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ON PROPERTIES OF $S^{\times\times}$-CONTINUOUS FUNCTIONS

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Abstract

In this work, we study $S^{\times\times}$-continuous functions, a function $f: X \to Y$ is called $S^{\times\times}$-continuous function if the inverse image of every semi- open set in Y is semi open in X.Several properties of these functions are proved.

1.Introduction and Preliminaries:

Let (X,T) be a topological space ,let $A \subseteq X$, closure of A and interior of A are denoted by clA, $IntA$ respectively A is called semi –open [3] if $A \subseteq cl\,\text{int}\,A$ every open set is semi open but the converse is not necessarily true .

Definition :1-1

Let $f:(X,T) \to (Y,\Omega)$ be a function ,we say that:

a) f is semi –continuous ($S-$continuous) [2] ,if the inverse of every open set in Y is semi open in X .

b) f is say $S^{\times}-$continuous ,if the inverse image of every semi –open in Y is open in X .

c) f is say $S^{\times\times}-$continuous ,if the inverse image of every semi –open in Y is semi-open in X .

2-Certain forms of $S^{\times\times}-$continuous functions

In this section, we introduce and study several forms of $S^{\times\times}-$ continuous functions.

We recall the following definitions.

Definition 2.1:[4]

a) Let $(X.T)$ be a topological space, let $B \subseteq X$, we say that B is semi –closed if B^c is semi open in X .

b) Let $B \subseteq X$, the semi-closure of B $(scl(B))$ is the intersection of all semi-closed sets in X containing B .

c) Let $F \subseteq X$, we say that F is semi – generalized closed in X (sg-closed) if $(F \subseteq O \Rightarrow scl(F) \subseteq O$ (O is semi –open in X)

Now we are ready to introduce a weak form of $S^{\times\times}-$ continuous function which we call $A-S^{\times\times}$-continuous function.

Definition 2.2:

Let $f:(X,T) \to (Y,\Omega)$ be a function, we say that f is $A-S^{\times\times}-$continuous function if $F \subseteq f^{-1}(O) \to scl(F) \subset f^{-1}(O)$ (O is semi open in Y, F is sg- closed in X) Of courses , if f is $S^{\times\times}-$continuous function then f is $A-S^{\times\times}-$continuous function .

Example 2.3:

Let $X=\{a,b\}, T=\{\Phi, X, \{a\}\}$, Define $f: X \to X$ as follows :

$f(a)=b, f(b)=a$

Now $A=\{a\}$ is open and hence semi open, consider $f^{-1}(A)=\{b\}$

Now $\{b\}$ is semi closed in X so the inverse of every semi open in X is semi closed which shows that f is $A-S^{\times\times}-$ continuous function.

$(F \subseteq f^{-1}(O) \to scl(F) \subseteq scl(f^{-1}(O) = f^{-1}(O))$,

But f is not $S^{\times\times}-$ continuous function because $\{a\}$ is semi open in X and

$f^{-1}(\{a\})=\{b\}$ .Which is not semi open in X $(\{b\}\not\subseteq cl\,\mathrm{int}\{b\})$

Before, we state the next theorem, we need the following definition.

Definition 2.4 [3]

Let $f:(X,T)\to(Y,\Omega)$ be a function, we say that f is Contra $-S^{xx}-$ continuous if the inverse of every semi -open in Y is semi - closed in X

Theorem 2.5:

Let $f:(X,T)\to(Y,\Omega)$ be contra $-S^{xx}-$ continuous function, then f is $A-S^{xx}-$ continuous function.

Proof :

Let O be a semi open in Y, let F be $sg-$ closed in X , let $F\subseteq f^{-1}(O)$, then $scl(F)\subseteq scl(f^{-1}(O)=f^{-1}(O)$, Because $f^{-1}(O)$ is semi closed in X which means that f is $A-S^{xx}-$ continuous function.

Theorem 2.6:

Let $f:(X,T)\to(Y,\Omega)$ be a function from a topological space (X,T) into a topological space (Y,Ω), If the semi open and semi closed sets of (X,T) coincide, the f is $A-S^{xx}-$ continuous function if and only if f is contra $-S^{xx}-$continuous function.

Proof :

Assume f is $A-S^{xx}-$ continuous function .Let A be an arbitrary subset of (X,T) such that $A\subseteq W$,Where W is semi open in X, then by hypothesis $scl(A)\subseteq scl(W)=W$, therefore all subset of (X,T) are $sg-$closed (and hence all are $sg-$open) so ,

for any O which is semi-open in Y, $f^{-1}(O)$ is $sg-$closed. $scl(f^{-1}(O))\subseteq f^{-1}(O)$.

Therefore $scl(f^{-1}(O))=f^{-1}(O)$, i.e. $f^{-1}(O)$ is semi closed in X, which means that f is contra $-S^{xx}-$ continuous function.

Corollary2.7 :

Let $f:(X,T)\to(Y,\Omega)$ be a function from a topological space (X,T) into a topological space (Y,Ω) , If the semi open and semi closed sets of (X,T) coincide ,then f is $A-S^{xx}-$ continuous if and only if f is $S^{xx}-$ continuous.

Proof:

Let f be $A-S^{xx}-$ continuous function, let O be semi –open in Y, we will show that f is $S^{xx}-$ continuous in X.

Now $f^{-1}(O)$ is $sg-closed$ (theorem2.6) , $f^{-1}(O)\subseteq f^{-1}(O)\to sclf^{-1}(O)\subseteq f^{-1}(O)$

Which means that $f^{-1}(O)$ is semi-closed in X , But the semi-open and semi–closed sets in X coincide, so $f^{-1}(O)$ is semi-open in X so f is $S^{xx}-$ continuous.

Another proof, According to theorem 2.6, If f is $A-S^{xx}-$ continuous then f contra $-S^{xx}-$continuous, let O be semi open in X, hence $f^{-1}(O)$ is semi-open in X , so f is $S^{xx}-$continuous.

Definition2.8 :

Let $f:(X,T)\to(Y,\Omega)$ be a function, we say that f is perfectly contra $-S^{xx}-$ continuous function if the inverse of every semi-open in Y is semi clopen in X (that is semi-open and semi-closed)

Before, we state the next theorem we need the following definition .

Definition 2.9:

Let $f:(X,T)\to(Y,\Omega)$ be a function we say that f is $A-$semi closed if $f(B)\subseteq A\Rightarrow f(B)\subseteq \sin t(A)$,

($\sin t(A)=$semi-interior of A, A is $sg-$ open subset of Y ,B is semi-closed subset of X).

Theorem2.10 :

Let $f:(X,T)\to(Y,\Omega)$ be $S^{xx}-$ continuous function and A –semi closed then $f^{-1}(A)$ is $sg-$ closed whenever A is $sg-$ closed subset of Y.

Proof :

Let A be $sg-$closed subset of Y, suppose $f^{-1}(A)\subseteq O$ (O is semi –open in X) .

Now $O^c\subseteq f^{-1}(A^c)$ hence $f(O^c)\subseteq A^c$, then $f(O^c)\subseteq \sin(A^c)=(scl(A))^c$ it follows that $O^c\subseteq (f^{-1}(scl(A))^c$ and hence,

$(f^{-1}(scl(A))\subseteq O$ since f is $S^{xx}-$ continuous, then $(f^{-1}(scl(A))$ is semi-closed.

Thus we have

$(f^{-1}(A))\subseteq scl(f^{-1}(scl(A)))=f^{-1}(scl(A))\subseteq O$, This implies that $f^{-1}(A)$ is $sg-$closed in X .

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المستخلص:

في هذا البحث درسنا الدوال المستمرة $-S^{xx}$.الدالة $f:X\to Y$ تسمى دالة مستمرة $-S^{xx}$ إذا كانت الصورة النظيرة لأي مجموعة شبه مفتوحة في Y مجموعة شبه مفتوحة في X . برهنا مجموعة خصائص لهذه الدوال .