

# Analyze and model data using a transmuted Lomax distribution

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**Abstract**—We discussed two distributions (The Lomax Distribution and The Transmuted Lomax Distribution) and we dealt with some functions such as the PDF, CDF function, and we also discussed some statistical properties such as moments, the moment generation function, the coefficient of variation, flattening, torsion, and ordered statistics. The research concluded the importance of the Transmuted Lomax Distribution on the Lomax Distribution and on all sample sizes in the practical aspect experimental.

**Keywords**—Lomax Distribution ; Transmuted Lomax Distribution; Probability Density Function ; Moments; Maximum likelihood Estimation

## I. INTRODUCTION

Recently, several families of transformed distributions have been proposed, which are extended models with one or more parameters to the basic model of continuous distributions, both (Shaw and Buckley) used the rank transformation method (RTM), which is looking for a tool to build new families of non-distributives Gaussian, as it was used to modify a specific fundamental distribution for the purposes of modulating moments, especially skewness, torsion and kurtosis, introduced the QRTM quadratic rank transformation map, which many authors have used to introduce new important distributions such as the Lomax Distribution Transformed by Lomax (1954), Where these new families provide more flexibility in modeling and analyzing real-life data in many applied fields, and this is why the statistical literature contains a good number of new families, where the distributions could not model some phenomena, and that is why they found shifting of the distributions in general or extending these distributions so that they may be compatible with some medical or biological phenomenon . In this article we have taken the Lomax Distribution and used the transform map approach to define a new model named the Transmuted Lomax Distribution (TLom), and according to the Quadratic Rank Transmutation Map (QRTM). Approach the Cumulative Distribution Function (CDF) satisfy the relationship:

$$F1(x)=(1+\lambda) F2(x) - \lambda (F2(x))^2 \tag{1}$$

which on differentiation yields,

$$f1(x) = f2(x) [(1+\lambda) - 2\lambda F2(x)] \tag{2}$$

where  $F2(x)$  is the CDF of the base distribution

## II. TRANSMUTED LOMAX DISTRIBUTION

The Lomax Distribution, conditionally also called the Pareto Type II distribution , is a heavy-tail probability distribution used in business, economics, actuarial science, queuing theory and internet traffic modeling . This distribution is named after (Lomax K . S.) [6].

The PDF of Lomax Distribution is given by [4] :

$$f(x) = \theta/\gamma(1 + x/\gamma )^{-(\theta+1)} \tag{3}$$

where  $\theta$  is shape parameter and  $\gamma$  is scale parameter

The CDF of Lomax distribution is given by [4] :

$$F(x)=1-(1 + x/\gamma )^{-\theta} \tag{4}$$

From equation (1) and (3) we have the CDF of the Transmuted Lomax Distribution (TLom) random variable may by written as[2]:

$$F(x) = [1-(1 + \gamma x)^{-\theta}][ (1+\lambda) - \lambda(1 - (1 + \gamma x)^{-\theta})] \tag{5}$$

where  $|\lambda| \leq 1, \theta > 0, x > 0, \gamma > 0$

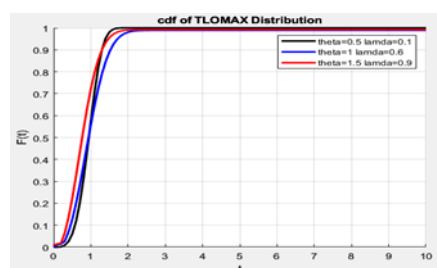


Figure (1.1) The PDF for T Lomax Distribution

$$f(x) = \frac{\theta\gamma}{(1+\gamma x)^{(\theta+1)}} [1 + \lambda - 2\lambda(1 - (1 + \gamma x)^{-\theta})] \quad (6)$$

where  $|\lambda| \leq 1, \theta > 0, x > 0, \gamma > 0$

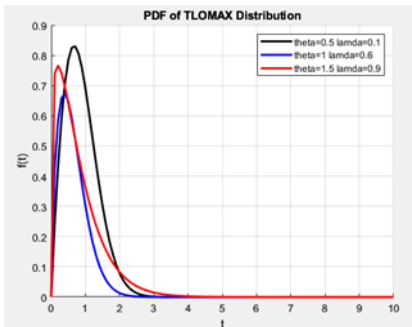


Figure (1.2) The CDF for T Lomax Distribution

**A. The Moments**

The  $r^{th}$  moments about origin of the Transmuted Lomax Distribution random variable  $x$  is given by<sup>[6]</sup>:

$$\begin{aligned} \mu_r &= E(X^r) = \int_0^\infty x^r f(x) dx \\ E(X^r) &= \mu = \frac{1+\lambda}{\gamma^r} \sum_{i=0}^r \binom{r}{i} (-1)^i \frac{\theta}{\theta+i-r} - \frac{2\lambda}{\gamma^r} \sum_{i=0}^r \binom{r}{i} (-1)^i \frac{\theta^2}{(2\theta+i-r)(\theta+i-r)} \end{aligned} \quad (7)$$

The first moment can be obtained for T Lom Distribution by taking  $r=1$

$$E(x) = \frac{\theta(2-\lambda)-1}{2\theta^2\gamma-3\theta\gamma+\gamma} \quad (8)$$

when  $r=2$ , we get the second moment of (T Lom) Distribution and it is in the following from:

$$E(X^2) = \frac{\theta(4-3\lambda)-2}{\gamma^2(2\theta^3-7\theta^2+7\theta-2)} \quad (9)$$

The variance for (T Lom) it can be obtained through the following formula :

$$\begin{aligned} Var(x) &= E(X^2) - (E(X))^2 \\ &= \frac{\theta^3(4-2\lambda-\lambda^2) + \theta^2(2\lambda^2-\lambda-4) + \theta(\lambda+1)}{\gamma^2(\theta-2)(2\theta^2-3\theta+1)^2} \end{aligned} \quad (10)$$

The Coefficient of Variation (CV) of transmuted Lomax distribution can be obtained by the following relationship :

$$CV = \frac{\sqrt{Var(x)}}{E(X)}$$

$$CV = \sqrt{\frac{\frac{\theta^3(4-2\lambda-\lambda^2) + \theta^2(2\lambda^2-\lambda-4) + \theta(\lambda+1)}{\gamma^2(\theta-2)(2\theta^2-3\theta+1)^2}}{\frac{\theta(2-\lambda)-1}{2\theta^2\gamma-3\theta\gamma+\gamma}}}$$

when  $r=3$ , we get the third moment of transmuted Lomax distribution it is in the following from :

$$E(X^3) = \frac{9\lambda(5\theta^2 - 9\theta + 2)}{\gamma^3(4\theta^5 - 32\theta^4 + 95\theta^3 - 130\theta^2 + 81\theta - 18)} \quad (12)$$

The Skewness (CS) of Transmuted Lomax Distribution can be obtained through the following formula :

$$\begin{aligned} CS &= \frac{E(X-\mu)^3}{(Var(x))^{3/2}} \\ E[(X-\mu)^3] &= E\left[\sum_{i=0}^3 \binom{3}{i} (-\mu)^i X^{3-i}\right] \\ &= E\left[\binom{3}{0}(-\mu)^0 X^3 + \binom{3}{1}(-\mu)X^2 + \binom{3}{2}(-\mu)^2 X + \binom{3}{3}(-\mu)^3 X^0\right] \\ &= E[X^3 - 3\mu X^2 + 3\mu^2 X - \mu^3] \\ &= E[X^3] - 3\mu E[X^2] + 3\mu^2 E[X] - \mu^3 \\ &= \mu_3 - 3\mu\mu_2 + 3\mu^2\mu - \mu^3 \\ &= \mu_3 - 3\mu\mu_2 + 2\mu^3 \\ CS &= \frac{\mu_3 - 3\mu\mu_2 + 2\mu^3}{(Var(x))^{3/2}} \end{aligned} \quad (13)$$

The fourth moment for the of Transmuted Lomax Distribution can be obtained by taking  $r=4$

$$\begin{aligned} E(X^4) &= \frac{1+\lambda}{\gamma^4} \left[ \binom{4}{0}(-1)^0 \frac{\theta}{\theta-4} + \binom{4}{1}(-1)^1 \frac{\theta}{\theta-3} + \binom{4}{2}(-1)^2 \frac{\theta}{\theta-2} + \binom{4}{3}(-1)^3 \left(\frac{\theta}{\theta-1}\right) + \binom{4}{4}(-1)^4 \right] - \frac{2\lambda}{\gamma^4} \left[ \binom{4}{0}(-1)^0 \frac{\theta^2}{(2\theta-4)(\theta-4)} + \binom{4}{1}(-1)^1 \frac{\theta^2}{(2\theta-3)(\theta-3)} + \binom{4}{2}(-1)^2 \frac{\theta^2}{(2\theta-2)(\theta-2)} + \binom{4}{3}(-1)^3 \frac{\theta^2}{(2\theta-1)(\theta-1)} + \binom{4}{4}(-1)^4 \frac{1}{2} \right] \\ E(X^4) &= \frac{1+\lambda}{\gamma^4} \left[ \frac{\theta}{\theta-4} - \frac{4\theta}{\theta-3} + \frac{4\theta}{\theta-2} - \frac{4\theta}{\theta-1} + 1 \right] - \frac{2\lambda}{\gamma^4} \left[ \frac{\theta^2}{(2\theta-4)(\theta-4)} - \frac{4\theta^2}{(2\theta-3)(\theta-3)} + \frac{4\theta^2}{(2\theta-2)(\theta-2)} - \frac{4\theta^2}{(2\theta-1)(\theta-1)} + \frac{1}{2} \right] \end{aligned} \quad (14)$$

The kurtosis (CK) of Transmuted Lomax Distribution can be obtained the following formula:

$$\begin{aligned} CK &= \frac{E(X-\mu)^4}{(Var(x))^2} \\ E[(X-\mu)^4] &= E\left[\sum_{i=0}^4 \binom{4}{i} (-\mu)^i X^{4-i}\right] \\ &= E\left[\binom{4}{0}(-\mu)^0 X^4 + \binom{4}{1}(-\mu)X^3 + \binom{4}{2}(-\mu)^2 X^2 + \binom{4}{3}(-\mu)^3 X + \binom{4}{4}(-\mu)^4 X^0\right] \\ &= E[X^4 - 4\mu X^3 + 6\mu^2 X^2 - 4\mu^3 X + \mu^4] \\ &= E[X^4] - 4\mu E[X^3] + 6\mu^2 E[X^2] - 4\mu^3 E[X] + \mu^4 \\ &= \mu_4 - 4\mu\mu_3 + 6\mu^2\mu_2 - 4\mu^3\mu + \mu^4 \\ &= \mu_4 - 4\mu\mu_3 + 6\mu^2\mu_2 - 3\mu^4 \end{aligned}$$

$$CK = \frac{\hat{\mu}_4 - 4\mu\hat{\mu}_4 + 6\mu^2\hat{\mu}_2 - 3\mu^4}{(Var(x))^2} \quad (15)$$

**B. Moment Generating Function**

The moment generation function (MGF) of the TLOm is [5]:

$$M_X(t) = \int_0^\infty e^{tx} f(x) dx$$

where  $f(x)$  is (PDF) of transmuted Lomax distribution.

$$M_X(t) = \int_0^\infty e^{tx} \frac{\theta\gamma}{(1+\gamma x)^{(\theta+1)}} [1 + \lambda - 2\lambda(1 - (1 + \gamma x)^{-\theta})] dx \quad (16)$$

$x > 0, \theta > 0, |\lambda| \leq 1$

**C. Survival Function**

The survival function  $S(x)$  of Transmuted Lomax Distribution is given by:

$$S(x) = 1 - F(x) = 1 - [(1 - (1 + \gamma x)^{-\theta})((1 + \lambda) - \lambda(1 - (1 + \gamma x)^{-\theta}))] \quad (17)$$

**C. ORDER STATISTICS**

Let  $X_1, X_2, \dots, X_n$  a random sample of size  $n$  with distribution function(cdf)  $F_X(x)$  and (pdf)  $f_X(x)$ . The corresponding order statistics are the  $X_i$  arranged in ascending order. The largest of the  $X_i$  is denoted by  $X_{(n)}$  and the smallest of the  $X_i$  is denoted by  $X_{(1)}$  [11], the pdf of  $X_{(i)}$  is given [1]:

$$f_{X_{(i)}}(x) = \frac{n!}{(i-1)!(n-i)!} f_X(x) [F_X(x)]^{i-1} [1 - F_X(x)]^{n-i} \quad (18)$$

The pdf of order statistics for the Transmuted Lomax Distribution is given by [7]:

$$f_{X_{(i)}}(x) = \frac{n!}{(i-1)!(n-i)!} \frac{\theta\gamma}{(1+\gamma x)^{(\theta+1)}} \{ (1 + \lambda - 2\lambda[1 - (1 + \gamma x)^{-\theta}]) [1 - (1 + \gamma x)^{-\theta}] [1 + \lambda - \lambda(1 + \gamma x)^{-\theta}] \}^{i-1} [1 - [1 - (1 + \gamma x)^{-\theta}] (1 + \lambda - \lambda(1 + \gamma x)^{-\theta})]^{n-j} \quad (19)$$

The pdf of the largest order statistic  $X_{(n)}$  is given by [7]:

$$f_{X_{(n)}}(x) = \frac{n\theta\gamma}{(1 + \gamma x)^{(\theta+1)}} [1 + \lambda - 2\lambda(1 - (1 + \gamma x)^{-\theta})] [1 - (1 + \gamma x)^{-\theta}] [(1 + \lambda) - \lambda(1 - (1 + \gamma x)^{-\theta})]^{n-1} \quad (19)$$

The pdf of the smallest order statistic  $X_{(1)}$  is given by [7]:

$$f_{X_{(1)}}(x) = \frac{n\theta\gamma}{(1+\gamma x)^{(\theta+1)}} [1 + \lambda - 2\lambda(1 - (1 + \gamma x)^{-\theta})] [1 - ((1 - (1 + \gamma x)^{-\theta})((1 + \lambda) - \lambda(1 - (1 + \gamma x)^{-\theta})))]^{n-1} \quad (20)$$

**D. Estimation Method of (TLOm)**

We used the Maximum Likelihood Method for estimating the parameters of TLOm. Let  $X_1, X_2, \dots, X_n$  is the sample size

$n$  at random from the TLOm. Then the likelihood function, we write as follows [7]:

$$L = (x_i, \theta, \gamma, \lambda) = \prod_{i=1}^n f(x_i, \theta, \gamma, \lambda)$$

$$L = \prod_{i=1}^n \left[ \frac{\theta\gamma}{(1 + \gamma x_i)^{(\theta+1)}} (1 + \lambda - 2\lambda(1 - (1 + \gamma x_i)^{-\theta})) \right]$$

$$L = \theta^n \gamma^n \prod_{i=1}^n (1 + \gamma x_i)^{-(\theta+1)} [1 + \lambda - 2\lambda(1 - (1 + \gamma x_i)^{-\theta})]$$

$$L = \ln L = n \ln \theta + n \ln \gamma + \sum_{i=1}^n \ln [(1 + \gamma x_i)^{-(\theta+1)} (1 + \lambda - 2\lambda(1 - (1 + \gamma x_i)^{-\theta}))]$$

$$L = \ln L = n \ln \theta + n \ln \gamma - (\theta + 1) \sum_{i=1}^n \ln(1 + \gamma x_i) + \sum_{i=1}^n \ln [1 + \lambda - 2\lambda(1 - (1 + \gamma x_i)^{-\theta})]$$

$$\frac{\partial L}{\partial \theta} = \frac{n}{\theta} - \sum_{i=1}^n \ln(1 + \gamma x_i) + \sum_{i=1}^n \frac{-2\lambda[0 - (1 + \gamma x_i)^{-\theta} \ln(1 + \gamma x_i)(-1)]}{1 + \lambda - 2\lambda[1 - (1 + \gamma x_i)^{-\theta}]} = 0$$

$$\frac{\partial L}{\partial \theta} = \frac{n}{\theta} - \sum_{i=1}^n \ln(1 + \gamma x_i) - \sum_{i=1}^n \frac{2\lambda \ln(1 + \gamma x_i)(1 + \gamma x_i)^{-\theta}}{1 + \lambda - 2\lambda[1 - (1 + \gamma x_i)^{-\theta}]} = 0 \quad (21)$$

$$\frac{\partial L}{\partial \gamma} = \frac{n}{\gamma} - (\theta + 1) \sum_{i=1}^n \frac{x_i}{1 + \gamma x_i} + \sum_{i=1}^n \frac{-2\lambda [0 + \theta(1 + \gamma x_i)^{-(\theta+1)} x_i]}{1 + \lambda - 2\lambda[1 - (1 + \gamma x_i)^{-\theta}]} = 0$$

$$\frac{\partial L}{\partial \gamma} = \frac{n}{\gamma} - (\theta + 1) \sum_{i=1}^n \frac{x_i}{1 + \gamma x_i} - 2\lambda\theta \sum_{i=1}^n \frac{(1 + \gamma x_i)^{-(\theta+1)} x_i}{1 + \lambda - 2\lambda[1 - (1 + \gamma x_i)^{-\theta}]} = 0 \quad (22)$$

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^n \frac{1 - 2[1 - (1 + \gamma x_i)^{-\theta}]}{1 + \lambda - 2\lambda[1 - (1 + \gamma x_i)^{-\theta}]} = 0 \quad (23)$$

By using numerical methods, we find the solution of nonlinear equations (21), (22), (23). The Newton-Raphson method is employed to evaluate the estimated values of the parameters  $\theta, \gamma, \lambda$ .

**III. SIMULATION**

We conducted Monte Carlo simulation study to assess on the finite sample behavior of the maximum likelihood and the simulations were carried out using the statistical software

Matlab 2022b program. This study is repeated 1000 times each with sample sizes  $n=25, 75, 100, 150$ .

Assuming that the default values for the parameters of Lom is  $(\theta=3, \gamma=2)$  and TLom is  $(\theta=3, \gamma=2, \lambda=0.5)$ , the estimation results can be summarized as shown in Table (1).

Table (1) showed that under theoretical values of parameters of distributions ((Lom is  $\theta=3, \gamma=2$ ), and TLom is  $(\theta=3, \gamma=2, \lambda=0.5)$ ),

Table (1).The values of the real density function and estimated according to the method of MLE of the distribution of Lom, TLom at the theoretical samples sizes.

N	$f(x)_{Lom}$	$\hat{f}(x)_{Lom}$	MSE	$f(x)_{TLom}$	$\hat{f}(x)_{TLom}$	MSE
Para	$\hat{\theta}=3.45948, \hat{\gamma}=2.47813$		$\hat{\theta}=3.3434, \hat{\gamma}=3.53422, \hat{\lambda}=0.53733$			
25	0.90737	0.89401	0.01291	0.73528	0.77378	0.00334
	0.85890	0.86574	0.01287	0.73297	0.76824	0.00334
	0.85822	0.86532	0.01245	0.73226	0.76690	0.00331
	0.72685	0.78072	0.01232	0.71085	0.76209	0.00265
	0.60939	0.69481	0.01221	0.71058	0.73534	0.00148
	0.58072	0.67227	0.01117	0.70433	0.72674	0.00124
	0.49644	0.60215	0.00893	0.70049	0.72178	0.00120
	0.36695	0.48197	0.00874	0.69600	0.71602	0.00060
	0.31033	0.42394	0.00838	0.68411	0.70111	0.00050
	0.30754	0.42099	0.00730	0.68020	0.69627	0.00045
IMSE			0.05912			0.00611
AIC			-42.36558			-56.70871
Best	$\hat{f}(x)_{TLom}$					
Para	$\hat{\theta}=3.35337, \hat{\gamma}=2.36223$		$\hat{\theta}=3.24246, \hat{\gamma}=3.35451, \hat{\lambda}=0.52345$			
75	0.51331	0.51822	0.00011	0.18172	0.22502	0.00004
	0.50592	0.51608	0.00010	0.15487	0.19835	0.00004
	0.49870	0.50842	0.00010	0.14853	0.19192	0.00001
	0.49419	0.49761	0.00010	0.14398	0.18728	0.00001
	0.48635	0.48950	0.00010	0.11754	0.15971	0.00001
	0.48291	0.48919	0.00010	0.11134	0.15308	0.00000
	0.48085	0.48549	0.00009	0.11018	0.15183	0.00000
	0.47290	0.47475	0.00009	0.04859	0.08034	0.00000
	0.46649	0.46788	0.00008	0.01219	0.02784	0.00000
	0.45393	0.45442	0.00007	0.00592	0.01608	0.00000
IMSE			0.00485			0.00024
AIC			-54.72579			-121.06341
Best	$\hat{f}(x)_{TLom}$					
Para	$\hat{\theta}=3.33373, \hat{\gamma}=2.26727$		$\hat{\theta}=3.22356, \hat{\gamma}=3.20886, \hat{\lambda}=0.52244$			
100	0.43463	0.42748	0.00384	0.14628	0.14740	0.00011
	0.42447	0.42660	0.00363	0.13845	0.14107	0.00007
	0.42037	0.42175	0.00345	0.13354	0.13708	0.00002
	0.41722	0.38765	0.00340	0.08926	0.10009	0.00001
	0.41155	0.37444	0.00318	0.08071	0.09266	0.00001
	0.41032	0.34062	0.00290	0.05722	0.07147	0.00000
	0.40749	0.33026	0.00267	0.05682	0.07110	0.00000
	0.39759	0.32295	0.00236	0.04446	0.05928	0.00000

	0.38690	0.31327	0.00195	0.01739	0.03019	0.00000
	0.35487	0.29321	0.00161	0.00573	0.01406	0.00000
IMSE			0.00304			0.00008
AIC			-71.84245			-155.92351
Best	$\hat{f}(x)_{TLom}$					
Para	$\hat{\theta}=3.24563, \hat{\gamma}=2.24997$			$\hat{\theta}=3.11342, \hat{\gamma}=3.189064, \hat{\lambda}=0.518899$		
150	0.15537	0.13868	0.00002	0.00497	0.00899	0.00000
	0.15238	0.13598	0.00002	0.00309	0.00706	0.00000
	0.14500	0.12932	0.00001	0.00320	0.00352	0.00000
	0.09417	0.08383	0.00001	0.00271	0.00276	0.00000
	0.09268	0.08250	0.00001	0.00241	0.00234	0.00000
	0.07894	0.07032	0.00000	0.00234	0.00225	0.00000
	0.05717	0.05109	0.00000	0.00081	0.00161	0.00000
	0.05302	0.04743	0.00000	0.00026	0.00105	0.00000
	0.03376	0.03046	0.00000	0.00015	0.00006	0.00000
	0.01541	0.01422	0.00000	0.00007	0.00005	0.00000
IMSE			0.00010			0.00005
AIC			-107.07716			-252.95456
Best	$\hat{f}(x)_{TLom}$					

under  $n=25, 75, 100, 150$  the transmuted Lomax distribution is the best of other distributions because the IMSE and AIC for  $\hat{f}(x)_{TLom}$  less than the Lomax distribution for all sample sizes.

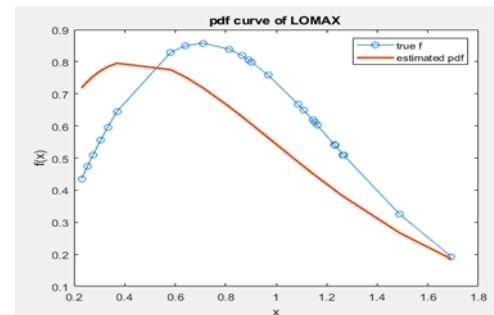


Figure (1.3) Real and Maximum Likelihood curve for Lomax distribution under  $n=25$

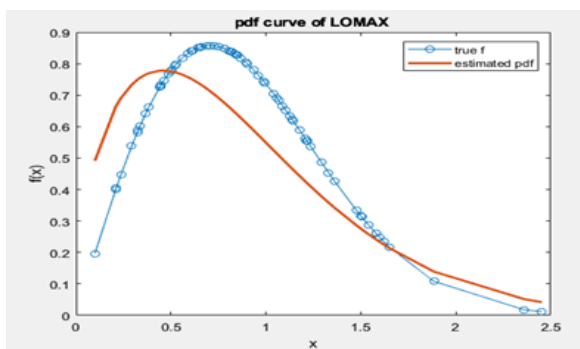


Figure (1.4) Real and Maximum Likelihood curve for Lomax distribution under n=75

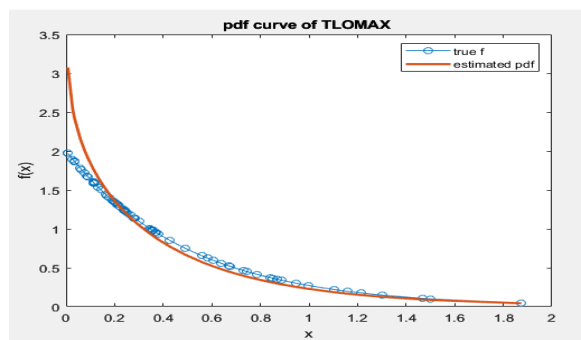


Figure (1.8) Real and Maximum Likelihood curve for TLomax distribution under n=75

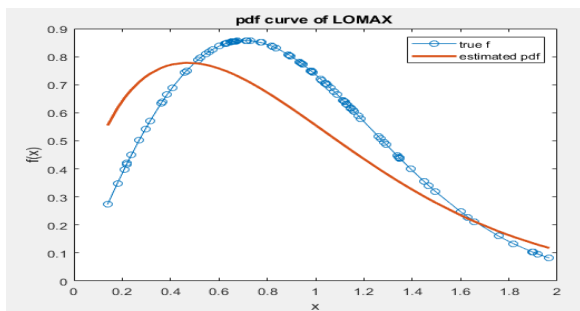


Figure (1.5) Real and Maximum Likelihood curve for Lomax distribution under n=100

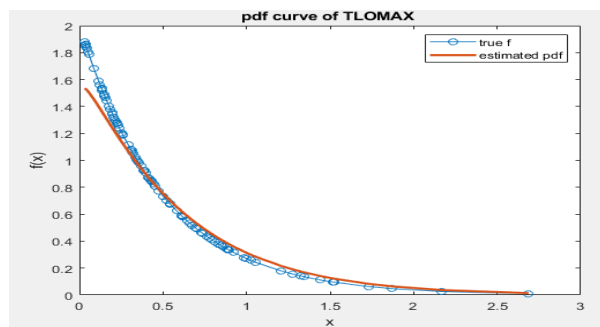


Figure (1.9) Real and Maximum Likelihood curve for TLomax distribution under n=100

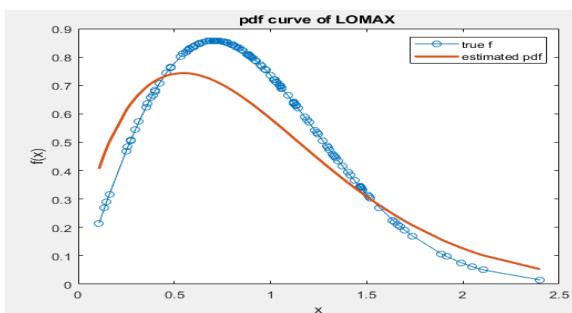


Figure (1.6) Real and Maximum Likelihood curve for Lomax distribution under n=150

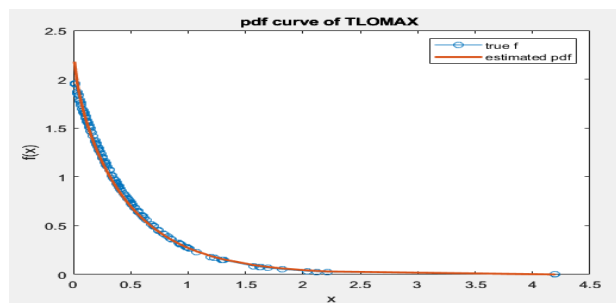


Figure (1.10) Real and Maximum Likelihood curve for TLomax distribution under n=150

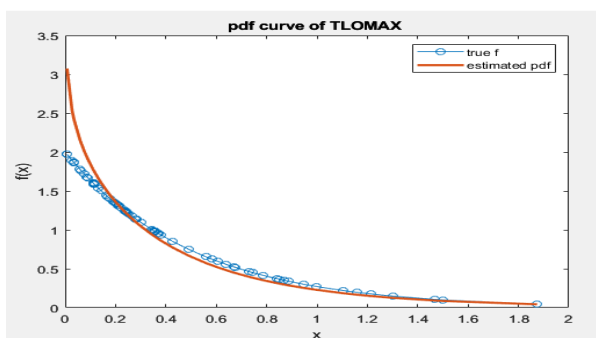


Figure (1.7) Real and Maximum Likelihood curve for TLomax distribution under n=25

We notice from the figures (1.3) to (1.10) the probability density function curve of the Transmuted Lomax Distribution is closer to the real probability density function of the distribution and it has more convergence than the un Transmuted Lomax Distribution for all sample sizes and that the larger the sample size, the closer the curve of the probability density function estimated by the maximum Likelihood method from the curve The real probability density function at the default parameters of the Lomax Distribution and the Transmuted Lomax Distribution .

#### IV. CONCLUSIONS

In this paper, we dealt with the generating extended models from classical models was addressed through the use of the quadratic transmutation map, by adding new parameters that make these distributions more flexible and appropriate, which have proven their importance in many researches and studies. Thus, the Transmuted Lomax Distribution was used and presented mathematical properties, and the behavior of the probability density function and the aggregate distribution function of the transmuted distribution was demonstrated. We discuss the maximum likelihood estimation of the model parameters. In addition to that, we conducted a simulation experiment and the obtained results display that the Quartic Rank Transmuted Lomax Distribution offers better appropriate than Lomax Distribution.

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