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On The Bifuzzy ideals of RG − algebra

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Abstract. *We consider the fuzzy fication of the concept ideal and some fundamental properties to RG-algebra are discussed and the image (pre − image) of ideal of RG − algebra, and investigate some of there properties and investigate some related properties.*

Keywords: RG-algebras, fuzzy q-ideals, bifuzzy q-ideal, image (pre-image) of bifuzzy q-ideal, Cartesian product of bifuzzy q-ideal.

1. Introduction

R.A.K. Omar, have introduced the notion of $RG-algebras$, $RG-ideals$, $RG-subalgebras$ and studied the relations among them and introdued the concept of homomorphism of $RG-algebras$ and investigated some related properties. $A.T.Hameed$ and et al. introduced and studied concepts of fuzzy $RG-subalgebras$ and fuzzy $RG-ideals$ of $RG-algebra$ and investigate some of its properties, several Theorems, properties are stated and proved. A.T. Hameed and $S.M.Abrahem$ introduced the notion of doubt fuzzy $RG-ideals$ of $RG-algebras$ and studied the homomorphism image and inverse image of doubt fuzzy $RG-ideals$ also prove that the Cartesian product of doubt fuzzy $RG-ideals$ are doubt fuzzy $RG-ideals$. In this paper, we introduce the notion of bifuzzy ideals of $RG-algebras$ and fuzzy image $(pre-image)$ of bifuzzy ideals of $RG-algebras$. We also introduce the intersection and union bifuzzyideals of $RG-algebras$ and investigate some results.

2. Preliminaries:

We review some definitions and properties that will be useful in our results.

Definition. 2.1 [24].
An algebra $(\partial; *, 0)$ is called an RG-algebra (RG-A) if the following axioms are satisfied: $\forall\, \sigma, \rho, \varepsilon \in \partial$,
(i) $\sigma * 0 = \sigma$,
(ii) $\sigma * \rho = (\sigma * \varepsilon) * (\rho * \varepsilon)$,
(iii) $\sigma * \rho = \rho * \sigma = 0$ imply $\sigma = \rho$.

Remark. 2.2 [24].
For brevity we also call ∂ RG-A, we can define a binary relation $(\leq)$ by putting $\sigma \leq \rho$ if and only if $\sigma * \rho = 0$.

Example. 2.3([24]).
Let $\partial = \{0, 1\}$ and let $*$ be defined by:

$*$	0	1
0	0	1
1	1	0

Then $(\partial; *, 0)$ is an RG-A.

Example. 2.4 [24].
Let ∂ ={0 , a, b, c} and $(\partial;*)$ be the pair given by the table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$(\partial;*,0)$ is an RG-A.

Proposition. 2.5 [15].
In any RG-A $(\partial;*,0)$, the following hold: $\forall\sigma,\rho\in\partial$,
i) $\sigma*\sigma=0$,
ii) $0*(0*\sigma)=\sigma$,
iii) $\sigma*(\sigma*\rho)=\rho$,
iv) $\sigma*\rho=0$ if and only if $\rho*\sigma=0$,
v) $\sigma*e=0$ implies $\sigma=0$,
vi) $0*(\rho*\sigma)=\sigma*\rho$
In any RG-A $(\partial;*,0)$, the following hold: $\forall\sigma,\rho,\varepsilon\in\partial$,
i) $(\sigma*\rho)*(0*\rho)=(\sigma*(0*\rho))*\rho=\sigma$,
ii) $\sigma*(\sigma*(\sigma*\rho))=\sigma*\rho$,
iii) $(\sigma*\rho)*\varepsilon=(\sigma*\varepsilon)*y$.
iv) $\sigma*\rho=(\varepsilon*\rho)*(\varepsilon*\sigma)$,
v) $((\sigma*\rho)*(\sigma*\varepsilon))*(\varepsilon*\rho)=0$.

Definition. 2.7([15]).
Let $(\partial;*,0)$ be a RG-A and let S be a nonempty subset of ∂. S is called an RG-subalgebra of ∂ (RG-SA) if $\sigma*\rho\in S$ whenever $\sigma,\rho\in S$.

Definition. 2.8([19]).
A nonempty subset I of a RG-A $(X;*,0)$ is called an ideal of ∂ (RG-I) if it satisfies the following conditions: for any $\sigma,\rho\in\partial$,
(I_1) $0\in I$;
(I_2) $\sigma*\rho\in I$ and $\sigma\in I$ imply $\rho\in I$.

Proposition . 2.9 ([15]).
Every RG-I of RG-A is an RG-SA.

Definition. 2.10 ([15]).
Let $(\partial;*,0)$be a RG-A, a fuzzy subset μ of ∂ is called a fuzzy RG-subalgebra of ∂ (FRG-SA) if $\forall\sigma,\rho\in\partial$,
$\pi(\sigma*\rho)\geq\min\{\pi(\sigma),\pi(\rho)\}$.

Definition. 2.11([15]).
Let $(\partial;*,0)$ be a RG-A, a fuzzy subset π of ∂ is called a fuzzy ideal of ∂ (FRG-I) if it satisfies the following conditions, $\forall\sigma,\rho\in\partial$,
(1) $\pi(0)\geq\pi(\sigma)$,
(2) $\pi(\rho)\geq\min\{\pi(\sigma*\rho),\pi(\sigma)\}$.

Lemma. 2.12([15]).
Let π be a FRG-I of RG-A $(\partial;*,0)$and if $\sigma\leq\rho$, then $\mu(\sigma)\geq\mu(\rho)$, $\forall\sigma,\rho\in\partial$,.

Proposition. 2.13([15]).
1- Let π be a fuzzy subset of RG-A $(\partial;*,0)$. π is a FRG-SA of ∂ if and only if for every t ∈ [0 ,1], π_t is an RG-SA of X.
2- Let π be a fuzzy subset of RG-A $(\partial;*,0)$, π is a FRG-I of ∂ if and only if for every t ∈ [0 ,1], π_t is RG-I of ∂.
3- Let A be a nonempty subset of a RG-A $(\partial;*,0)$ and π be a fuzzy subset of ∂ such that π is into {0 , 1}, so that π is the characteristic function of A. Thenπ is a FRG-I of ∂ if and only if A is RG-I of ∂.

Proposition. 2.14([15]).
Every FRG-I of RG-A is a FRG-SA.

Definition. 2.15[6,7]:
Let ∂ be a nonempty set and π be a fuzzy subset of ∂ and let $\alpha\in[0,T]$. A mapping $\pi^T_\alpha:\partial\to[0,1]$ is called a $\boldsymbol{\alpha}$-translation fuzzy sub-set of π if it satisfies:
$\pi^T_\alpha(\sigma)=\pi(\sigma)+\alpha$, for all $\sigma\in\partial$, where$T=1-\sup\{\pi(\sigma):\sigma\in\partial\}$.

Definition. 2.16([10]).
Let $(\partial;*,0)$ be an RG-A, a fuzzy subset π in ∂ is called a fuzzy q-ideal of ∂ (Fq-I) if it satisfies the following conditions: $\forall\sigma,\rho,\varepsilon\in\partial$,
(1) $\pi(0)\geq\pi(\sigma)$,
(2) $\pi(\sigma*\varepsilon)\geq\min\{\pi((\sigma*\rho)*\varepsilon),\pi(\rho)\}$.

Proposition . 2.17([10]).
Every Fq-I of RG-A $(\partial;*,0)$ is a FRG-I of ∂.

3. The Bifuzzy RG-subalgebras of RG-algebra

In this section, we shall define the notion of bifuzzy RGsubalgebra of RGalgebra and we study its properties when the bifuzzy RGalgebra is replace with bifuzzy RGsubalgebra of RGalgebra.

Definition 3.1.
Let $(\partial;*,0)$ be RG-algebra, a fuzzy sub-set v of ∂ is called an antifuzzy RG-subalgebra of ∂ if $\forall\sigma,\rho\in\partial$,
$v(\sigma*\rho)\leq\max\{v(\sigma),v(\rho)\}$.

Definition 3.2.
A bifuzzy sub-set A in a nonempty-set ∂ is an object having the form $A=\{(\sigma,\pi_A(\sigma),\nu_A(\sigma))\mid\sigma\in\partial\}$ where the functions $\pi_A:\partial\to[0,1]$ denote the fuzzy function and $\nu_A:\partial\to[0,1]$ denote the degree of antifuzzy function and $e\leq\mu_A(\sigma)+\nu_A(\sigma)\leq1$, for all $\sigma\in\partial$.

Definition 3.3.

Let $A = \{(\sigma, \pi_A(\sigma), \nu_A(\sigma)) \mid \sigma \in \partial\}$ be a bifuzzy sub-set of an RG-algebra $(\partial; *, 0)$. A is said to be bifuzzy RGsubalgebra of ∂ if : $\forall \sigma, \rho \in \partial$,

(IFS$_1$) $\pi_A(\sigma * \rho) \geq \min\{\pi_A(\sigma), \pi_A(\rho)\}$,

(IFS$_2$) $\nu_A(\sigma * \rho) \leq \max\{\nu_A(\sigma), \nu_A(\rho)\}$.

i.e., π_A is fuzzy RG-subalgebra of RG-algebra and ν_A is antifuzzy RG-subalgebra of RG-algebra.

Example 3.4.

Let $\partial = \{0, 1, 2, 3\}$ in which $(*)$ be a defined by the following table:

$*$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Then $(\partial; *, 0)$ is an RG-algebra . $S_1 = \{0, 1\}$, $S_2 = \{0, 2\}$, $S_3 = \{0, 3\}$ and $S_4 = \{0, 1, 2, 3\}$ are RG-subalgebras of ∂.

Define a fuzzy sub-set $\pi_A: \partial \to [0, 1] \ni \pi_A(0) = 0.7$, $\pi_A(1) = \pi_A(2) = 0.6$, $\pi_A(3) = 0.4$,

ν_A: $\partial \to [0, 1] \ni \nu_A(0) = 0.3$, $\nu_A(1) = \nu_A(2) = 0.4$, $\nu_A(3) = 0.6$.

Routine calculation gives that π_A is a fuzzy RG-subalgebra of ∂ and that ν_A is antifuzzy RG-subalgebra of ∂.

Proposition 3.5.

Every bifuzzy RG-subalgebra $A = \{(\sigma, \pi_A(\sigma), \nu_A(\sigma)) \mid \sigma \in \partial\}$ of RG-algebra $(\partial; *, 0)$ satisfies the inequalities $\pi_A(0) \geq \pi_A(\sigma)$ and $\nu_A(0) \leq \nu_A(\sigma)$, $\forall \sigma \in \partial$.

Proof:

$\pi_A(0) = \pi_A(\sigma * \sigma) \geq \min\{\pi_A(\sigma), \pi_A(\sigma)\} = \pi_A(\sigma)$ and $\nu_A(0) = \nu_A(\sigma * \sigma) \leq \max\{\nu_A(\sigma), \nu_A(\sigma)\} = \nu_A(\sigma)$. ⌂

Definition 3.6.

For a fuzzy sub-set s π_A and ν_A of an RG-algebra $(\partial; *, 0)$ and $t \in \mathrm{Im}(\nu_A)$ with $t \geq \alpha$, let $U(\pi_A, t) = \{\sigma \in \partial \mid \pi_A(\sigma) \geq t\}$ and $s \in \mathrm{Im}(\nu_A)$ with $s \leq \varepsilon$, $L(\nu_A, s) = \{\sigma \in \partial \mid \nu_A(\sigma) \leq s\}$.

Remark 3.7.

1- If π_A is a fuzzy RG-subalgebra of RG-algebra $(\partial; *, 0)$, then It's that $U(\pi_A, t)$ is an RG-subalgebra of ∂, for all $t \in \mathrm{Im}(\pi)$. Let $\sigma, \rho \in U(\pi_A, t)$, then $\pi_A(\sigma) \geq t$, and $\pi_A(\rho) \geq t$, then $\min\{\pi_A(\sigma), \pi_A(\rho)\} \geq t$, since π_A is a fuzzy RG-subalgebra, then $\pi_A(\sigma * \rho) \geq \min\{\pi_A(\sigma), \pi_A(\rho)\} \geq t$, therefore $\sigma * \rho \in U(\pi_A, t)$.

2- If ν_A is antifuzzy RG-subalgebra of X, then It's that $L(\nu_A, s)$ is an RG-subalgebra of X, for all $s \in \mathrm{Im}(\nu)$. Let $\sigma, \rho \in L(\nu_A, s)$, then $\nu(\sigma) \leq s$, and $\nu(\rho) \leq s$, then $\max\{\nu_A(x), \nu_A(y)\} \leq s$, since ν_A is antifuzzy RG-subalgebra, then $\nu_A(\sigma * \rho) \leq \max\{\nu_A(\sigma), \nu_A(\rho)\} \leq s$, therefore $\sigma * \rho \in L(\nu_A, s)$.

3- But if we do not give a condition that π_A is a fuzzy RG-subalgebra of ∂, then $U(\pi_A, t)$ is not an RG-subalgebra of ∂ or ν_A is antifuzzy RG-subalgebra of ∂, then $L(\nu_A, s)$ is not an RG-subalgebra of ∂ as seen in the following example.

Example 3.8.

Consider $\partial = \{0, 1, 2, 3\}$ is an RG-algebra which is given in Example (3.4) . Define a fuzzy sub-set λ of ∂:

∂	0	1	2	3
π_A	0.7	0.6	0.4	0.3

Then π_A is not a fuzzy RG-subalgebra of ∂.

Since $\pi_A(1*2) = 0.3 < 0.4 = \min\{\pi_A(1), \pi_A(2)\}$.

For $t = 0.5$, we obtain $U(\pi_A, t) = \{0, 1, 2\}$ which is not an RG-subalgebra of ∂ since $1*2 = 3 \notin U(\pi_A, t)$.

Proposition 3.9.

Let $A = \{(\sigma, \pi_A(\sigma), \nu_A(\sigma)) \mid \sigma \in \partial\}$ be a bifuzzy sub-set of an RGalgebra $(\partial; *, 0)$. If A is a bifuzzy RG-subalgebra of ∂, then it satisfies: for any $t, s \in [0, 1]$, $U(\pi_A, t) \neq \emptyset$ implies $U(\pi_A, t)$ is an RG-subalgebra of ∂ and $L(\nu_A, s) \neq \emptyset$ implies $L(\nu_A, s)$ is an RG-subalgebra of ∂.

Proof:

Assumethat π is a fuzzy RG-subalgebra of ∂, let $t \in [0, 1]$ be $\ni U(\pi_A, t) \neq \emptyset$, and let $\sigma, \rho \in \partial$ be $\ni \sigma, \rho \in U(\pi, t)$, then $\pi_A(\sigma) \geq t$ and $\pi_A(\rho) \geq t$, so $\pi_A(\sigma * \rho) \geq \min\{\pi_A(\sigma), \pi_A(\rho)\} \geq t$, so that $(\sigma * \rho) \in U(\pi_A, t)$. Hence $U(\pi_A, t)$ is an RG-subalgebra of ∂.

Assumethat ν_A is an antifuzzy RG-subalgebra of ∂, let $s \in [0, 1]$ be $\ni L(\nu_A, s) \neq \emptyset$, and let $w, u \in X$ be $\ni \sigma, \rho \in L(\nu_A, s)$, then $\nu_A(\sigma) \leq s$ and $\nu_A(\rho) \leq s$, so $\nu_A(\sigma * \rho) \leq \max\{\nu_A(\sigma), \nu_A(\rho)\} \leq s$, so that $(\sigma * \rho) \in L(\nu_A, s)$.

Hence $L(\nu_A, s)$ is an RG-subalgebra of ∂. ⌂

Proposition 3.10 .

Let $A = \{(\sigma, \pi_A(\sigma), \nu_A(\sigma)) \mid \sigma \in \partial\}$ be a bifuzzy sub-set of an RG-algebra $(\partial; *, 0)$. If $U(\pi_A, t)$ and $L(\nu_A, s)$ are RG-subalgebras of ∂, for all $t, s \in [0, 1]$, $U(\pi_A, t) \neq \emptyset \neq L(\nu_A, s)$, then A is a bifuzzy RG-subalgebra of ∂ .

Proof:

Suppose that A is not bifuzzy RG-subalgebra of ∂, satisfies $U(\pi_A, t)$ is an RG-subalgebra of ∂. Now, assume $\pi_A(\sigma * \rho) < \min\{\pi_A(\sigma), \pi_A(\rho)\}$, taking $t_e = \frac{1}{2}(\pi_A(\sigma * \rho) + \min\{\pi_A(\sigma), \pi_A(\rho)\})$, we have $t_e \in [0, 1]$ and $\min\{\pi_A(\sigma), \pi_A(\rho)\} > t_e > \pi_A(\sigma * \rho)$, it follows that $\sigma, \rho \in U(\pi_A, t_e)$ and $\sigma * \rho \notin U(\pi_A, t_e)$, this is a contradiction since $U(\pi_A, t_e)$ is an RG-subalgebra of ∂ ,

And $L(\nu_A, s)$ is an RG-subalgebra of ∂, assume $\nu_A\,(\sigma * \rho) > max\{\nu_A(\sigma), \nu_A\,(\rho)\}$,
taking $t_e = \frac{1}{2}(\nu_A\,(\sigma * \rho) + max\{\nu_A\,(\sigma), \nu_A\,(\rho)\})$, we have $s_e \in [0\,, 1]$ and
$max\{\nu_A(\sigma), \nu_A\,(\rho)\} < t_e < \nu_A\,(\sigma * \rho)$, it follows that $\sigma, \rho \in L(\nu_A, s_e)$ and
$\sigma * \rho \notin L(\pi, t_e)$, this is a contradiction since $L(\nu_A, s_e)$ is an RG-subalgebra of X. Therefore A is a bifuzzy RG-subalgebra of ∂. ⌂

Corollary 3.11.

Let $A = \{(\sigma, \pi_A(\sigma), \nu_A\,(\sigma)) \,|\sigma \in \partial\}$ be a bifuzzy sub-set of an RGalgebra $(\partial;*\,,0)$. If A is a bifuzzy an RG-subalgebra of ∂, then for every $t \in Im(\pi_A)$, $L(\pi_A, t)$is an RG-subalgebra of ∂ and $s \in Im(\nu_A)$, $L(\nu_A, s)$ is an RG-subalgebra of ∂.

Definition 3.12.

Let $A = \{(\sigma, \pi_A(\sigma), \nu_A\,(\sigma)) \,|\sigma \in \partial\}$ be a bifuzzy sub-set of an RG-algebra $(\partial;*\,,0)$ where i $\in \Lambda \ni \pi_{Ai}$ are a fuzzy RG-subalgebras of ∂ and ν_{Ai} are antifuzzy RG-subalgebras of ∂, for any $\sigma \in \partial$, then

1-The R_intersection of any set of bifuzzy sub-set of ∂ is $(\cap\, \pi_{Ai})(\sigma) = \inf\,(\pi_{Ai})(\sigma)$ and $(\cup\, \nu_{Ai})(\sigma) = \sup\,(\nu_{Ai})(\sigma)$.
2-The P_intersection of any set of bifuzzy sub-set of X is $(\cap\, \pi_{Ai})(\sigma) = \inf\,(\pi_{Ai})(\sigma)$ and $(\cap\, \nu_{Ai})(\sigma) = \inf\,(\nu_{Ai})(\sigma)$.
3-The R_union of any set of bifuzzy sub-set of ∂ is $(\cup\, \pi_{Ai})(\sigma) = \sup(\pi_{Ai})\,(\sigma)$ and $(\cap\, \nu_{Ai})(\sigma) = \inf\,(\nu_{Ai})(\sigma)$.
4-The P_union of any set of bifuzzy sub-set of ∂ is $(\cup\, \pi_{Ai})(\sigma) = \sup(\pi_{Ai})\,(\sigma)$ and $(\cup\, \nu_{Ai})(\sigma) = \sup\,(\nu_{Ai})(\sigma)$.

Proposition 3.13.

The R_intersection of any set of bifuzzy RG-subalgebra of $(\partial;*\,0)$ is also bifuzzy RG-subalgebra of ∂.

Proof:

Let $A_i = \{(\sigma, \pi_{Ai}\,(\sigma), \nu_{Ai}\,(\sigma)) \mid \sigma \in \partial\}$ where i $\in \Lambda$, be a set of bifuzzy RG-subalgebra of ∂ and $\sigma, \rho \in \partial$, then

$$\begin{aligned}(\cap\, \pi_{Ai})(\sigma * \rho) &= inf\,(\pi_{Ai})(\sigma * \rho)\\ &\geq inf\,\{min\{(\pi_{Ai})(\sigma), (\pi_{Ai})(\rho)\}\}\\ &= min\{inf(\pi_{Ai}(\sigma)), inf(\pi_{Ai}(\rho))\}\\ &= \min\{(\cap\, \pi_{Ai})(\sigma), (\cap\, \pi_{Ai})(\rho)\}.\end{aligned}$$

Hence $(\cap\, \pi_{Ai})$ is a fuzzy RGsubalgebra of ∂.

$$\begin{aligned}(\cup\, \nu_{Ai})(\sigma * \rho) &= \sup\,(\nu_{Ai})(\sigma * \rho)\\ &\leq \sup\{\max\{(\nu_{Ai})(\sigma), (\nu_{Ai})(\rho)\}\}\\ &= \max\{\sup\,(\nu_{Ai}(\sigma)), \sup\,(\nu_{Ai}(\rho))\}\\ &= \max\{(\cup\, \nu_{Ai})(\sigma), (\cup\, \nu_{Ai})(\rho)\}.\end{aligned}$$

Hence $(\cup\, \nu_{Ai})$ is antifuzzy RG-subalgebra of ∂.
Therefore, R_intersection of A_i is a bifuzzy RG-subalgebra of ∂. ⌂

Remark 3.14.

The P-intresection of any sets of bifuzzy RG-subalgebra need not be a bifuzzy RG-subalgebra, for example:

Example 3.15.

Let $X = \{0\,, a, b, c, d\}$ be a set with the following table:

*	0	a	b	c	d
0	0	a	b	c	d
a	a	b	c	d	0
b	b	c	d	0	a
c	c	d	0	a	b
d	d	0	a	b	c

Then $(X;*,0)$ is an RG-algebra. I $=\{0\,, c\}$ and J $=\{0\,, d\}$ are RG-subalgebras of X.

We defined two cubic set
$A_1 = \{(x, \mu_{A1}\,(x), \nu_{A1}\,(x)) \mid x \in X\}$ and
$A_2 = \{(x, \mu_{A2}\,(x), \nu_{A2}\,(x)) \mid x \in X\}$ of X by :-

$$\pi_{A1}(x) = \begin{cases} 0.8\,, & \text{if } x \in \{0\,, c\},\\ 0.7, & \text{if } x \in \{a, b\},\\ 0.6, & \text{otherwise}\end{cases} \qquad \nu_{A1}(x) = \begin{cases} 0.2\,, & \text{if } x \in \{0\,, c\},\\ 0.6, & \text{if } x \in \{a, b\},\\ 0.4, & \text{otherwise}\end{cases}$$

$$\pi_{A2}(x) = \begin{cases} 0.7\,, & \text{if } x \in \{0\,, d\},\\ 0.2, & \text{otherwise.}\end{cases} \quad \text{and} \quad \nu_{A2}(x) = \begin{cases} 0.1\,, & \text{if } x \in \{0\,, d\},\\ 0.4, & \text{otherwise.}\end{cases}$$

Then A_1 and A_2 are bifuzzy RG-subalgebra of ∂, but P_intersection of $A_1 \cap A_2$ is not bifuzzy RG-subalgebra of X. Since
$(\cap\, \pi_{Ai})(c * d) = \min\{0.7, 0.2\} = 0.2 \ngeq 0.6 = \min\{(\cap\, \pi_{Ai})(c), (\cap\, \pi_{Ai})(d)\} = \min\{\min\{0.8, 0.2\}, \min\{0.1, 0.6\}\}$and
$(\cup\, \nu_{Ai})(c * d) = \max\{0.6, 0.4\} = 0.6 \nleq 0.4 = \max\{(\cup\, \nu_{Ai})(c), (\cup\, \nu_{Ai})(d)\} = \max\{\max\{0.2, 0.4\}, \max\{0.4, 0.1\}\}$.

Proposition 3.16.

Let $A_i = \{(\sigma, \pi_{Ai}\,(\sigma), \nu_{Ai}\,(\sigma)) \mid \sigma \in \partial\}$ where i $\in \Lambda$, be a set of bifuzzy RG-subalgebra of RG-algebra $(X;*\,,0)$, where i $\in \Lambda$,
$inf\{max\{\pi_{Ai}(\sigma), \pi_{Ai}(\rho)\}\} = max\{inf\,\pi_{Ai}(\sigma), inf\,\pi_{Ai}(\rho)\}$, for all $\sigma \in \partial$, then the P_intresection of A_i is also a bifuzzy RG-subalgebra of ∂.

Proof:

Let $A_i = \{(\sigma, \pi_{Ai}\,(\sigma), \nu_{Ai}\,(\sigma)) \mid \sigma \in \partial\}$ where i $\in \Lambda$, be a set of bifuzzy RG-subalgebra of ∂ and $\sigma, \rho \in \partial$, then

$$\begin{aligned}(\cap\, \pi_{Ai})(\sigma * \rho) &= inf\,(\pi_{Ai})(\sigma * \rho)\\ &\geq inf\,\{min\{(\pi_{Ai})(\sigma), (\pi_{Ai})(\rho)\}\}\\ &= min\{inf(\pi_{Ai}(\sigma)), inf(\pi_{Ai}(\rho))\}\\ &= \min\{(\cap\, \pi_{Ai})(\sigma), (\cap\, \pi_{Ai})(\rho)\}.\end{aligned}$$

Hence $(\cap\, \pi_{Ai})$ is a fuzzy RG-subalgebra of ∂.

$$\begin{aligned}(\cap\, \nu_{Ai})(\sigma * \rho) &= inf\,(\nu_{Ai})(\sigma * \rho)\\ &\leq \inf\{\max\{(\nu_{Ai})(\sigma), (\nu_{Ai})(\rho)\}\}\\ &= \max\{\inf\,(\nu_{Ai}(\sigma)), \inf\,(\nu_{Ai}(\rho))\}\\ &= \max\{(\cap\, \nu_{Ai})(\sigma), (\cap\, \nu_{Ai})(\rho)\}.\end{aligned}$$

Hence $(\cap \nu_{Ai})$ is antifuzzy RG-subalgebra of ∂.
Therefore, P_ntersection of A_i is a bifuzzy RG-subalgebra of ∂. ⌂

Proposition 3.17.

Let $A_i = \{(\sigma, \pi_{Ai}(\sigma), \nu_{Ai}(\sigma)) \mid \sigma \in \partial\}$ where i ∈ Λ, be a set of bifuzzy RG-subalgebra of RG-algebra $(\partial;*,0)$, where i ∈ Λ,
$sup\{min\{\pi_{Ai}(\sigma), \pi_{Ai}(\rho)\}\} = min\{sup\, \pi_{Ai}(\sigma), sup\, \pi_{Ai}(\rho)\}$, for all $\sigma \in \partial$, then the P_union of A_i is also a bifuzzy RG-subalgebra of ∂.

Proof:

Let $A_i = \{(\sigma, \pi_{Ai}(\sigma), \nu_{Ai}(\sigma)) \mid \sigma \in \partial\}$ where i ∈ Λ, be a set of bifuzzy RG-subalgebra of X and $\sigma, \rho \in \partial$, then

$$\begin{aligned}(\cup \pi_{Ai})(\sigma*\rho) &= sup\,(\pi_{Ai})(\sigma*\rho)\\ &\geq sup\,\{min\{(\pi_{Ai})(\sigma), (\pi_{Ai})(\rho)\}\}\\ &= min\{sup(\pi_{Ai}(\sigma)), sup(\pi_{Ai}(\rho))\}\\ &= \min\{(\cup \pi_{Ai})(\sigma), (\cup \pi_{Ai})(\rho)\}.\end{aligned}$$

Hence $(\cup \pi_{Ai})$ is a fuzzy RGsubalgebra of ∂.

$$\begin{aligned}(\cup \nu_{Ai})(\sigma*\rho) &= \sup(\nu_{Ai})(\sigma*\rho)\\ &\leq \sup\{\max\{(\nu_{Ai})(\sigma), (\nu_{Ai})(\rho)\}\}\\ &= \max\{\sup(\nu_{Ai})(\sigma), \sup(\nu_{Ai})(\rho)\}\\ &= \max\{(\cup \nu_{Ai})(\sigma), (\cup \nu_{Ai})(\rho)\}.\end{aligned}$$

Hence $(\cup \nu_{Ai})$ is antifuzzy RG-subalgebra of ∂.
Therefore, P_union of A_i is a bifuzzy RG-subalgebra of ∂. ⌂

Remark 3.18.

The R_union of any sets of bifuzzy RG-subalgebra need not be a bifuzzy RG-subalgebra, for example:

Example 3.19.

Let ∂ = {0 , a, b, c, d} be a set with the following table:

*	0	a	b	c	d
0	0	a	b	c	d
a	a	b	c	d	0
b	b	c	d	0	a
c	c	d	0	a	b
d	d	0	a	b	c

Then $(\partial;*,0)$ is an RG-algebra. It's easy to show that I ={0, 3} and J ={0 , 4} are RG-subalgebras of ∂.

We defined two cubic set
$A_1 = \{(\sigma, \pi_{A1}(\sigma), \nu_{A1}(\sigma)) \mid \sigma \in \partial\}$ and
$A_2 = \{(\sigma, \pi_{A2}(\sigma), \nu_{A2}(\sigma)) \mid \sigma \in \partial\}$ of X by :-

$$\pi_{A1}(\sigma) = \begin{cases} 0.7, & \text{if } \sigma \in \{0,3\},\\ 0.6, & \text{if } \sigma \in \{1,2\},\\ 0.5, & \text{otherwise}\end{cases}$$

$$\nu_{A1}(\sigma) = \begin{cases} 0.1, & \text{if } \sigma \in \{0,3\},\\ 0.5, & \text{if } \sigma \in \{1,2\},\\ 0.3, & \text{otherwise}\end{cases}$$

$$\pi_{A2}(\sigma) = \begin{cases} 0.6, & \text{if } \sigma \in \{0,4\},\\ 0.1, & \text{otherwise.}\end{cases} \quad \text{and}$$

$$\nu_{A2}(x) = \begin{cases} 0.1, & \text{if } x \in \{0,4\},\\ 0.3, & \text{otherwise.}\end{cases}$$

Then A_1 and A_2 are bifuzzy RG-subalgebra of ∂, but R_union of $A_1 \cup A_2$ are not bifuzzy RG-subalgebras of ∂. Since
$(\cup \pi_{Ai})(3*4) = \max\{0.6, 0.1\} = 0.6 \ngeq 0.7 = \max\{(\cup \pi_{Ai})(3), (\cup \pi_{Ai})(4)\} = \max\{\max\{0.7, 0.1\}, \max\{0.1, 0.5\}\}$ and
$(\cap \nu_{Ai})(3*4) = \max\{0.6, 0.4\} = 0.6 \nleq 0.2 = \max\{(\cap \nu_{Ai})(3), (\cap \nu_{Ai})(4)\} = \max\{\min\{0.2, 0.4\}, \min\{0.4, 0.1\}\}$.

Proposition 3.20 .

Let $A_i = \{(\sigma, \pi_{Ai}(\sigma), \nu_{Ai}(\sigma)) \mid \sigma \in \partial\}$ where i ∈ Λ, be a set of bifuzzy RG-subalgebra of RGalgebra $(X;*,0)$, where i ∈ Λ,
$sup\{min\{\pi_{Ai}(\sigma), \pi_{Ai}(\rho)\}\} = min\{sup\, \pi_{Ai}(\sigma), sup\, \pi_{Ai}(\rho)\}$ and
$inf\{max\{\pi_{Ai}(\sigma), \pi_{Ai}(\rho)\}\} = max\{inf\, \pi_{Ai}(\sigma), inf\, \pi_{Ai}(\rho)\}$, for all $\sigma, \rho \in \partial$, then the R_union of A_i is also a bifuzzy RG-subalgebra of ∂.

Proof:

Let $A_i = \{(\sigma, \pi_{Ai}(\sigma), \nu_{Ai}(\sigma)) \mid \sigma \in \partial\}$ where i ∈ Λ, be a set of bifuzzy RG-subalgebra of ∂ and $\sigma, \rho \in \partial$, then

$$\begin{aligned}(\cup \pi_{Ai})(\sigma*\rho) &= sup\,(\pi_{Ai})(\sigma*\rho)\\ &\geq sup\,\{min\{(\pi_{Ai})(\sigma), (\pi_{Ai})(\rho)\}\}\\ &= min\{sup(\pi_{Ai}(\sigma)), sup(\pi_{Ai}(\rho))\}\\ &= \min\{(\cup \pi_{Ai})(\sigma), (\cup \pi_{Ai})(\rho)\}.\end{aligned}$$

Hence $(\cup \pi_{Ai})$ is a fuzzy RGsubalgebra of ∂.

$$\begin{aligned}(\cap \nu_{Ai})(\sigma*\rho) &= \inf(\nu_{Ai})(\sigma*\rho)\\ &\leq \inf\{\max\{(\nu_{Ai})(\sigma), (\nu_{Ai})(\rho)\}\}\\ &= \max\{\inf(\nu_{Ai})(\sigma), \inf(\nu_{Ai})(\rho)\}\\ &= \max\{(\cup \nu_{Ai})(\sigma), (\cup \nu_{Ai})(\rho)\}.\end{aligned}$$

Hence $(\cup \nu_{Ai})$ is antifuzzy RGsubalgebra of ∂.
Therefore, R_union of A_i is a bifuzzy RGsubalgebra of ∂. ⌂

4. The Bifuzzyideals of RGalgebra

In this section, we shall define the notion of bifuzzy ideals on RG -algebra and we study its properties when the bifuzzy RG-algebra is replace with bifuzzy ideal of RG-algebra.

Definition 4.1.

Let $(\partial;*,0)$ be an RG-algebra, a fuzzy sub-set ν of ∂ is called **an anti-fuzzy ideal of *X*** if it satisfies the following conditions, for all $\sigma, \rho \in \partial$,
(AFq-I_1) $\nu(0) \leq \nu(\sigma)$,
(AFq-I_2) $\nu(\rho) \leq max\{\nu(\sigma*\rho), \nu(\sigma)\}$.

Definition 4.2.

Let $A = \{(\sigma, \pi_A(\sigma), \nu_A(\sigma)) \mid \sigma \in \partial\}$ be a bifuzzy sub-set of an RG-algebra $(\partial;*,0)$. A is said to be a **bifuzzy ideal of** ∂ if for all $\sigma, \rho \in \partial$,,

(1) $\pi_A(0) \geq \pi_A(\sigma)$ and $\nu_A(0) \leq \nu_A(\sigma)$,
(2) $\pi_A(y) \geq min\{\pi_A(\sigma*\rho), \pi_A(\sigma)\}$ and
(3) $\nu_A(y) \leq \max\{\nu_A(\sigma*\rho), \nu_A(\sigma)\}$.

i.e., μ_A is fuzzy ideal of RG-algebra and ν_A is anti-fuzzy ideal of RG-algebra.

Example 4.3.

Let ∂ ={0 ,1,2, 3} in which ($*$) be a defined by the following table:

$*$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Then (∂; $*$,0) is an RG-algebra. It's easy to show that I_1 ={0 ,1}, I_2 ={0 ,2}, I_3={0 ,3} and I_4 ={0 , 1, 2, 3} are ideals of ∂. Define a fuzzy sub-set

π_A: $\partial \to [0,1] \ni \pi_A(0) = 0.7$, $\pi_A(1) = \pi_A(2) = 0.6$, $\pi_A(3) = 0.4$,

ν_A: $\partial \to [0,1] \ni \nu_A(0) = 0.3$, $\nu_A(1) = \nu_A(2) = 0.4$, $\nu_A(3) = 0.6$.

Routine calculation gives that μ_A is a fuzzy ideal of ∂ and ν_A is anti-fuzzy ideal of ∂.

Remark 4.4.

1- If π_A is a fuzzy ideal of an RGalgebra (∂; $*$,0), then It's that $U(\pi_A, t)$ is an ideal of ∂, for all $t \in \mathrm{Im}(\pi)$. Let $\sigma, \rho \in U(\pi_A, t)$, then $\pi_A(\sigma * \rho) \geq t$, and $\pi_A(\sigma) \geq t$, then min { $\pi_A(x * y)$, $\pi_A(x)$}$\geq$ t , since π_A is a fuzzy ideal , then $\mu_A(y) \geq min\{\pi_A(\sigma * \rho), \pi_A(\sigma)\} \geq t$,therefore $y \in U(\pi_A, t)$.

2- If ν_A is anti-fuzzyideal of X, then It's that $L(\nu_A, s)$is an ideal of ∂, for all $s \in \mathrm{Im}(\nu)$. Let $\sigma, \rho \in L(\nu_A, s)$, then $\nu_A(\sigma * \rho) \leq s$, and $\nu_A(\sigma) \leq s$, then max { $\nu_A(\sigma * \rho)$, $\nu_A(\sigma)$}$\leq$ s , since ν_A is anti-fuzzy ideal , then $\nu_A(\rho) \leq max\{\nu_A(\sigma * \rho), \nu_A(\sigma)\} \leq s$,therefore $\rho \in L(\nu_A, s)$.

3- But if we do not give a condition that π_A is a fuzzy ideal of ∂, then $U(\pi_A, t)$ is not an ideal of ∂ or ν_A is anti-fuzzy ideal of ∂, then $L(\nu_A, s)$is not an ideal of ∂as seen in the following example.

Example 4.5.

Consider ∂ = {0 , 1, 2, 3 } is an RGalgebra which is given in Example (3.3) . Define a fuzzy sub-set π_A of ∂:

X	0	1	2	3
μ_A	0 .7	0 .6	0.4	0 .3

Then π_A is not a fuzzy ideal of X, since $\pi_A(1*2)$ = 0 .3 < 0 .4 = min{ $\pi_A(1), \pi_A(2)$}. For t = 0 .5, we obtain $U(\mu_A, t)$ = {0 , 1, 2} which is not an ideal of ∂ since 1*2 = 3 $\notin U(\mu_A, t)$.

Proposition 4.6.

Let $A = \{(\sigma, \pi_A(\sigma), \nu_A(\sigma)) \mid \sigma \in \partial\}$ be an bifuzzy sub-set of an RG-algebra (X;$*$,0) $\ni$ A is a bifuzzy ideal of ∂, then $U(\pi_A, t)$ and $U(\pi_A, t)$ are fuzzy ideals of ∂, for any $t \in \mathrm{Im}(\mu)$, $s \in \mathrm{Im}(\nu)$.

Proof:

Assume that A is a bifuzzy ideal , then by Definition (4.2) π_A is a fuzzy ideal of ∂ and ν_A is an anti-fuzzy ideal of ∂, then by Remark (4.4), $U(\mu_A, t)$ and $L(\nu_A, s)$ are ideals of ∂, for any $t \in \mathrm{Im}(\pi)$, $s \in \mathrm{Im}(\nu)$. ⌂

Proposition 4.7.

Let $A = \{(\sigma, \pi_A(\sigma), \nu_A(\sigma)) \mid \sigma \in \partial\}$ be an bifuzzy sub-set of an RGalgebra (∂; $*$,0) $\ni U(\pi_A, t)$ and $L(\nu_A, s)$ are fuzzyideals of ∂, for all $t \in \mathrm{Im}(\pi)$, $s \in \mathrm{Im}(\nu)$, then A is a bifuzzyideal of ∂.

Proof:

1- $\pi_A(0) \geq \pi_A(\sigma)$ and $\nu_A(0) \leq \nu_A(\sigma)$.

2- Assume that $x, y \in U(\pi_A, t)$ and π_A of μ is not a fuzzy ideal of ∂, therefore $\pi_A(y) < t \leq min\{\pi_A(\sigma * \rho), \pi_A(\sigma)\}$, then $(\pi_A)(\sigma * \rho) \geq$ t and $(\pi_A)(\sigma) \geq$ t , but $(\pi_A)(\rho) <$ t . This shows that $y \notin U(\pi_A, t)$ **.** This is a contradiction, and so $\pi_A(y) \geq min\{\pi_A(\sigma * \rho), \pi_A(\sigma)\}$, for all $\sigma, \rho \in \partial$. Hence π_A is a fuzzy ideal of ∂.
$\nu_A(\rho) > s \geq max\{\nu_A(\sigma * \rho), \nu_A(\sigma)\}$, then $(\nu_A)(\sigma * \rho) \leq$ s and $(\nu_A)(\sigma) \leq$ s, but $(\nu_A)(\rho) >$ s . This shows that $y \notin L(\nu_A, s)$**.** This is a contradiction, and so $\nu_A(\rho) \leq max\{\nu_A(\sigma * \rho), \nu_A(\sigma)\}$, for all $\sigma, \rho \in \partial$. Therefore, ν_A is anti-fuzzy ideal of X.

Hence A is a bifuzzy ideal of ∂. ⌂

Proposition 4.8.

Every bifuzzy ideal of RG-algebra (∂; $*$,0) is a bifuzzy RG-subalgebra of ∂.

Proof:

Let (∂ ;$*$, 0) be an RG-algebra and $A = \{(\sigma, \pi_A(\sigma), \nu_A(\sigma)) \mid \sigma \in \partial\}$ is a bifuzzy ideal of ∂.

Since A is an bifuzzy ideal of ∂, then by Proposition (4.6) , $U(\pi_A, t)$ and $L(\nu_A, s)$ are fuzzy ideals of X, for all $t \in \mathrm{Im}(\pi)$, $s \in \mathrm{Im}(\nu)$. By Proposition (2.9), $U(\pi_A, t)$ and $L(\nu_A, s)$ are RGsubalgebras of ∂, for all $t \in \mathrm{Im}(\pi)$, $s \in \mathrm{Im}(\nu)$.

Therefore, π_A is fuzzy RG-subalgebra of RG-algebra and ν_A is antifuzzy RG-subalgebra of RG-algebra by Proposition (3.10).

Hence A is a bifuzzy RG-subalgebra of ∂ by Definition (3.2). ⌂

Proposition 4.9.

The R_intersection of any set of bifuzzy ideal of RG-algebra (∂;$*$,0) is also bifuzzy ideal of ∂.

Proof:

Let $A_i = \{(\sigma, \pi_{Ai}(\sigma), \nu_{Ai}(\sigma)) \mid \sigma \in \partial\}$ where $i \in \Lambda$**,** be a set of bifuzzy ideal of ∂ , then

1- For any $\sigma \in \partial$, $\cap \pi_{Ai}(0) = inf\{\pi_{Ai}(0)\} \geq inf\{\pi_{Ai}(\mathrm{x})\} = \cap \pi_{Ai}(x)$ and $\cup \nu_{Ai}(0) = sup\{\nu_{Ai}(0)\} \leq sup\{\nu_{Ai}(\sigma)\} = \cup \nu_{Ai}(\sigma)$.

2- for all $\sigma, \rho \in \partial$,

$$\begin{aligned}(\cap \pi_{Ai})(\rho) &= inf\,(\pi_{Ai})(\rho)\\ &\geq inf\{min\{(\pi_{Ai})(\sigma * \rho), (\pi_{Ai})(\sigma)\}\}\\ &= min\{inf(\pi_{Ai}(\sigma * \rho), inf(\mu_{Ai}(\sigma))\}\\ &= \min\{(\cap \pi_{Ai})(\sigma * \rho), (\cap \pi_{Ai})(\sigma)\}\end{aligned}$$ and

Hence $(\cap \pi_{Ai})$ is a fuzzyideal of ∂.

3- $(\cup\, v_{Ai})(\rho) = \sup\,(v_{Ai})(\rho)$
$\leq \sup\{\max\{(v_{Ai})(\sigma * \rho)\,, (v_{Ai})(\sigma)\}\}$
$= \max\{\sup\,(v_{Ai}\,(\sigma * \rho)), \sup\,(v_{Ai}(\sigma))\}$
$= \max\{(\cup\, v_{Ai})(\sigma * \rho)\,, (\cup\, v_{Ai})(\sigma)\}$.

Hence $(\cup\, v_{Ai})$ is anti-fuzzyideal of ∂.

Therefore, R_intersection of A_i is a bifuzzy ideal of ∂. ⌂

Remark 4.10 .

The P-intresection of any sets of bifuzzy ideal need not be a bifuzzyideal, for example:

Example 4.11.

Let∂ = {0 , a, b, c, d} be a set with the following table:

*	0	a	b	c	d
0	0	a	b	c	d
a	a	b	c	d	0
b	b	c	d	0	a
c	c	d	0	a	b
d	d	0	a	b	c

Then $(\partial; *, 0)$ is an RG-algebra. It's easy to show that I ={0, c} and J ={0 , d} are ideals of ∂ .

We defined two cubic set
$A_1 = \{(\sigma, \pi_{A1}\,(\sigma), v_{A1}\,(\sigma)) \mid \sigma \in \partial\}$ and
$A_2 = \{(\sigma, \pi_{A2}\,(\sigma), v_{A2}\,(\sigma)) \mid \sigma \in \partial\}$ by :-

$$\pi_{A1}(\sigma) = \begin{cases} 0.8\,, & \text{if } \sigma \in \{0\,, c\}, \\ 0.7, & \text{if } \sigma \in \{a, b\}, \\ 0.6, & \text{otherwise} \end{cases}$$

$$v_{A1}(\sigma) = \begin{cases} 0.2\,, & \text{if } \sigma \in \{0\,, c\}, \\ 0.6, & \text{if } \sigma \in \{a, b\}, \\ 0.4, & \text{otherwise} \end{cases}$$

$$\pi_{A2}(\sigma) = \begin{cases} 0.7\,, & \text{if } \sigma \in \{0\,, d\}, \\ 0.5, & \text{otherwise.} \end{cases} \quad \text{and}$$

$$v_{A2}(\sigma) = \begin{cases} 0.1\,, & \text{if } \sigma \in \{0\,, d\}, \\ 0.4, & \text{otherwise.} \end{cases}$$

Then A_1 and A_2 are bifuzzy ideal of ∂ , but P_intersection of $A_1 \cap A_2$ is not bifuzzy ideals of X . Since $(\cap\, \pi_{Ai})(b + d) = (\cap\, \pi_{Ai})(a) = \min\{0.7, 0.2\} = 0.2 \ngeq 0.5 = \min\{(\cap\, \pi_{Ai})(b + a), (\cap\, \pi_{Ai})(d - a)\} = \min\{(\cap\, \pi_{Ai})(c)\,, (\cap\, \pi_{Ai})(d)\} =$ $\{\min\{\min\{0.8, 0.5\}\,, \min\{0.6, 0.7\}\}$and $(\cup\, v_{Ai})(b + d) = (\cup\, v_{Ai})(a) = \max\{0.6, 0.4\} = 0.6 \nleq 0.4 = \max\{(\cup\, v_{Ai})(b + a), (\cup\, v_{Ai})(d - a)\} = \max\{(\cup\, v_{Ai})(c), (\cup\, v_{Ai})(d)\} = \max\{\max\{0.2, 0.4\}\,, \max\{0.4, 0.1\}\}$.

Proposition 4.12.

Let $A_i = \{(\sigma, \pi_{Ai}\,(\sigma), v_{Ai}\,(\sigma)) \mid \sigma \in \partial\}$ where i ∈ Λ, be a set of bifuzzy ideal of RG-algebra $(\partial; * , 0)$, where i ∈ Λ, $inf\{max\{\pi(\sigma), \pi_{Ai}(\rho)\}\} = max\{inf\, \pi_{Ai}(\sigma), inf\, \pi_{Ai}(\rho)\}$, for all $\sigma \in \partial$, then the P-intresection of A_i is also a bifuzzy ideal of ∂.

Proof:

Let $A_i = \{(\sigma, \pi_{Ai}\,(\sigma), v_{Ai}\,(\sigma)) \mid \sigma \in \partial\}$ where i ∈ Λ, be a set of bifuzzy ideal of ∂ and $\sigma, \rho \in \partial$, then

1- $\pi_{Ai}\,(0) \geq \pi_{Ai}(\sigma)$ and $v_{Ai}\,(0) \leq v_{Ai}(\sigma)$ implies that $(\cap\, \pi_{Ai})(0) \geq (\cap\, \pi_{Ai})(\sigma)$ and $(\cap\, v_{Ai})\,(0) \leq (\cap\, v_{Ai})(\sigma)$.

2- $(\cap\, \pi_{Ai})(\rho) = inf\,(\mu_{Ai})(\rho)$
$\geq inf\,\{min\{(\pi_{Ai})(\sigma * \rho)\,, (\pi_{Ai})(\sigma)\}\}$
$= min\{inf(\pi_{Ai}(\sigma * \rho\,), inf(\pi_{Ai}(\sigma))\}$
$= \min\{(\cap\, \pi_{Ai})(\sigma * \rho)\,, (\cap\, \pi_{Ai})(\sigma)\}$.

Hence $(\cap\, \pi_{Ai})$ is a fuzzyideal of X.

3- $(\cap\, v_{Ai})(\rho) = inf\,(v_{Ai})(\rho)$
$\leq \inf\{\max\{(v_{Ai})(\sigma * \rho)\,, (v_{Ai})(\sigma)\}\}$
$= \max\{\inf\,(v_{Ai}\,(\sigma * \rho)), \inf\,(v_{Ai}(\sigma))\}$
$= \max\{(\cap\, v_{Ai})(\sigma * \rho)\,, (\cap\, v_{Ai})(\sigma)\}$.

Hence $(\cap\, v_{Ai})$ is anti-fuzzy ideal of ∂.

Therefore, P_intersection of A_i is a bifuzzy ideal of ∂. ⌂

Proposition 4.13.

Let $A_i = \{(\sigma, \pi_{Ai}\,(\sigma), v_{Ai}\,(\sigma)) \mid \sigma \in \partial\}$ where i ∈ Λ, be a set of bifuzzyideal of RGalgebra $(\partial; * , 0)$, where i ∈ Λ, $sup\{min\{\pi_{Ai}(\sigma), \pi_{Ai}(\rho)\}\} = min\{sup\, \pi_{Ai}(\sigma), sup\, \pi_{Ai}(\rho)\}$, for all $\sigma \in \partial$, then the P_union of A_i is also a bifuzzy ideal of ∂.

Proof:

Let $A_i = \{(\sigma, \pi_{Ai}\,(\sigma), v_{Ai}\,(\sigma)) \mid \sigma \in \partial\}$ where i ∈ Λ, be a set of bifuzzy ideal of X and $x, y \in X$, then

1- $\pi_{Ai}\,(0) \geq \pi_{Ai}(\sigma)$ and $v_{Ai}\,(0) \leq v_{Ai}(\sigma)$ implies that $(\cup\, \pi_{Ai})(0) \geq (\cup\, \pi_{Ai})(\sigma)$ and $(\cup\, v_{Ai})\,(0) \leq (\cup\, v_{Ai})(\sigma)$.

2- $(\cup\, \pi_{Ai})(\rho) = sup\,(\pi_{Ai})(\rho)$
$\geq sup\,\{min\{(\pi_{Ai})(\sigma * \rho)\,, (\pi_{Ai})(\sigma)\}\}$
$= min\{sup(\pi_{Ai}(\sigma * \rho)), sup(\pi_{Ai}(\sigma))\}$
$= \min\{(\cup\, \pi_{Ai})(\sigma * \rho)\,, (\cup\, \pi_{Ai})(\sigma)\}$ and

Hence $(\cup\, \mu_{Ai})$ is a fuzzy ideal of ∂.

3- $(\cup\, v_{Ai})(\rho) = \sup\,(v_{Ai})(\rho)$
$\leq \sup\{\max\{(v_{Ai})(\sigma * \rho)\,, (v_{Ai})(\sigma)\}\}$
$= \max\{\sup\,(v_{Ai})\,(\sigma * \rho), \sup\,(v_{Ai})(\sigma)\}$
$= \max\{(\cup\, v_{Ai})(\sigma * \rho)\,, (\cup\, v_{Ai})(\sigma)\}$.

Hence $(\cup\, v_{Ai})$ is anti-fuzzy ideal of ∂.

Therefore, P_union of A_i is a bifuzzy ideal of ∂. ⌂

Remark 4.14.

The R_union of any sets of bifuzzy ideal need not be a bifuzzyideal, for example:

Example 4.15.

Let X = {0 , a, b, c, d} be a set with the following table:

*	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Then $(\partial; *, 0)$ is an RG-algebra. It's easy to show that I $=\{0, 3\}$ and J $=\{0, 4\}$ are ideals of ∂ .

Then A_1 and A_2 are bifuzzy ideal of ∂ , but R_union of $A_1 \cup A_2$ are not bifuzzy ideals of ∂ . We defined two cubic set $A_1 = \{(\sigma, \pi_{A1}(\sigma), v_{A1}(\sigma)) \mid \sigma \in \partial\}$ and $A_2 = \{(\sigma, \pi_{A2}(\sigma), v_{A2}(\sigma)) \mid \sigma \in \partial\}$ of X by :-

$$\pi_{A1}(\sigma) = \begin{cases} 0.7, & \text{if } \sigma \in \{0,3\}, \\ 0.6, & \text{if } \sigma \in \{1,2\}, \\ 0.5, & \text{otherwise} \end{cases}$$

$$v_{A1}(\sigma) = \begin{cases} e.1, & \text{if } \sigma \in \{0,3\}, \\ e.5, & \text{if } \sigma \in \{1,2\}, \\ e.3, & \text{otherwise} \end{cases}$$

$$\pi_{A2}(\sigma) = \begin{cases} 0.6, & \text{if } \sigma \in \{0,4\}, \\ 0.1, & \text{otherwise.} \end{cases} \quad \text{and}$$

$$v_{A2}(x) = \begin{cases} 0.1, & \text{if } \sigma \in \{0,4\}, \\ 0.3, & \text{otherwise.} \end{cases}$$

$(\cap \pi_{Ai})(2*4) = (\cap \pi_{Ai})(1) = \min\{0.7, 0.2\} = 0.2 \ngeq 0.5 = \min\{(\cap \pi_{Ai})((2*1)*1), (\cap \pi_{Ai})(1)\} = \min\{(\cap \pi_{Ai})(3), (\cap \pi_{Ai})(4)\} = \{\min\{\min\{0.8, 0.5\}, \min\{0.6, 0.7\}\}$ and

$(\cup v_{Ai})(2*4) = (\cup v_{Ai})(1) = \max\{0.6, 0.4\} = 0.6 \nleq 0.4 = \max\{(\cup v_{Ai})((2*1)*1), (\cup v_{Ai})(1)\} = \max\{(\cup v_{Ai})(3), (\cup v_{Ai})(4)\} = \max\{\max\{0.2, 0.4\}, \max\{0.4, 0.1\}\}$.

<u>Proposition 4.16.</u>

Let $A_i = \{(\sigma, \pi_{Ai}(\sigma), v_{Ai}(\sigma)) \mid \sigma \in \partial\}$ where $i \in \Lambda$, be a set of bifuzzy ideal of RG-algebra $(\partial; *, 0)$, where $i \in \Lambda$, $sup\{min\{\pi_{Ai}(\sigma), \pi_{Ai}(\rho)\}\} = min\{sup\, \pi_{Ai}(\sigma), sup\, \pi_{Ai}(\rho)\}$ and $inf\{max\{\pi_{Ai}(\sigma), \pi_{Ai}(\rho)\}\} = max\{inf\, \pi_{Ai}(\sigma), inf\, \pi_{Ai}(\rho)\}$, for all $\sigma \in \partial$, then the R_union of A_i is also a bifuzzy ideal of ∂.

<u>Proof:</u>

Let $A_i = \{(\sigma, \pi_{Ai}(\sigma), v_{Ai}(\sigma)) \mid \sigma \in \partial\}$ where $i \in \Lambda$, be a set of bifuzzy ideal of X and $x, y \in X$, then

1- $\pi_A(0) \geq \pi_A(x)$ and $v_A(0) \leq v_A(\sigma)$.

2- $(\cup \pi_{Ai})(\rho) = sup\,(\pi_{Ai})(\rho)$
$\geq sup\{min\{(\pi_{Ai})(\sigma * \rho), (\pi_{Ai})(\sigma)\}\}$
$= min\{sup(\pi_{Ai}(\sigma * \rho)), sup(\pi_{Ai}(\sigma))\}$
$= \min\{(\cup \pi_{Ai})(\sigma * \rho), (\cup \pi_{Ai})(\sigma)\}$ and

Hence $(\cup \pi_{Ai})$ is a fuzzy ideal of ∂.

3- $(\cap v_{Ai})(\rho) = \inf(v_{Ai})(\rho)$
$\leq \inf\{\max\{(v_{Ai})(\sigma * \rho), (v_{Ai})(\sigma)\}\}$
$= \max\{\inf(v_{Ai})(\sigma * \rho), \inf(v_{Ai})(\sigma)\}$
$= \max\{(\cup v_{Ai})(\sigma * \rho), (\cup v_{Ai})(\sigma)\}$.

Hence $(\cup v_{Ai})$ is anti-fuzzy ideal of ∂.

Therefore, R_union of A_i is a bifuzzy ideal of ∂. ⌂

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