αC – CONTINUOUS FUNCTION

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ABSTRACT

The main aim of this paper is to study new class of continuous function is called αc – continuous function. for this aim , the nation of α - open and pre – open sets and α - compact space are introduced. and we shall study their relation ship between αc – continuous and pc – continuous function .

1- Introduction and preliminaries

Let (X,τ) be a topological space and $A \subseteq X$, we denoted to the complement of A in X by A^c, the closure and the interior of A respectively by $\overline{A}$ and A^o we recall the following function :-

Definition 1-1 :- Let (X,τ) be a topological space , A subset $A \subseteq X$ is called

1) **a semi open set [5]** if $A \subseteq \overline{(A^o)}$ and semi - closed if $(\overline{A})^o \subseteq A$.
2) **a pre – open set [10]** if $A \subseteq (\overline{A})^o$ and pre – closed set if $\overline{(A^o)} \subseteq A$.
3) **an α - open set [13]** if $A \subseteq \left(\overline{(A^o)}\right)^o$ and α - close set if $\overline{((\overline{A})^o)} \subseteq A$.
4) **a regular open set** if $A = (\overline{A})^o$ and regular closed set if $A = \overline{(A^o)}$.

The complement of an α - open (resp. semi – open , pre – open , regular open) set is called α - closed (resp. semi – closed , pre – closed , regular closed) set . The smallest α - closed (resp. semi – closed , pre – closed , regular closed) set containing $A \subseteq X$ is called α - closure (resp. semi – closure , pre – closure , regular – closure) of A and shall be denoted by $\overline{A}^{\alpha}$ (resp. $\overline{A}^{s}$, $\overline{A}^{p}$, $\overline{A}^{R}$) .we denote the family of α - open (resp. semi – open , pre – open , regular open) sets by T^{α} ,(resp. T^s ,T^p ,T^R) sets ,it is shown in [ ] that $T \subset T^{\alpha} \subset T^s$ and T^{α} is a topology for X ,It is also shown that $T^{\alpha} = T^s \cap T^p$. an α - open set is precisely semi – open and pre – open [14, Lemma 1]

In 1970 , Gentry and Hoyle [4] introduce the concept of **C –continuous function** , A function f: $X \to Y$ is called **C-continuous function** if and only if for each $x \in X$ and each open set V containing f(x) such that V^c is compact ,there exists an open set U containing x such that $f(U) \subset V$,in 1987 ,F. cammroto and T.noiri [3] define new concept of function is called **WC - continuous function** by replacing **compact** in the definition of C- continuous function with

weakly – compact relative to Y. and A function $f : X \to Y$ is called **PC – continuous function [5]** if and only if for each $x \in X$, and each open set V containing f(x) , such that V^c is **P - compact relative to Y** , there exists an open set U containing x such that $f(U) \subset V$.

2- α - compact space.

Definition 2.1 :- A space (X , τ)is said to be α **- compact space** [9] if every α - open cover of X has a finite sub cover .

It is shown in [9] that every α - compact space is compact but not conversely , the following is obvious *from **definition 2.1*** . the subspace (A , τ / A) is α - compact , where τ / A denotes the induced topology on A .

Proposition 2.2 :- A space (X , τ) is α **- compact** if and only if **(X , τ^{α}) is compact** .

Definition 2.3 :- A subset A of space (X , τ) is said to be α **- compact relative to(X ,τ)** , if every cover of A by α **- open sets** of (X , τ) has a finite sub cover .

Theorem 2.4 :- A subset A of a space (X , τ) is α **- compact relative to (X , τ)** if and only if is **compact in (X , τ^{α})** .

Proof :-

Necessity : Let $A = \cup \{ W_\lambda \mid \lambda \in \Omega \}$ and $W_\lambda \in \tau^{\alpha} / A$ for each $\lambda \in \Omega$.

For each $\lambda \in \Omega$, there exists $V_\lambda \in \tau^{\alpha}$, such that $W_\lambda = V_\lambda \cap A$. since $\{ V_\lambda \mid \lambda \in \Omega \}$ is a cover of A , there exists a finite subset Ω_o of Ω such that $A \subset \cup \{ V_\lambda \mid \lambda \in \Omega_o \}$. Therefore , we obtain $A = \cup \{ W_\lambda \mid \lambda \in \Omega_o \}$.

Hence A is compact set of (X , τ^{α}) .

Sufficiency : Let $A \subset \cup \{ W_\lambda \mid \lambda \in \Omega \}$ and $W_\lambda \in \tau^{\alpha}$ for each $\lambda \in \Omega$.

Then , we have $A = \cup \{ W_\lambda \cap A \mid \lambda \in \Omega \}$ and $W_\lambda \cap A \in \tau^{\alpha} / A$ for each $\lambda \in \Omega$.

Therefore , for some finite subsets Ω_o of Ω .

$A = \cup \{ W_\lambda \cap A \mid \lambda \in \Omega_o \} \subset \cup \{ W_\lambda \mid \lambda \in \Omega_o \}$.

This is shows that A is α - compact relative to (X , τ^{α}) .

Proposition 2.5 :- Let A be a subset of space (X , τ) if A is α **- compact relative to (X , τ)**, then is α **- compact** .

Proof :- Let $A = \cup \{ W_\lambda \mid \lambda \in \Omega \}$and $W_\lambda \in (\tau / A)^\alpha$ for each $\lambda \in \Omega$. it follows from proposition 2.1 of [8] that $(\tau / A)^\alpha \subset \tau^\alpha / A$.
For each $\lambda \in \Omega$, there exists , $V_\lambda \in \tau^\alpha$ such that $W_\lambda = V_\lambda \cap A$.
Since $\{V_\lambda \mid \lambda \in \Omega \}$ s a cover of A , there exists Ω_o of Ω , such that $A \subset \cup \{ W_\lambda \mid \lambda \in \Omega_o\}$.
Therefore , we obtain $A = \cup \{ W_\lambda \mid \lambda \in \Omega \}$.
So A is α - compact of (X , τ).

Lemma 2.6 :- [8] , A space (X , τ) is α **- compact** if and only if every **proper** α **- closed set** of (X , τ) is α **- compact relative to (X , τ)** .

Corollary 2.7 :- if a space (X , τ) is α **- compact** and A is an α **- closed set** of (X , τ) , then A is α **- compact** .

Proof :-
This is an immediate consequence of lemma 2.6 and proposition 2.5

By corollary 2.7 , we abserve that the assumption “ open “ of statement (a) in the following corollary is super fluous .

Corollary 2.8 :- [9] Let (X , τ) be an α - compact space and A a subset of (X , τ) , then we have

a) if A is open and α - closed set in (X , τ) , then it is α - compact .
b) if A is α - closed in (X , τ) , then it is compact .

3- αc– continuous function

Definition 3.1 :- A function $f : X \to Y$ is called α**c – continuous function** if and only if for each $x \in X$, and each open set V containing f(x) , such that V^c is α **- compact relative to Y** , there exists an open set U containing x such that $f(U) \subset V$.

Theorem 3.2 :- for a function $f : (X, \tau) \rightarrow (Y, \tau')$ the following are equivalent :-

1) f is α**c – continuous** function .
2) if V is **open** in Y and V^c is α **- compact relative toY** then $f^{-1}(V)$ is **open** set in X.
3) if F is **closed** in Y and α **- compact relative to Y** then $f^{-1}(F)$ is **closed** set in X

proof :- of $1 \rightarrow 2$

Let V an open set of Y such that V^c is α - compact relative to Y .

Let $x \in f^{-1}(V)$, then $f(x) \in V$, and there exists an open set U of X , such that $f(U) \subset V$.

So , we have $x \in U \subset f^{-1}(V)$.

Then $f^{-1}(V)$ is open set in (X, τ) .

of $2 \rightarrow 3$

let F is closed set and α - compact relative to Y .

then F^c is open , and since $F = (F^c)^c$ is α - compact relative to Y .

then $f^{-1}(F^c)$ is open set

so $f^{-1}(F)$ is closed .

of $3 \rightarrow 1$

let $x \in X$, and V an open containing f (x) , such that V^c is α - compact relative to Y.

so $f^{-1}(V^c)$ is closed set in X .

and hence $U = f^{-1}(V)$is an open set containing x , such that $f(U) \subset V$.

so f is αc – continuous function .

Proposition 3.3 :- if $A \subseteq X$ is **p –compact relative to X** then it is α **- compact relative to X**.

proof :-

let $A \subset \cup \{ W_\lambda \mid \lambda \in \Omega \}$, such that W_λ is α - open set .

then W_λ is pre – open set .

since A is P – compact relative to X .

so there exists a finite subset Ω_o of Ω , such that $A \subset \cup \{ W_\lambda \mid \lambda \in \Omega_o \}$.

There for A is α -compact relative to X .

Proposition 3.4 :- if A_1 and A_2 are α **- compact relative to X** then $A_1 \cup A_2$ is α **- compact relative to X** .

proof:- let $A_1 \cup A_2 \subset \cup \{ W_\lambda \mid \lambda \in \Omega \}$, such that W_λ is α - open set .
then $A_1 \subset \cup \{ W_\lambda \mid \lambda \in \Omega \}$ and $A_2 \subset \cup \{ W_\lambda \mid \lambda \in \Omega \}$.
So for each i = 1 , 2 , there exists a finite subset Ω_i of Ω ,
such that $A_i \subset \cup \{ W_\lambda \mid \lambda \in \Omega_i \}$
so , we have $A_1 \cup A_2 \subset \cup \{ W_\lambda \mid \lambda \in \Omega_i \}$.
Then $A_1 \cup A_2$ is α - compact relative to X .

Let (X , τ) be a topological space , it follows from **proposition 3 . 4** that the family of open sets having the complement α - compact relative to X may be used as a base for a topology $\tau_{\alpha c}$. it has been shown that the family of open sets having the complement P – compact relative to X may be used as base for a topology τ_{PC} and the family of open sets having the complement compact relative to X may be used as base for a topology τ_c and the family of open sets having the complement almost – compact relative to X may be used as base for a topology τ_A and the family of open sets having the complement weakly – compact relative to X may be used as base for a topology τ_{WC} .

Remark 3.5 :- for space X, we have $\tau_{\alpha c} \subset \tau_{PC} \subset \tau_C \subset \tau_A \subset \tau_{WC} \subset \tau$.

Definition 3.6 :- A function f : (X , τ) $\rightarrow$ (Y, τ') is said to be

1) α **- continuous** function [12] if $f^{-1}(V)$ is α - open in X for every V is open in Y.
2) **Weakly** α **- continuous** function if for each $x \in X$, and each open set V in Y, such that $f(x) \in V$, there exists an α - open set U containing x such that $f(U) \subset \overline{V}$.

Theorem 3.7 :- A function f : (X , τ) $\rightarrow$ (Y , $\tau'_{\alpha c}$) is **continuous** if and only if f : (X , τ) $\rightarrow$ (Y , $\tau'_{\alpha c}$) α **- continuous** function .

Proof :- **Necessity** :let V is open in Y , since f : (X , τ) $\rightarrow$ (Y , $\tau'_{\alpha c}$) is continuous .

Then $f^{-1}(V)$ is open in X .

So , $f^{-1}(V)$ is α - open in X .

Therefore $f : (X , \tau) \to (Y , \tau'_{\alpha c})$ is α - continuous function .

Sufficiency : let V is open in Y .

Then V^c is α - compact relative to Y .

From **proposition 3.2** , Then $f^{-1}(V)$ is open in X .

Then $f : (X , \tau) \to (Y , \tau'_{\alpha c})$ is continuous function .

Theorem 3.8 :- A function $f : (X , \tau) \to (Y , \tau'_{\alpha c})$ is α**c – continuous** function if and only if $f : (X , \tau) \to (Y , \tau'_{\alpha c})$ is **continuous** function .

Proof :- **Necessity** : let V is open in Y .

Then V^c is α - compact relative to Y .

From **proposition 3.2** , Then $f^{-1}(V)$ is open in X .

Then $f : (X , \tau) \to (Y , \tau'_{\alpha c})$ is continuous function .

Sufficiency : Let $x \in X$, and each open set V containing f (x) , such that V^c is α - compact relative to Y .

Then $f^{-1}(V)$ is open set in X containing x .

Let $f^{-1}(V) = U$, then $f (U) \subset V$.

Therefore $f : (X , \tau) \to (Y , \tau'_{\alpha c})$ is αc - continuous function .

Corollary 3.9 :- A function $f : (X , \tau) \to (Y , \tau'_{\alpha c})$ is αc – continuous function if and only if $f : (X , \tau) \to (Y , \tau'_{\alpha c})$ is α - continuous function .

Proof :- this is an immediate from **theorem 3.8** .

Theorem 3.10 :- if $f : (X , \tau) \to (Y , \tau'_{\alpha c})$ is α**c – continuous** function then $f : (X , \tau) \to (Y , \tau'_{\alpha c})$ is **weakly** α **– continuous** function .

Proof :- let $x \in X$, and each an open set V in Y .

Then V^c is α - compact relative to Y .

Since $f : (X , \tau) \rightarrow (Y , \tau'_{\alpha c})$ is αc – continuous function , then there exists an open set U containing x , such that $f (U) \subset V$.

So , $f (U) \subset \overline{V}$.

Therefore $f : (X , \tau) \rightarrow (Y , \tau'_{\alpha c})$ is weakly α – continuous function.

Theorem 3.11:-

Let (X , τ) , (Y , τ') be tow topological spaces ,if $f : (X , \tau) \rightarrow (Y , \tau')$ is α **- continuous function** then $f : (X , \tau) \rightarrow (Y , \tau')$ is **PC – continuous function** .

Proof :- let $x \in X$ and V is open set containing f (x) ,such that V^c is P – compact space relative to Y

Then V^c is α - compact relative to Y .

Since $f : X \rightarrow Y$ is αc – continuous function .

So there exists open set U containing x , such that $f (U) \subset V$.

Therefore $f : X \rightarrow Y$ is PC – continuous function .

Theorem 3.12 :- if $f : (X , \tau) \rightarrow (Y , \tau')$ is α**c – continuous** function , and A is subset of X, then restriction $f /A : A \rightarrow Y$ is αc – continuous function .

Theorem 3.13 :- if $f : (X , \tau) \rightarrow (Y , \tau')$ is **continuous** function and $g : (Y , \tau') \rightarrow (Z , \sigma)$ is α**c – continuous** function , then the composition $g \circ f : X \rightarrow Z$ is αc – continuous .

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الدوال المستمرة - αc

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الخلاصة

ان الهدف الرئيسي من هذا البحث هو دراسة نوع جديد من الدوال المستمرة يسمى الدوال المستمرة - αc. ولتحقيق هذا الهدف قدمنا المصطلحات α - open و pre – open و α - compact . كما سنقوم بدراسة العلاقة بين الدوال المستمرة - αc والدوال المستمرة-pc.