \ldots, p_m$ are distinct primes and $\alpha_1, \alpha_2, .., \alpha_m$ are positive integers, then ;

$$K(D_{nh}) = \bigoplus_{i=1}^{2} K(D_n) \quad \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)-2} C_2 \quad \oplus \quad K(C_4).$$

And we find the rational valued characters table of the group D_{nh}.

1. Introduction

Let G be a finite group ,two elements of G are said to be Γ- ***conjugate*** if the cyclic subgroups they generate are conjugate in G, this defines an equivalence relation on G .Its classes are called Γ- ***classes*** . The Z - valued class function on the group G , which is constant on the Γ- ***classes*** forms a finitely generated abelian group cf(G,Z) of a rank equal to the number of Γ- ***classes*** .

The intersection of cf(G,Z) with the group of all generalized characters of G ,R(G) is a normal subgroup of cf(G,Z) denoted by $\overline{R}(G)$,then cf(G,Z)/$\overline{R}(G)$ is a finite abelian factor group which is denoted by K(G).

Each element in $\overline{R}(G)$ can be written as $u_1\theta_1 + u_2\theta_2 + \ldots\ldots + u_l\theta_l$, where l is the number of Γ - ***classes*** , u_1 , u_2 , ... , $u_l \in Z$ and $\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$, where χ_i is an irreducible character of the group G and σ is any element in Galios group $Gal(Q(\chi_i)/Q)$.

Let ≡*(G) denotes the $l \times l$ matrix which corresponds to the θ_i's and columns correspond to the Γ- ***classes*** of G .The matrix expressing $\overline{R}(G)$ basis in terms of the cf(G,Z) basis is ≡*(G).

We can use the theory of invariant factors to obtain the direct sum of the cyclic Z-module of orders the distinct invariant factors of ≡*(G) to find the cyclic decomposition of K(G).In1982 M.S.Kirdar[13] studied the K(C_n).In 1994 H.H. Abass[3]studied the K(D_n) and found $\equiv^*(D_n)$.In 1995 N.R. Mahamood [15] studied the factor group cf(Q_{2m},Z) / $\overline{R}$(Q_{2m}).In 2005 N.S. Jasim [16] studied the factor group cf(G,Z)/$\overline{R}(G)$ for the special linear group SL(2,p).

In this paper we study K(D_{nh}) and find $\equiv^*(D_{nh})$ when n is an odd number .

2. Preliminaries

In this section we review definitions and some results which will be used in later section.

Definition (1.1):[1]

The set of all $l \times l$ non-singular matrices over the field F which forms a group under the operation of the matrix multiplication is called ***the general linear group*** of the dimension l over the field F, denoted by GL (l,F).

Definition (1.2): [1]

A matrix representation of a group G is a group homomorphism T of G into GL (l,F), l is called ***the degree of matrix representation T*** .

Definition (1.3): [1]

The trace of an $l \times l$ matrix is the sum of the main diagonal elements , denoted by tr(A) .

Definition (1.4): [3]

A matrix representation T: G→GL (l, F) is said to be ***reducible*** if there exists a non-singular matrix A over F such that:

$$A^{-1}\ T(g)\ A=\begin{bmatrix} T_1(g) & E(g) \\ o & T_2(g) \end{bmatrix}\ \text{, for all } g\in G.$$

Where $T_1(g)$, $T_2(g)$ are matrices of representations T_1 and T_2 of a group over F of the dimension r×r, s×s respectively and E(g) is a matrix of the dimension r×s such that $0< r < l$ and $r+s= l$. If no such reducible matrix exists then T is called ***an irreducible matrix representation***.

Theorem (1.5):[1]

Let $T_1: G_1 \rightarrow GL(V_1)$ and $T_2: G_2 \rightarrow GL(V_2)$ be two irreducible representations of the groups G_1 and G_2 respectively, then $T_1 \otimes T_2$ is irreducible representations of the group G_1 Ï G_2 .

Definition (1.6): [3]

Let T be a matrix representation of a group G over the field F, ***the character*** χ of a matrix representation T is the mapping χ: G→F defined by $\chi(g)=Tr(T(g))$ for all $g\in G$ where Tr(T(g)) refers to the trace of the matrix T(g) and $\chi(1)$ is the degree of χ .

Remark (1.7):

(i)A finite group G has a finite number of conjugacy classes and a finite number of distinct irreducible character, the group character of a group representation is constant on a conjugacy class , the values of irreducible characters can be written as a table whose columns are the conjugacy class and rows the value of irreducible characters on each conjugacy class , this table of the group G, denoted by ≡(G).

(ii) If $G = C_n =<r>$ is the cyclic group of order n generated by r .If $\omega = e^{2\pi i/n}$ is the primitive n-th root of unity , the

≡(C_n)=

CL_α	1	r	r^2	…	r^{n-1}
$\lvert CL_\alpha \rvert$	1	1	1	…	1
$\lvert C_G(C_\alpha) \rvert$	n	n	n	…	n
χ_1	1	1	1	…	1
χ_2	1	ω	ω^2	…	ω^{n-1}

χ_3	1	ω^2	ω^4	$\cdots$	ω^{n-2}
j	j	j	j	l	j
χ_n	1	ω^{n-1}	ω^{n-2}	$\cdots$	ω

Definition (1.8):[3]

Let χ and ψ as characters of a group G, then :

1. The sum of characters is defined by:

$(\chi+\psi)(g) = \chi(g)+\psi(g)$, for all $g\in G$

2. The product of characters is defined by : $(\chi.\psi)(g) = \chi(g).\psi(g)$, for all $g\in G$.

Theorem (1.9):[3]

Let T_1: $G_1 \rightarrow GL(n,K)$ and T_2: $G_2 \rightarrow GL(m,K)$ are two matrix representations of the groups G_1 and G_2 , χ_1 and χ_2 be two characters of T_1 and T_2 respectively, then the character of $T_1 \otimes T_2$ is $\chi_1\chi_2$.

Definition (1.10):[1]

A rational valued character θ of G is a character whose values are in the set of integers Z, which is $\theta(g) \in Z$, for all $g \in G$.

Proposition (1.11):[4]

The rational valued characters $\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$ form basis for $\overline{R}(G)$, where χ_i are the irreducible characters of G and their numbers are equal to the number of all distinct Γ- classes of G.

2. The factor group K(G)

In this section , we study the factor K(G) and discuss the cyclic decomposition of the factor groups $K(C_n)$ and $K(D_n)$.

Definition (2.1):[4]

Let M be a matrix with entries in a principal ideal domain R , a k-minor of M is the determinate of $k\times k$ sub matrix preserving rows and columns order.

Definition (2.2):[4]

A k-th determinant divisor of M is the greatest common divisor (g.c.d) of all the k-minors of M . This is denoted by D_k(M).

Lemma (2.3):[4]

Let M, P and W be matrices with entries in a principal ideal domain R , if P and W are invertible matrices, then D_k(P M W)= D_k(M) modulo the group of unites of R.

Theorem (2.4):[4]

Let M be an $l\times l$ matrix entries in a principal ideal domain R, then there exists matrices P and W such that:

1- P and W are invertible.
2- P M W = D.
3- D is diagonal matrix.
4- if we denote D_{ii} by d_i then there exists a natural number m ;

$0 \le m \le l$ such that $j > m$ implies $d_j = 0$ and $j \le m$ implies $d_j \ne 0$ and $1 \le j \le m$ implies $d_j \mid d_{j+1}$.

Definition (2.5):[4]

Let M be a matrix with entries in a principal ideal domain R be equivalent to a matrix D=diag $\{d_1, d_2, \dots, d_m, 0, 0, \dots, 0\}$ such that $d_j \mid d_{j+1}$ for $1 \le j < m$.

We call D ***the invariant factor matrix of*** M and $d_1, d_2, \dots, d_m$ the invariant factors of M.

Theorem (2.6):[4]

Let K be a finitely generated module over a principal ideal domain R, then K is the direct sum of a cyclic sub modules with an annihilating ideal $<d_1>, <d_2>, \dots ,<d_m>$, $d_j \mid d_{j+1}$ for j = 1, 2, … , K-1.

Proposition(2.7):[4]

Let A and B be two non-singular matrices of the rank n and m respectively, over a principal ideal domain R . Then the invariant factor matrices of A $\otimes$ B equals D(A) $\otimes$ D(B) , where D(A)and D(B) are the invariant factor matrices of A and B respectively .

Theorem(2.8):[4]

Let H and L be p_1-group and p_2-group respectively ,where p_1 and p_2 are distinct primes . Then , $\equiv^*(H \times L) = \equiv^*(H) \otimes \equiv^*(L)$.

Remark (2.9):[4]

Suppose cf(G,Z) is of the rank l , the matrix expressing the $\overline{R}$ (G) basis in terms of the cf(G,Z) = Z^l basis is $\equiv^*(G)$.

Hence by theorem (2.4),we can find two matrices P and Q with a determinant ± 1 such that $P.\equiv^*(G).Q = D(\equiv^*(G)) = diag\{d_1,d_2,\dots\dots\dots,d_l\}$, $d_i = \pm D_i(\equiv^*(G)) / \pm D_{i-1}(\equiv^*(G))$.

this yields a new basis for $\overline{R}$ (G) and Cf(G,Z),$\{v_1,v_2,\dots\dots, v_l\}$ and $\{u_1,u_2,\dots\dots, u_l\}$ respectively with the property $v_j = d_j u_j$.

Hence by theorem (2.6) the Z-module K(G) is the direct sum of cyclic submodules with annihilating ideals $<d_1>, <d_2>, \dots ,<d_l>$.

Theorem(2.10) :[4]

Let p be a prime number , then :

$$K(G) = \oplus \sum_{i=1}^{s} C_{d_i} \quad \text{such that } d_i = \pm D_i(\equiv^*(G)) / \pm D_{i-1}(\equiv^*(G)) .$$

Theorem (2.11):[4]

$|K(G)| = \det(\equiv^*(G))$.

Proposition (2.3.1):[14]

The ***rational valued characters table of the cyclic group*** C_{p^s} of the rank s+1 where p is a prime number which is denoted by ($\equiv^*(C_{p^s})$), is given as follows:

Γ-classes	[1]	$[r^{p^{s-1}}]$	$[r^{p^{s-2}}]$	$[r^{p^{s-3}}]$	…	$[r^{p^2}]$	$[r^p]$	[r]
θ_1	$p^{s-1}(p-1)$	$-p^{s-1}$	0	0	…	0	0	0
θ_2	$p^{s-2}(p-1)$	$P^{s-2}(p-1)$	$-p^{s-2}$	0	…	0	0	0
θ_3	$p^{s-3}(p-1)$	$P^{s-3}(p-1)$	$p^{s-3}(p-1)$	$-p^{s-3}$	…	0	0	0
					⋮			
θ_{s-1}	p(p-1)	p(p-1)	p(p-1)	p(p-1)	…	p(p-1)	-p	0
θ_s	p-1	p-1	p-1	p-1	…	p-1	p-1	-1
θ_{s+1}	1	1	1	1	…	1	1	1

where its rank s+1 represents the number of all distinct Γ-classes.

Example (2.13):

Consider the cyclic group C_{25} by using table (2.3), we can find the rational valued characters table of C_{25} as follows:

$\equiv^*(C_{25}) = \equiv^*(C_{5^2}) =$

Γ-classes	[1]	$[r^5]$	$[r]$
θ_1	20	-5	0
θ_2	4	4	-1
θ_3	1	1	1

Remark (2.14):

In general, for $n = p_1^{\alpha 1} . p_2^{\alpha 2} . \dots . p_m^{\alpha m}$ where g.c.d $(p_i, p_j) = 1$, if $i \neq j$, p_i's are prime numbers and $\alpha_i \in Z^+$, then we have the following formula :

$\equiv^*(C_n) = \equiv^*(C\ p_1^{\alpha 1}) \otimes \equiv^*(C\ p_2^{\alpha 2}) \otimes \dots \otimes \equiv^*(C\ P_m^{\alpha m})$.

Proposition (2.14):[4]

If p is a prime number , then $D(\equiv^*(C_{p^s})) = diag\{p^s, p^{s-1}, \dots\dots, p, 1\}$.

Remark (2.15):[4]

For $n = p_1^{\alpha 1} . p_2^{\alpha 2} . \dots . p_m^{\alpha m}$ where p_i's are distinct primes and $\alpha_i \in Z^+$, then :

$D(\equiv^*(C_n)) = D(\equiv^*(C\ p_1^{\alpha 1})) \otimes D(\equiv^*(C\ p_2^{\alpha 2})) \otimes \dots \otimes D(\equiv^*(C\ P_m^{\alpha m}))$.

Theorem(2.16) :[4]

Let p be a prime number , then :$K(C_{p^s}) = \oplus \sum_{i=1}^{s} C_{P^i}$.

Example(2.17):-

$K(C_{25}) = K(C_{5^2}) = C_5 \oplus C_{5^2}$.

Proposition(2.18):[4]

Let $n = \prod_{i=1}^{k} P_i^{a_i}$,where p_i's are distinct primes and $a_i \in Z^+$,then :

$$K(C_n) = \oplus \sum_{i=1}^{k} \left(\oplus \sum K\left(C_{P_i^{a_i}} \right)\right) \left[\prod_{\substack{j \neq i \\ j=1}}^{k} (a_j + 1) \right] \ time .$$

Example(2.19) :

Find $K(C_{540})$

$$K(C_{540}) = K(C_{2^2.3^3.5}) = \underbrace{K(C_{2^2}) \oplus \cdots \oplus K(C_{2^2})}_{(1+3).(1+1)\ times} \oplus \underbrace{K(C_{3^3}) \oplus \cdots \oplus K(C_{3^3})}_{(2+1).(1+1)\ times} \oplus \underbrace{K(C_{5}) \oplus \cdots \oplus K(C_{5})}_{(2+1).(3+1)\ times}$$

By theorem (2.16) we can find :

$K(C_{2^2}) = C_{2^2} \oplus C_2$, $K(C_{3^3}) = C_{3^3} \oplus C_{3^2} \oplus C_3$ and $K(C_5) = C_5$

Then $K(C_{540}) = \overset{(8)}{C_{2^2}} \oplus \overset{(8)}{C_2} \oplus \overset{(6)}{C_{3^3}} \oplus \overset{(6)}{C_{3^2}} \oplus \overset{(6)}{C_3} \oplus \overset{(12)}{C_5}$.

Definition (2.20):[3]

For a fixed positive integer $n \geq 3$,***the dihedral group D_n*** is a certain non- abelian group of the order 2n . In general can write it as: $D_n=\{ S^j r^k : 0 \leq k \leq n-1 , 0 \leq j \leq 1\}$ which has the following properties : $r^n = 1$, $S^2 = 1$, $Sr^kS^{-1} = r^{-k}$.

Definition (2.21):[3]

The group D_{nh} is the direct product group **D_n Ï C_2** .

Lemma (2.22):[2]

The rational valued characters table of D_n when n is an odd number is given as follows:

≡*(D_n)=

	Γ- classes of C_n	[S]
θ_1	≡*(C_n)	0
	1 1 1 … 1 1	
θ_{l-1}		0
θ_l		1
θ_{l+1}	1 1 1 … 1 1	-1

Where l is the number of Γ- classes of C_n .

Example (2.23):

To find $\equiv^{*}(D_{25})$,From example (2.13), we obtain $\equiv^{*}(C_{25})$ and by using lemma (2.22) , we have

$\equiv^{*}(D_{25}) = \equiv^{*}(D_{5^2})$

Γ-classes	[1]	$[r^5]$	$[r]$	[S]
θ_1	$\equiv^{*}(C_{5^2})$			0
θ_2	1	1	1	0
θ_3				1
θ_4	1	1	1	-1

=

Γ-classes	[1]	$[r^5]$	$[r]$	[S]
θ_1	20	-5	0	0
θ_2	4	4	-1	0
θ_3	1	1	1	1
θ_4	1	1	1	-1

Proposition(2.24):[2]

$D(\equiv^{*}(D_n))=\left[\begin{array}{c|c} D\left(\equiv^{*}(C_n)\right) & 0 \\ \hline 0 & -2 \end{array}\right]$ Where $D(\equiv^{*}(D_n))$ and $D(\equiv^{*}(C_n))$ are the invariant factors matrices of $\equiv^{*}(D_n)$ and $\equiv^{*}(C_n)$ respectively .

Theorem(2.25) :- [2]

Let $n=\prod_{i=1}^{k} P_i^{a_i}$,where P_i are distinct primes , $a_i \in Z^{+}$ and $P_i \neq 2$ for all i ,then :

$K(D_n)= K(C_n) \oplus C_2$

Example(2.26):-

To find $K(D_{540})$

$K(D_{540}) = K(C_{540}) \oplus C_2$

From example(2.19) we find K(C_{540}) , then we hav

K(C_{540}) = $\overset{(8)}{C_{2^2}} \oplus \overset{(8)}{C_2} \oplus \overset{(6)}{C_{3^3}} \oplus \overset{(6)}{C_{3^2}} \oplus \overset{(6)}{C_3} \oplus \overset{(12)}{C_5}$.

3. The Main Results

This section is devoted to study the rational valued characters table of the group D_{nh} and to find the cyclic decomposition of K(D_{nh}) where n is an odd number .

Theorem(3.1) :-

The rational valued characters table of the group D_{nh} when n is an odd number is given as follows: $\equiv^*(D_{nh}) = \equiv^*(D_n) \otimes \equiv^*(C_2)$

Proof :-

since

$\equiv C_2 =$

	g'_1	g'_2
χ'_1	1	1
χ'_2	1	-1

and by proposition (2.12),

$\equiv^*(C_2)$ =

	g'_1	g'_2
θ'_1	1	1
θ'_2	1	-1

Table (3.5)

then, $\chi'_1(g'_1) = \chi'_1(g'_2) = \theta'_1(g'_1) = \theta'_1(g'_2) = 1$ $\chi'_2(g'_1) = \theta'_2(g'_1) = 1$, $\chi'_2(g'_2) = \theta'_2(g'_2) = -1$.

from the definition of D_{nh} , (Theorems (1.5) and (1.9)), $\equiv D_{nh} = \equiv D_n \otimes \equiv C_2$, each element in D_{nh} $g_{kh} = g_k \cdot g'_h$ $\forall$ $g_k \in D_n$, $g'_h \in C_2$, $k = 1,2,3,\ldots\ldots,2n$, $h = 1,2$ and each irreducible character of D_{nh} is $\chi_{ij} = \chi_i . \chi'_j$ where χ_i is an irreducible character of D_n and χ'_j is the irreducible character of C_2 ,then

$$\chi_{ij}(g_{kh}) = \begin{cases} \chi_i(g_k) & if \quad j=1 \quad and \quad h=1,2 \\ \chi_i(g_k) & if \quad j=2 \quad and \quad h=1 \\ -\chi_i(g_k) & if \quad j=2 \quad and \quad h=2 \end{cases}$$

from proposition (1.11) $\theta_{ij} = \sum_{\sigma\in cal(Q(\chi_{ij})/Q)} \sigma(\chi_{ij})$ where θ_{ij} is the rational valued character of D_{nh}

Then, $\theta_{ij}(g_{kh}) = \sum_{\sigma\in Gal(Q(\chi_{ij}(g_{kh}))/Q)} \sigma(\chi_{ij}(g_{kh}))$

(I) If j=1 and h= 1,2 $\theta_{ij}(g_{kh}) = \sum_{\sigma\in Gal(Q(\chi_i(g_k))/Q)} \sigma(\chi_i(g_k)) = \theta_i(g_k) \, . \, 1 = \theta_i(g_k) \, . \, \theta'_j(g'_h)$ where θ_i is the rational valued character of D_n .

(II) (a) If j=2 and h=1

$$\theta_{ij}(g_{kh}) = \sum_{\sigma\in Gal(Q(\chi_i(g_k))/Q)} \sigma(\chi_i(g_k)) = \theta_i(g_k) \, . \, 1 = \theta_i(g_k) \, . \, \theta'_j(g'_h)$$

(b) If j=2 and h=2

$$\theta_{ij}(g_{kh}) = \sum_{\sigma\in Gal(Q(\chi_i(g_k))/Q)} \sigma(-\chi_i(g_k)) = -\sum_{\sigma\in Gal(Q(\chi_i(g_k))/Q)} \sigma(\chi_i(g_k))$$

$$= \sum_{\sigma\in Gal(Q(\chi_i(g_k))/Q)} \sigma(\chi_i(g_k)) \, . \, -1 = \theta_i(g_k) \, . \, \theta'_j(g'_h) \, .$$

From [I] and [II] we have $\theta_{ij} = \theta_i \, . \, \theta'_j$ .Then $\equiv^*(D_{nh}) = \equiv^*(D_n) \otimes \equiv^*(C_2)$.

Example(3.2):-

To find the rational valued characters table of D_{25h} , we can use theorem (3.1) .

By lemma (2.23) and proposition (2.12), we have

$\equiv^*(D_{5^2}) =$

Γ-classes	[1]	$[r^5]$	$[r]$	[S]
θ_1	20	-5	0	0
θ_2	4	4	-1	0
θ_3	1	1	1	1
θ_4	1	1	1	-1

$\equiv^*(C_2) =$

	1	r^*
θ'_1	1	1
θ'_2	1	-1

Then**,** by theorem (3.1) $\equiv^*(D_{25h}) = \equiv^*(D_{25}) \otimes \equiv^*(C_2) \equiv^*(D_{25h})$

Γ- Classes	[1,1]	$[1,r^*]$	$[r^5,1]$	$[r^5,r^*]$	$[r,1]$	$[r,r^*]$	[S,1]	$[S,r^*]$
$\|CL_\alpha\|$	1	1	2	2	2	2	5	5
$\|C_G(CL_\alpha)\|$	20	20	10	10	10	10	4	4
θ_{11}	20	20	-5	-5	0	0	0	0
θ_{12}	20	-20	-5	5	0	0	0	0
θ_{21}	4	4	4	4	-1	-1	0	0
θ_{22}	4	-4	4	-4	-1	1	0	0
θ_{31}	1	1	1	1	1	1	1	1
θ_{32}	1	-1	1	-1	1	-1	1	-1
θ_{41}	1	1	1	1	1	1	-1	-1
θ_{42}	1	-1	1	-1	1	-1	-1	1

Theorem(3.3) :-

For a fixed positive odd integer n such that $n = p_1^{\alpha 1} \cdot p_2^{\alpha 2} \cdots p_m^{\alpha m}$ where $p_1, p_2, \ldots, p_m$ are distinct primes and $\alpha_1, \alpha_2, .., \alpha_m$ are positive integers, then ;

$$K(D_{nh}) = \bigoplus_{i=1}^{2} K(D_n) \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)-2} C_2 \bigoplus K(C_4) .$$

Proof:-

By theorem (3.1) and proposition(2.7) $D(\equiv^*(D_{nh})) = D(\equiv^*(D_n)) \otimes D(\equiv^*(C_2))$ By proposition(2.24)

$$D(\equiv^*(D_n)) = \left[\begin{array}{c|c} D\left(\equiv^*(C_n)\right) & 0 \\ \hline 0 & -2 \end{array}\right]$$

Then ,

$$D(\equiv^*(D_{nh})) = \begin{bmatrix} D(\equiv^*(C_n)) & 0 \\ 0 & -2 \end{bmatrix} \otimes \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} = \left[\begin{array}{cc|cc} 2D(\equiv^*(C_n)) & 0 & & \\ 0 & -4 & \multicolumn{2}{c}{0} \\ \hline & & -D(\equiv^*(C_n)) & 0 \\ \multicolumn{2}{c|}{0} & 0 & 2 \end{array}\right]$$

$$= \text{diag}\{2d_1, 2d_2, \ldots, 2d_{((\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1))}, -4, -d_1, -d_2, \ldots, -d_{((\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1))}, 2\}$$

where d_i is the invariant factor of $\equiv^*(C_n)$. Then , by theorem (2.10)

$$K(D_{nh}) = \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)} C_{2d_i} \oplus C_4 \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)} C_{d_i} \oplus C_2$$

$$= \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)} C_{d_i} \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)} C_2 \oplus C_4$$

$$\bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)} C_{d_i} \oplus C_2$$

$$= \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)} C_{d_i} \oplus C_2 \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)-2} C_2 \oplus C_2 \oplus C_4$$

$$\bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)} C_{d_i} \oplus C_2$$

$$= \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)} C_{d_i} \oplus C_2 \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)} C_{d_i} \oplus C_2$$

$$\bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)-2} C_2 \oplus C_4 \oplus C_2$$

By theorems(2.16) and (2.25) , we have :

$$K(D_{nh}) = \bigoplus_{i=1}^{2} K(D_n) \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)-2} C_2 \oplus K(C_4) .$$

Example(3.4):-

To find the cyclic decomposition $K(D_{25h})$, $K(D_{35h})$, $K(D_{1125h})$, $K(D_{1157625h})$ and $K(D_{15015h})$ by theorem (3.3)

$$K(D_{25h})= K(D_{5^2 h}) = \bigoplus_{i=1}^{2} K(D_{5^2}) \bigoplus_{i=1}^{(2+1)-2} C_2 \oplus K(C_4) = \bigoplus_{i=1}^{2} K(D_{25}) \oplus C_2 \oplus K(C_4)$$

$$K(D_{35h})= K(D_{5.7h})= \bigoplus_{i=1}^{2} K(D_{5.7}) \bigoplus_{i=1}^{(1+1).(1+1)-2} C_2 \oplus K(C_4)= \bigoplus_{i=1}^{2} K(D_{35}) \bigoplus_{i=1}^{2} C_2 \oplus K(C_4)$$

$$K(D_{1125h}) = K(D_{3^2.5^3 h}) = \bigoplus_{i=1}^{2} K(D_{3^2.5^3}) \bigoplus_{i=1}^{(2+1).(3+1)-2} C_2 \oplus K(C_4)$$

$$= \bigoplus_{i=1}^{2} K(D_{1125}) \bigoplus_{i=1}^{10} C_2 \oplus K(C_4)$$

$$K(D_{1157625h}) = K(D_{3^3.5^3.7^3 h}) = \bigoplus_{i=1}^{2} K(D_{3^3.5^3.7^3}) \bigoplus_{i=1}^{(3+1).(3+1)(3+1)-2} C_2 \oplus K(C_4)$$

$$= \bigoplus_{i=1}^{2} K(D_{1157625}) \bigoplus_{i=1}^{62} C_2 \oplus K(C_4)$$

$$K(D_{15015h})=K(D_{3.5.7.11.13\, h})= \bigoplus_{i=1}^{2} K(D_{3.5.7.11.13}) \bigoplus_{i=1}^{(1+1).(1+1)(1+1)(1+1).(1+1)-2} C_2 \oplus K(C_4)$$

$$= \bigoplus_{i=1}^{2} K(D_{15015}) \bigoplus_{i=1}^{30} C_2 \oplus K(C_4) .$$

References

[1] C.Curits and I.Reiner ,"Methods of Representation Theory with Application to Finite Groups and Order " , John wily& sons, New York, 1981.

[2] H.H. Abass," On The Factor Group of Class Functions Over The Group of Generalized Characters of D_n ", M.Sc thesis, Technology University,1994.

[3] J. P. Serre, " Linear Representation of Finite Groups " , Springer- Verlage, 1977.

[4] M.S. Kirdar , " The Factor Group of The Z-Valued Class Function Modulo The Group of The Generalized Characters " , Ph.D . thesis , University of Birmingham ,1982 .

[5] N. R. Mahamood " The Cyclic Decomposition of the Factor Group $cf(Q_{2m},Z)/\overline{R}(Q_{2m})$", M.Sc. thesis, University of Technology, 1995.

[6] N. S. Jasim, " Results of The Factor Group $cf(G,Z)/\overline{R}(G)$", M.Sc. thesis, University of Technology, 2005 .

التجزئة الدائرية للزمرة الكسرية $cf(D_{nh},Z)/ R(D_{nh})$ عندما n عدد فردي

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الخلاصــــــــــــة

لعدد صحيح موجب $n \geq 3$ ، لتكن D_n زمرة ثنائي السطوح، $D_{nh} = D_n \times C_2$ و $cf(D_{nh},Z)$ الزمرة الابدالية لدوال الصف ذات القيم الصحيحة للزمرة D_{nh} . ان تقاطع $cf(D_{nh},Z)$ مع زمرة كل الشواخص العمومية للزمرة D_{nh} ، $R(D_{nh})$ تكون زمرة جزئية سوية من $cf(D_{nh},Z)$ ويرمز لها بالرمز $\overline{R}(D_{nh})$ ، إذن $K(D_{nh})$ تكون زمرة كسرية ابدالية منتهية والتي يرمز لها بالرمز $cf(D_{nh},Z)/\overline{R}(D_{nh})$، حددنا في هذا البحث ، التجزئة الدائرية للزمرة الابدالية المنتهية $cf(D_{nh},Z)/R(D_{nh})$ عندما n عدد فردي، وجدنا إن التجزئــة للزمــرة $K(D_{nh})$ تعتمد على القواسم الأولية للعدد n ، إذا كان $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots\ldots p_m^{\alpha_m}$ حيث ان $p_1^{\alpha_1}, p_2^{\alpha_2}, \ldots\ldots, p_m^{\alpha_m}$ أعــداد أوليــة مختلفــة $\alpha_1, \alpha_2, \ldots\ldots, \alpha_m$ اعـــــــــداد صـــــــــحيحة موجبـــــــــة فـــــــــان:

$$K(D_{nh}) = \bigoplus_{i=1}^{2} K(D_n) \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)-2} C_2 \oplus K(C_4)$$

وكذلك وجدنا الشواخص النسبية للزمرة D_{nh}.