Whole Even Sum Domination in Graphs

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***Abstract*—:** *In this paper, a new concept of domination which is called whole even sum domination is presented, many bounded, properties, remarks, examples, are introduced. Furthermore, some operations as join and corona graphs of sum graphs are discussed. Finally, determined this number of the graph obtained from a graph G by the removal of all edges between any pair of neighbours of a vertex.*

Keywords: Whole Even sum dominating set, whole even sum domination number, some operations in graphs.

1. Introduction

Let $G = (V, E)$ be a finite undirected simple graph, The subject of domination received a formal mathematical definition, first by C. Berge in [1] and second by Ore [2] in1962.

Domination in graphs is considered a key concept on which most practical applications are based, whether scientific or otherwise. Therefore, researchers were very interested in providing different formulas and definitions to meet the needs of society to solve its problems and to find solutions that may include the least cost, time, distance, etc. The importance of this concept is that it has been used to find solutions in most sciences, such as medicine, engineering, physics, chemistry, statistics, and others. Since constructing a graph depends on mathematical foundations, this concept has entered most fields of mathematics, such as topological graph [10-13], fuzzy graph [14], labeled graph [15], general graph [2-17],, and others. . A.A.Omran and T.A.Ibrahim introduced the definition whole domination in graphs

, they are discussed many properties of this concept [3]. A.A.Omran and I.M.Rasheed introduced the definition Even Sum Domination in graphs, they are founded many bounded, properties, and theorems with their algorithm [4]. After that Ahmed A. Omran and Ahllam A. Alfatlawi, study the Even Sum Domination Number of Join Two Paths and get many results about path and related path [5]. In this paper, a new definition of domination is initiate which is called Whole Even Sum Domination (WESD). Many properties of this concept are discussed from the observation, properties, and theorems. Also, the domination numbers of WESD of the operation on two graphs are calculated such as the join graph, corona graph. Furthermore, the effect of deletion a vertex is proved with the observation $G \circledast v$ (which is the graph obtained from G by the removal of all edges between any pair of neighbours of v) and get the result of this operation. All concept not founded here can be founded in [6-8].

2. Whole Even Sum Domination in Graphs

Definition 2.1: Let G be a graph and D is a dominating set, the set D is called even sum dominating set (ESDS) if, $\forall u \in V - D$ there is a vertex $v \in D$ and v adjacent to u such that $deg(v) + deg(u)$ is even.

Definition 2.2: Let G be a simple connected graph, a proper subset D ⊂ V is called Whole Even Sum domination set (WESDS), if D is even sum dominating set and every vertex in the set $V - D$ is adjacent to all vertices in the set D.

Definition 2.3: The set D is called minimal WESDS (MWESDS) if it has no proper WESDS. The whole even sum domination number denoted by $\gamma_{we}(G)$ for simplicity γ_{we} is the minimum cardinality of a MWESDS. The MWESDS has γ_{we} is called γ_{we}- set

Remark 2.4

i) Suppose that the graph G is disconnected, then this graph has no WESDS

ii) The graph G has a WESDS if and only if there is a spanning subgraph isomorphic to complete bipartite.

iii) Suppose that the graph G a WESDS, then $daim\ (G)\ =\ 2$.

iv) The subset D of the vertex set V is a MWESDS iff the subset D is a WESDS and there is no vertex in D joining with the all others vertices in D

v) If the neighborhood of each vertex has an odd (even) in G has an even (odd) degree, then this vertex belong to D .

Observation 2.5

i) $\gamma_{we}(P_n)$ = 1 if $n = 2$, Otherwise P_n has no WEDS

ii) $\gamma_{we}(C_n) = \begin{cases} 1 & if\ n = 3 \\ 2 & if\ n = 4 \end{cases}$. Otherwise C_n has no WEDS.

iii) $\gamma(K_n) = \gamma_{we}(K_n) = 1$

iv) $\gamma_{we}(\overline{N}_n) = \gamma_{we}(K_n) = 1$

Example: let G be the graph shown in Figure 1

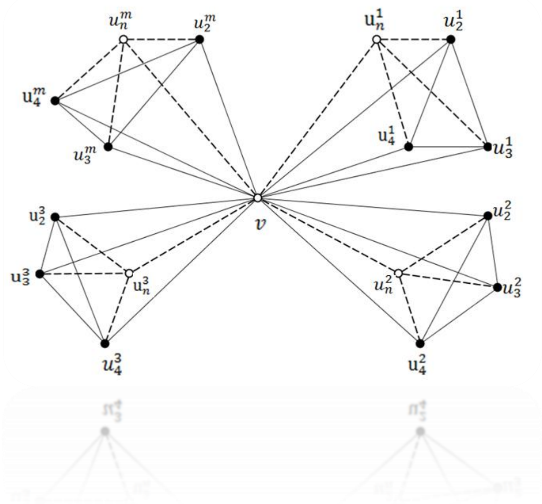

Figure 1: Windmill graph

$D_1 = \{\ v\}$ is minimum whole even sum dominating set of $G\ \gamma_{we}\ (G) = 1$

Proposition 2.6: If G = (V, D) has WDS then G has ESDS .The opposite is not true.

proof: If G has WDS, then all vertices in V-D are adjacent to all vertices in D. If $v\ \in V - D$ and v has odd(even) degree, then there are $u\ \in D\ such\ that\ \deg(v) + \deg(u) = even$. If u is not adjacent to any vertex in G with the same degree, then u must belong to D and G has ESDS.

Nevertheless, if G has a single isolated vertex, according to Remark 1.4 (i), it has an even sum dominating set but no WDS. ■

Proposition 2.7. A tree has WESDS if and only if it is a star S_n and n is even

Proof. In the event that G is a tree and that tree possesses a WESDS, then that WESDS is denoted by the letter D and has the smallest possible cardinality. If "D" is bigger than one ($|D|\ > 1$), then "G" must have a cycle of order greater than or equal to four; this is in direct opposition to our working assumption. As a result, 1 corresponds to the value of D.

Now, if $G[V\ - D]$ is not isomorphic to null graph, which would indicate that it has at least one edge, then the vertices that are incident on this edge with the vertex that belongs to D form a cycle, which is again in conflict with our assumption. If $G[V - \ D]$ is isomorphic to null graph, then it would not have any edges. Therefore, since $|D|$ equals 1 and G[V- D] is a null graph, the answer to the question is that G is a star. On the other hand, the claim is quite open and obvious

Conversely, the assertion is clear by Proposition 1.6. ■

Proposition 2.8. If G is a complete bipartite graph $K_{m,n}$, then

$\gamma_{we}(G) = \{n, \quad if\ n\ and\ m\ \ are\ both\ even\ or\ odd, m \geq n\ \}$, Otherwise G has no Whole Even sum domination set .

Proof. Suppose that $V_1 = \{u_{i,}, i = 1,2, \dots, n\}$ and $V_2 = \{v_{j,}, j = 1,2, \dots, m\}$ are the bipartite sets of the graph G. Then three different cases are obtained as follows.

Case 1. If $n = 1$ and m is odd, then G is isomorphic to the S_n, by Proposition 1.7. Then one can easily conclude that $D = \{u_1\}$ is γ_{es} −set. Thus,

$\gamma_{we}(G) = 1$.

Case 2. If both n and m are even, it is clear that let $D = \{V_1\ or\ V_2\}$ is γ_{we}-set, since all of the vertices in$V_1 = \{u_{i,}, i = 1,2, \dots, n\}$ are neighbors of all of the vertices in $V_2 = \{v_{j,}, j = 1,2, \dots, m\}$. Similarly, if both n and m are odd numbers. Therefore, $\gamma_{es}(G) = n$.

Case 3 . . If n is odd and m is even, then there are two different degrees between any pair of neighboring vertices. According to Remark 2.1(vi), there are no WESDS in graph G, hence $D = \emptyset$. The same holds true if n is even and m is odd numbers. ■

Proposition 2.9. If the graphs $G_n\ and\ G_m$ have $\gamma_{we}(G_n)\ and\ \gamma_{we}(G_m)$, $\forall n, m\ \geq 2$ respectively , then $\gamma_{we}(G_n +\ G_m) \leq \{\ \gamma_{we}(G_n) + \gamma_{we}\ (G_m)\}$.

Proof. Since both $G_1\ and\ G_2$ contain entire whole even sum dominating sets, we will refer to these sets as D_1 and D_2, respectively, where D_1 and $D_2$are the whole even sum dominating sets with the smallest cardinality in Graph .Since every vertex in either D_1 or D_2 will be joining with every vertex in $G_1 + G_2$and the degrees of all vertices will be changing, it follows that we $\gamma_{we}(G_1 + G_2) \leq \{\gamma_{we}(G_1) + \gamma_{we}(G_2)\}$. ■

Proposition 2.10. Let G be a graph isomorphic to the graph $C_n + C_m$; then =.

$$\gamma_{we}(C_n + C_m) = \begin{cases} 3 & if\ n = 3\ and\ m = 4\ or\ vice\ versa \\ 2 & if n\ and\ m\ \text{both are odd (even)} \\ \{\gamma_{we}(C_1) + \gamma_{we}(C_2)\}\ if\ n\ is\ odd \\ and\ m\ is\ even\ or\ vice\ versa \end{cases}$$

Proof. Let $\{u_1, u_2, \dots, u_n\}$ represent a collection of vertices, and let $\{v_1, v_2, \dots, v_m\}$represent the sets of vertices along routes C_n and C_m, respectively.

Part 1: In this Case , the degree of dominance will be three if the graph G1 takes the shape of a circle with an order of three. Conversely, according to Observation 1.5, the degree of dominance for a circle with an order of four is equal to two. By employing the technique of combining them, it can be observed that each vertex in the triangular circle will be interconnected with every vertex of the quadrilateral circle, resulting in a degree of connectivity. Each vertex within the triangle circle possesses an equivalent value of six and is interconnected with the remaining vertices of the same degree. However, in the circle with rank four, the degree of each vertex will be equal to five, and since every two vertices are near to the other vertices, the resulting dominance is equal to two. Upon completion of the procedure of combining the two claims, it can be observed that the resulting dominance will be equivalent to three .

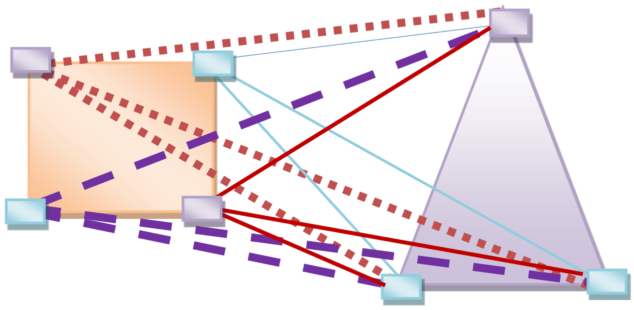

Figure :2
$D = \{v_1, v_3, u_1\}, \gamma_{se}(G_1 + G_2) = 3$

Part 2:If n and m are odd(even) then all vertex in n adjacent with all vertices in m and $u_i + v_j =$ even $(\forall\ i = 1,2, \dots n\ and\ j = 1,2, \dots m)$ such that $\gamma_{we}(G_1 + G_2) = 2$.

In this case, if the number of vertices in both graphs is even or odd, then their sum in both cases is even, and each vertex is adjacent to the first graph with two vertices of the same degree, and from the second graph there are vertices that are vertex in degree or lower than it in degree. In this case, we will take two vertices. One of them is from the first graph, let it be u_1, and the second is from the second graph, let it be v_1. Since the operation is an Addition, each vertex of the graph will be connected to the rest of the other vertices. In this case, the two conditions of the definition are met. Strong Even Sum. The first condition is, that the sum of each two vertices is equal. An even number, and the other condition is that there is a vertex adjacent to the vertex that belongs to the dominant group and is less than it or equal to it in degree, that is, it must be

Part 3:In this case, the graph consists of two groups, one of which has all the vertices with an odd degree, and its dominance is D_1, and the other group that all the vertices with an even degree, so its dominance is D_2, meaning each group of them has its own dominance. To know the dominance of all the graphs$(G_1 + G_2)$, the original group will have an original dominance equal to $D = D_1 + D_1$ ■

Proposition 2.11. Let G be a graph isomorphic to the graph $P_n + C_m$,

$$\gamma_{we}(P_n + C_m) = \begin{cases} 1 & if\ n = 2\ and\ m\ is\ odd \\ 3 \quad if\ n = 2, 3\ where\ m = 4\ and\ if\ n = 3\ where\ m\ is\ odd \\ m + 1 & if\ n = 2,3\ and\ m > 4 \end{cases}$$

.Otherwise $P_n + C_m$ has on WESDS.

Proof: Three parts discussed as follows.

Part 1. Let $n = 2$ such that $\{u_1, u_2\}$ represent a collection of vertices and $\{v_1, v_2, \dots, v_m\}$ represent the collections of vertices along routes $P_2$and C_m, respectively. If m is odd, then all vertices in P_2 that are adjacent to all vertices in m and $u_i + v_j =$ even $(\forall\ i = 1,2\ and\ j = 1,2, \dots m)$ are such that $\gamma_{we}\ (P_2 + C_m) = min\{\gamma_{we}(G_1), \gamma_{we}(G_2)\}$. Since γ_{we} P_2 equals 1, $\gamma_{we}(P_2 + C_m) = 1$.

Part 2. In this part there are two cases

Case 1) Combining the two graphs P_2 and C_4 results in a new statement with two sets of vertices, the first of which is odd and the second of which is even. According to Proposition 1.12 , the degrees of all vertices between the two statements are distinct. In this instance, we treat it as a single statement and calculate the minimum between P_2 and c4 from P_2. We will select the vertex u1 that has a degree of five and is dominant over the other vertex. Its degree is likewise five, and u_1 is connected to the remaining vertices. Even, it is only possible to dominate it with two heads because each vertex of even degree is connected to the remaining even-degree vertices, so γ_{we} $(P_2 + C_m)$ will equal three. $\{u_1, u_2\}$

Case 2) Let $n = 3$ so that each set of vertices is denoted by u_1, u_2 and $, u_3$, and each set of vertices along P_2 and C_m is denoted by $v_1, v_2, \dots and\ v_3$.

In this case, the three vertices in P_3 will be connected to all of the vertices in C_4, but the degree of u_2 in P_3 is equal to 6. The degree of each vertex in the remainder of the graph is unusual. According to the observation, γ_{we} $(C_m) = 2$ if the graph is a quadrilateral, and since all of the graph's vertices are odd, when data for any two degrees for any two vertices are compared, the result is even . Except for u_2, it is equal to an even number if the number of vertices in the dominant graph is u_2 and $v_1\ and\ v_2$, and the product is equal to 3.

Similarly, if m exclusively contains odd numbers, that is, circles with an odd number of vertices, the probability are the same.

Part 3. In this case, each vertex in the graphC_m will have an even degree, but none of them will be adjacent to all other vertex in the other graph. As for the two vertices in the graph $P_2\ or\ P_3$, their degree will be odd, so it is possible to choose any of the two vertices within the dominance D, as each vertex is adjacent to the other vertices of the graph and the sum of its degree and the degree of the vertex $u_1\ in\ P_2\ and\ u_1,\ u_3\ in\ P_3$ will be odd . The outcome will be even, but in C_m we take all vertices of the statement with a value of m, so the result in group D is m plus 1. Then $\gamma_{we}(P_n + C_m) = m + 1. \forall n = 2, 3$ ■

Proposition 2.12. If $G_1$is a Null graph of order 1 and G_2be a cycle or star of order m, then

$$\begin{cases} \gamma_{we}(N_1 \odot C_m) \equiv \gamma_{we}(W_m) \\ = \begin{cases} 1 & if\ m\ is\ odd \\ 3 & if\ m = 4\quad O.W\ \ has\ no\ \text{WESDS} \end{cases} \\ \gamma_{we}(N_1 \odot S_m) = \begin{cases} 1 & \text{if m is } even \\ m & if\ m\ is\ odd \end{cases} \end{cases}$$

Proof . Let $N_1 = \{v_1\}$ and $C_m = S_m = \{u_1, u_2, \dots, u_m\}$, so there are three parts.

Part 1. Two cases discussed in this part.

Case1: In this case, if the graph resembles the graph of the empty graph with the circle in the Corona process, since the degree of each vertex in the circle is equal to 2, then when it is linked with the empty graph, the degree of the vertices changes, so that they all become an odd degree and equal to 3. Therefore, the only vertex, v_1, that represents the empty graph, is the dominant vertex of The entire graph is because its degree is odd, and when it is combined with any degree of the rest of the vertices, it will also remain odd. Likewise, all vertices in the circle will be adjacent to v_1, according to the definition of Corona. Therefore, every vertex in the circle will be adjacent tov_1, so the dominance in this case is equal to 1.

Case 2:that if the graph forms the circles graph, then the degrees of all the vertices in the circle will be even. If we use the Corona process and combine the two graphs, the degree of v 1 will be even, and all the degrees of the vertices within the circle will be odd. In this case, the condition, that every vertex is in the dominant group, will not be met. It is connected with the rest of the other vertices and dominates them. The exception to this case is that, if the circle is equal to C4, then the dominance here is equal to three.

Part 2: In this part there are two cases

Case1: LetG_2 be a graph isomorphic to Star graph order m such that m is even. The degree of each vertex in the star graph will be odd if the number of vertices is an even number. As a result of performing the corona operation by merging the empty graph v_1 with the remaining vertices, the degree of v_1 will be altered so that all vertices have a degree of even. Consequently, the value of dominance will be limited to v_1, as its combination with any vertex results in an even set, and every non-dominant vertex $V - D$ is adjacent to v_1.

Case 2 :If the number of vertices in the star graph is odd, then the degree of each vertex will be odd, except for the central vertex, in which case its degree will be even. When vertex $v_1$is merged in the corona process with the rest of the other vertices in the star graph, the degree of each vertex will change, so that all vertices become even except for vertex v_1 and the central vertex. For the star statement, the dominant set is equal to m except v_1 ■ .

Proposition 2.13 P_m is a path graph of order m if N_1 is a null graph. Then

$$\gamma_{we}(N_1 \odot P_m) = \begin{cases} 1 & if\ m = 1,2 \\ m & O.W \end{cases}$$

Proof. By definition of corona then N_1 adjacent with all vertex in graph G_m there are two cases

Case1: Let $\{u_1\}$ and $\{v_1\}$ be the sets of vertices of the paths N_1 and P_1 respectively. By Observation 1.5 (i) $\gamma_{we}(P_n)$ = 1 if $n = 2$ and if $\{v_1, v_2\} \in P_m$. such that $\{u_1\}$ adjacent with $\{v_1, v_2\}$ we will have graph isomorphic to C_3 by Observation 1.5 (i) $\gamma_{we}(C_3)$ = 1

Case2:When performing the corona operation between the path and the empty graph, we will have a graph that resembles an fun graph, which will be a dominant set equal to all of m, because if m is even, then the vertex v 1 will dominate only the vertices u1 and un, and the rest of the vertices will have a different degree and not adjacent to other vertices, so it must contain the set that dominates all m vertices. In this case, we guarantee that all vertices of the graph are adjacent to v 1 and satisfy both identity properties. In the same way, if m is odd, it will dominate all the internal vertices, except for the two vertices u1 and un, which have an even degree. Therefore, all m must be included within the dominant set, because it is not possible to take only the two vertices u1 and um with the vertex v1, because they are not adjacent to each other. The vertices that remain outside the dominant group, i.e. in the group V-D. ■

Proposition 2.14 Let G be a graph has whole Even sum domination number γ_{we} and $v \in G$ if $\gamma_{we}(G-v) \geq 1$, then $V^i \neq \emptyset \; \forall \; i = +,-,0$.

Proof. There are two cases into which the graph G might be placed.

Case 1: let G has $\gamma_{we}(G) = 1$. In this case, the proof is divided into two partial cases for further Clarification.

Sub cease 1: is when there is exactly one vertex in a graph G (let's call it v) that is adjacent to every other vertex in G and same degree all these vertices, in this case, $G - v$ has a $WESDS$ if there are two or more vertices in $G - v$ that are adjacent to every other vertex in $G - v,$ and thus $\gamma_{we}(G-v) > \gamma_{we}(G)$. Otherwise, there is no WESDS in $G - v$.

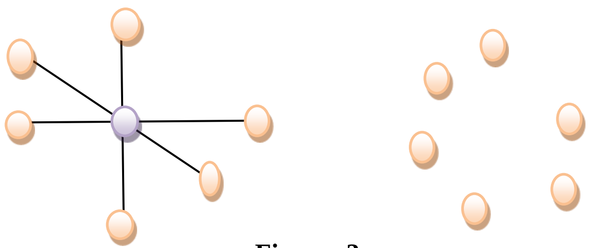

Figure: 3

$D_1 = \{7\}$ $\gamma_{we}(G) = \{1\}$	$D_1 = \{1,2,3,4,5,6\}$

Sub case 2: if there is a vertex $s \neq v$ such that deg(s) = n - 1, then $\gamma_{we}(G-v) = \gamma_{we}(G) = 1$.

Case 2: let G has $\gamma_{we}(G) > 1$ .there are two sub case

Sub case 1: If we delete the vertex $v \in V - D$, we acquire the two subcases shown below.

i) If |D| = |V- D| = we, then $\gamma_{we}(G-v) < \gamma_{we}(G)$, as the cardinal number of vertex set of induced subgraph $G[(V-D) - \{v\}]$ is less than the cardinal number of vertex set of induced subgraph. In addition, the vertices in are adjacent to all vertices in (V- D) – {v} , and the vertices in are adjacent to all vertices in D.

ii) If |D| is greater than |V-D|, then set D remains the MWEDS, and $\gamma_{we}(G-v) = \gamma_{we}(G)$ Delete the vertex v where $v \in D$. In this instance, the set D v remains the MWEDS in G, while $\gamma_{we}(G-v) < \gamma_{we}(G)$. ■

Proposition 2.15 : Let $G = (V, E)$ be a graph and $v \in V$.

Then, $\gamma_{we}(G) - deg(v) - 1 \leq \gamma_{we}(G-v) \leq \gamma_{we}(G) + deg(v) - 1$.

Proof. Suppose that v ∈ V and $D_{we}(G)$ is the whole even sum dominating set of G. First we find the upper bound for $\gamma_{we}(G-v)$. We consider the following cases:

(i) $v \notin D_{we}(G)$. Then at least two of v's neighbors must be members of $D_{we}(G)$. By placing all of v's other neighbors in $D_{we}(G)$, we now have a complete even-sum dominating set for Gv. This set has a maximum size of $\gamma_{we}(G)$ 1 + deg(v).

(ii) $v \in D_{we}(G)$ is adjacent to every vertex in (G) and has the same edger as . By removing v and its associated edges, some of the neighboring vertices may no longer be dominant with any other vertex. Therefore, by placing all of v's neighbors v in $D_{we}(G) - \{v\}$, we obtain a complete whole even sum dominating set for G -v. So $\gamma_{we}(G-v) \leq \gamma_{we}(G) + deg(v) - 1$.

(iii) $v \in D_{we}(G)$ is adjacent to every vertex in (G) and has the different edger as . Consequently, per Remark 1.4 Now, using the same logic as in case (ii), we have

$$\gamma_{we}(G-v) \leq \gamma_{we}(G) + deg(v) - 1.$$

Now we determine the lower bound for $\gamma_{we}(G-v)$. We begin by removing v and all of its corresponding edges. Now, a complete whole even sum dominating set is discovered for G -v .

Consider this set to be $D_{we}(G-v)$. It is straightforward to verify that $D_{we}(G-v) \cup N[v]$ is a whole even sum dominating set of G ,the result is $\gamma_{we}(G) \leq \gamma_{we}(G-v) + deg(v-1)$.

Now we consider to G ⊛ v and find upper and lower bounds for the co-even domination number of that ■

Proposition 2.16: Assume that $v \in V$ and that $G = (V, E)$ is a graph. This leads to the following: $\gamma_{we}(G) + deg(v) + 1 \leq \gamma_{we}(G \circledast v) \leq \gamma_{we}(G) + deg(v) - 1$.

Proof : Consider the case where $v \in V$ and $D_{we}(G)$ is the Whole even some dominating set of G. Initially, we determine the upper limit for $\gamma_{we}(G \odot v)$. The following cases are examined:

(i) $v \in V - D$ In this case, v must be adjacent to every vertex in G and there must exist an $u \in N_G(v)$ such that $u \in D_{we}(G)$. According to the definition of $(G \circledast v)$, the removal of all edges between any two neighbors of v may alter their degree to an odd number, but v's degree remains even. $D_{we}(G) \cup N_G(v)$ is therefore a Whole even sum dominating set for $(G \odot v)$, and since u is adjacent to v and $u \in D_{we}(G)$,

$\gamma_{we}(G \odot v) \leq \gamma_{we}(G) + deg(v) - 1$.holds.

(ii) $v \in D_{we}(G)$ In this instance, we consider to the set $D_{we}(G) - \{V\} \cup N_G(v)$. we assume that v has an even degree and dominates a group of vertices, so that the sum of their degrees is also even. When all adjacent edges v are deleted, their degree will change, but v's degree will remain unchanged, making it simple to maintain v's dominance as follows:

$\gamma_{we}(G \circledast v) \leq \gamma_{we}(G) + deg(v) - 1$.

iii) $v \in D_{we}(G)$, and deg(v) is an odd number. First, presume that at least one vertex exists in $D_{we}(G) \cap N_G(v)$. $D_{we}(G) \cup N_G(v)$ is then Whole for any dominating set for G v with size at most $\gamma_{we}(G) + deg(v) - 1$.

Now, let's assume that $D_{we}(G) \cap N_G(v) = \{\,\}$

All vertices in $N_G(v)$ cannot have an even degree and be connected to an odd number of vertices in $N_G(v)$ at the same time, as demonstrated by our proof. Suppose this occurs and all vertices in $N_G(v)$ have an even degree and are connected to an odd number of other vertices in $N_G(v)$simultaneously. By removing v and all vertices in G that are not in $N_G(v)$ and considering this subgraph of G, we obtain a graph with an odd number of vertices and each vertex has an odd degree, which contradicts the Handshake theorem. Therefore, this cannot occur. Therefore, there exists at least one vertex $w \in N_G(v)$adjacent to an even number of vertices. Note that deg(w) remains in $(G \circledast v)$ as well. By considering $D_{we}(G) - \{w\} \cup N_G(v)$ to be our dominating set, we have a whole even sum dominating set of size$\gamma_{we}(G) + deg(v) - 1$. $\gamma_{we}(G \circledast v) \leq \gamma_{we}(G) + deg(v) - 1$. is the result.

As our dominating set, we have a set of size $\gamma_{we}(G) + deg(v) - 1$ with whole even sum dominating set as its sum. $\gamma_{we}(G \odot v) \leq \gamma_{we}(G) + deg(v) - 1$. is the result.

Now, the lower bound for $\gamma_{we}(G \circledast v)$ is determined. First, $(G \circledast v)$ is formed, and then a complete whole even sum dominating set is found for it. This set is denoted by $D_{we}(G \circledast v)$. The following cases are examined:

$i^{\sim}$) $v \notin D_{we}(G \circledast v)$This indicates that v is adjacent to all vertices of G and that there is a vertex in D, such that the result of the plural of their degrees is an even number and there exists $u \in D_{we}(G \circledast v) \cap N_G(v)$. By considering $D_{we}(G \odot v) \cup N_G(v)$ as our dominating set, we have a whole even sum dominating set for G, and since u is adjacent to v and $u \in D_{we}(G \circledast v)$, then $\gamma_{we}(G) \leq \gamma_{we}(G \circledast v) + deg(v) - 1$.

$ii^{\sim}$) $v \in D_{we}(G \circledast v)$each vertex in $V - D$ is associated with it and its degree is even. When combined with the degrees of other vertices , the plural will also be even.. In this case we consider to the set $D_{we}(G \circledast v) - \{v\} \cup N_G(v)$.It is easy to see that this set is a whole even sum dominating set for G and since deg(v) is even after the collection process , we do not need it in our dominating set. So $\gamma_{we}(G) \leq \gamma_{we}(G \circledast v) + deg(v) - 1$.

$iii^{\sim}$) $v \in D_{we}(G \circledast v)$ and deg(v) is an odd quantity. Assume first that $D_{we}(G \circledast v) \cap N_G(v)$ contains at least one vertex. $D_{we}(G \circledast v) \cup N_G(v)$is then Whole for any dominating set for G v with size no larger than $\gamma_{we}(G) + deg(v) - 1$.

Now, let's suppose that $D_{we}(G) \cap N_G(v) = \{\,\}$

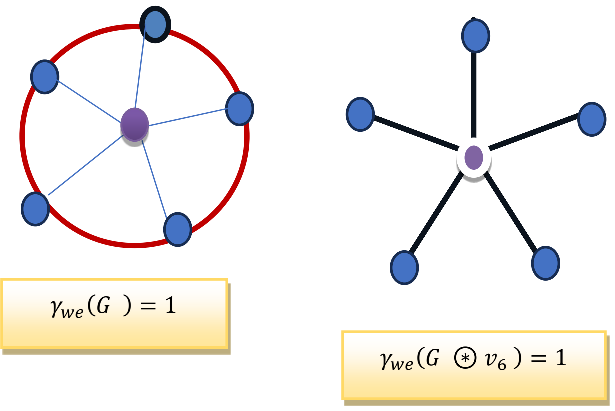

Figure : 4
$\gamma_{we}(G\ ) = \gamma_{we}(G \circledast v_6)$

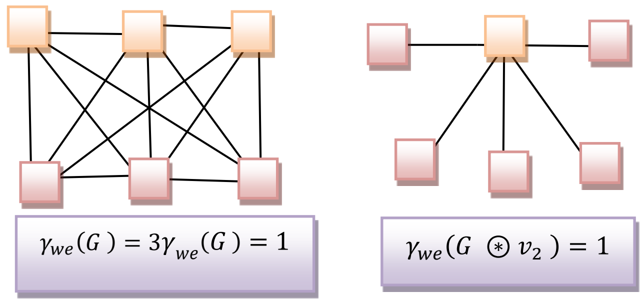

Figure : 5
$\gamma_{we}(G \circledast v_2\ ) \leq \gamma_{we}(G)$

Our proof demonstrates that no vertex in $N_G(v)$ can have an even degree and be connected to an odd number of vertices in the same graph. Assume that this occurs and that all vertices in $N_G(v)$have an even degree and are simultaneously connected to an odd number of other vertices in $N_G(v)$. By removing v and all vertices in G that are not in $N_G(v)$and contemplating this subgraph of G, we obtain a graph with an odd number of vertices and each vertex having an odd degree, which is inconsistent with the Handshake theorem. Consequently, this is impossible. There exists, therefore, at least one vertex s $\in N_G(v)$adjacent to an even number of vertices. Note that deg(s) is still present in $(G \circledast v)$. By considering $(D_{we}(G\ ) - \{s\}) \cup N_G(v)$ to be our dominating set, we have a dominating set of size $\gamma_{we}(G \circledast v) + deg(v) - 1$. that has a whole even sum. The result of $\gamma_{we}(G\ ) \leq \gamma_{we}(G \circledast v) + deg(v) - 1$. ■

3 .Acknowledgments

New definition of domination is presented and from the above results, many properties of this number are discussed and get a new results different from a previous domination. Also, some operation of some graphs are determined. Additionally the effect of deletion a vertex or removal of all edges between any pair of neighbours of a vertex are calculated.

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