$(\beta,\alpha)-$ magnified Translation of Fuzzy ideals on $RG-$ algebra

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Abstrac_ *In this paper, the concepts of $(\beta,\alpha)-$ magnified translation of fuzzy $RG-$ subalgebras and $(\beta,\alpha)-$ magnified translation fuzzy $q-$ ideals of $RG-$ algebras are given, some properties and some examples that illustrate certain cases, such as reverse some of the theorems and others, on $RG-$ subalgebra and q-ideals, each of which had its own proofs, characteristics and relationships.*

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1. Introduction

A $BCK-algebra$ is an important class of logical algebras introduced by [1,6] and was extensively investigated by several researchers. [7,23] was introduced the notion of fuzzy sets and gave some properties of it. In [16], them applied the concept of fuzzy set to $BCK-algebras$ and investigated further properties of fuzzy BCK-algebras and fuzzy ideals, they introduce Fuzzy translations and fuzzy multiplications of BCK/BCI-algebras. S.M. Mostafa, and A.T. Hameed [11,22] introduced $KUS-ideals$ in $KUS-algebras$ and introduced the notion fuzzy KUS-subalgebras, fuzzy $KUS-ideals$ of $KUS-algebras$ investigated relations among them. In [12-14], A.T. Hameed, I.H. Ghazi and A.H. Abed introduced the notion fuzzy α-translation $AB-ideal$ of $AB-algebras$, and A.T. Hameed and N.H. Malik [1-5] introduced magnified translation of intuitionistic fuzzy $AT-ideals$ on AT-algebra and (β, α)-fuzzy magnified translations of $AT-algebra$. In [10], A.T. Hameed, A.A. Abboodi Al-Koofee and A.H. Abed introduced the notion α-translation fuzzy q-ideals of $KK-algebra$ and β- multiplications fuzzy q-ideals of $KK-algebra$. A.T. Hameed and et al. , [19] have introduced the notion of $SA-algebras$, SA-ideals, SA-subalgebras and studied the relations among them . A.T. Hameed and et al. introduced the notion of $KK-algebra$ and $KK-ideals$ and fuzzy KK-ideal of KK-algebras and studied the homomorphism image and inverse image of fuzzy $KK-ideals$. In[18-21], have introduced the notion of $RG-algebras$, $RG-ideals$, $RG-subalgebras$ and studied the relations among them and P. Patthanangkoor, gave the concept of homomorphism of $RG-algebras$ and investigated some related properties. A.T. Hameed and et al. introduced and studied new concepts of fuzzy $RG-subalgebras$ and fuzzy $RG-ideals$ of $RG-algebra$ and investigate some of its properties, several Theorems, properties are stated and proved. A.T. Hameed and S.M. Abrahem introduced the notion of doubt fuzzy $RG-ideals$ of $RG-algebras$ and studied the homomorphism image and inverse image of doubt fuzzy $RG-ideals$ also prove that the Cartesian product of doubt fuzzy $RG-ideals$ are doubt fuzzy $RG-ideals$. In this paper, we introduce the notions of $(\beta,\alpha)-$ magnified translation fuzzy $RG-subalgebras$, $(\beta,\alpha)-$ magnified translation fuzzy ideals and $(\beta,\alpha)-$ magnified translation fuzzy $q-ideals$ of $RG-algebras$ and gave some properties of it.

2. Preliminaries:

We review some definitions and properties that will be useful in our results.

Definition 2.1 [24].
An algebra $(\partial;*,0)$ is called an RG-algebra (RG-A) if the following axioms are satisfied: $\forall\ \sigma,\rho,\varepsilon \in \partial$,
(i) $\sigma * 0 = \sigma$,
(ii) $\sigma * \rho = (\sigma * \varepsilon) * (\rho * \varepsilon)$,
(iii) $\sigma * \rho = \rho * \sigma = 0$ *imply* $\sigma = \rho$.

Remark 2.2 [24].
For brevity we also call ∂ RG-A, we can define a binary relation $(\leq)$ by putting $\sigma \leq \rho$ if and only if $\sigma * \rho = 0$.

Example 2.3([24]).
Let ∂ = {0, 1} and let $*$ be defined by:

$*$	0	1
0	0	1
1	1	0

Then $(\partial;\ *,0)$ is an RG-A.

Example 2.4 [24].
Let ∂ ={0, a, b, c} and $(\partial;*)$ be the pair given by the table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$(\partial;*,0)$ is an RG-A.

Proposition 2.5 [15].
In any RG-A $(\partial;*,0)$, the following hold: $\forall\sigma,\rho \in \partial$,
i) $\sigma * \sigma = 0$,
ii) $0 * (0 * \sigma) = \sigma$,
iii) $\sigma * (\sigma * \rho) = \rho$,
iv) $\sigma * \rho = 0$ *if and only if* $\rho * \sigma = 0$,
v) $\sigma * 0 = 0$ *implies* $\sigma = 0$,
vi) $0 * (\rho * \sigma) = \sigma * \rho$
In any RG-A $(\partial;*,0)$, the following hold: $\forall\sigma,\rho,\varepsilon \in \partial$,
i) $(\sigma * \rho) * (0 * \rho) = (\sigma * (0 * \rho)) * \rho = \sigma$,
ii) $\sigma * (\sigma * (\sigma * \rho)) = \sigma * \rho$,
iii) $(\sigma * \rho) * \varepsilon = (\sigma * \varepsilon) * y$.
iv) $\sigma * \rho = (\varepsilon * \rho) * (\varepsilon * \sigma)$,
v) $((\sigma * \rho) * (x * \varepsilon)) * (\varepsilon * \rho) = 0$.

Definition 2.7([15]).
Let $(\partial;\ *,0)$ be a RG-A and let S be a nonempty subset of X. S is called an RG-subalgebra of ∂ (RG-SA) if $\sigma * \rho \in S$ whenever $\sigma,\rho \in S$.

Definition 2.8([19]).
A nonempty subset I of a RG-A $(X;\ *,0)$ is called an ideal of X (RG-I) if it satisfies the following conditions: for any $x, y \in X$,
(I_1) $0 \in I$;
(I_2) $\sigma * \rho \in I$ *and* $\sigma \in I$ *imply* $\rho \in I$.

Proposition 2.9 ([15]).
Every RG-I of RG-A is an RG-SA.

Definition 2.10([15]).
Let $(\partial;\ *,0)$be a RG-A, a fuzzy subset μ of X is called a fuzzy RG-subalgebra of ∂ (FRG-SA) if $\forall\sigma,\rho \in \partial$,
$\pi(\sigma * \rho) \geq min\{\pi(\sigma),\pi(\rho)\}$.

Definition 2.11([15]).
Let $(\partial;\ *,0)$ be a RG-A, a fuzzy subset π of ∂ is called a fuzzy ideal of ∂ (FRG-I) if it satisfies the following conditions, $\forall\sigma,\rho \in \partial$,
(1) $\pi(0) \geq \pi(x)$,
(2) $\pi(y) \geq min\{\pi(\sigma * \rho),\pi(\sigma)\}$.

Lemma 2.12([15]).
Let π be a FRG-I of RG-A $(\partial;\ *,0)$and if $\sigma \leq \rho$, then $\mu(\sigma) \geq \mu(\rho)$, $\forall\sigma,\rho \in \partial$,.

Proposition 2.13([15]).
1- Let π be a fuzzy subset of RG-A $(\partial;\ *,0)$. π is a FRG-SA of ∂ if and only if for every t $\in$ [0,1], π_t is an RG-SA of X .
2- Let π be a fuzzy subset of RG-A $(\partial;*,0)$, π is a FRG-I of ∂ if and only if for every t $\in$ [0,1], π_t is RG-I of X .
3- Let A be a nonempty subset of a RG-A $(\partial;\ *,0)$ and π be a fuzzy subset of ∂ such that π is into {0, 1}, so that π is the characteristic function of A. Thenπ is a FRG-I of ∂ if and only if A is RG-I of ∂ .

Proposition 2.14([15]).
Every FRG-I of RG-A is a FRG-SA.

Definition 2.15[6,7]:
Let ∂ be a nonempty set and π be a fuzzy subset of X and let $\alpha \in [0,T]$. A mapping $\pi_\alpha^T : \partial \to [0,1]$ is called a $\boldsymbol{\alpha}$-translation fuzzy subset of π if it satisfies:
$\pi_\alpha^T(\sigma) = \pi(\sigma) + \alpha$, for all $\sigma \in \partial$, where$T = 1 - \sup\{\pi(\sigma): \sigma \in \partial\}$.

Definition 2.16([10]).
Let $(X;*,0)$ be an RG-A, a fuzzy subset μ *in* X is called a fuzzy q-ideal of ∂ (Fq-I) if it satisfies the following conditions: $\forall\sigma,\rho,\varepsilon \in \partial$,
(1) $\pi(0) \geq \pi(\sigma)$,
(2) $\pi(\sigma * \varepsilon) \geq min\{\pi((\sigma * \rho) * \varepsilon),\pi(\rho)\}$.

Proposition 2.17([10]).
Every Fq-I of RG-A $(X;\ *,0)$ is a FRG-I of X.

Definition 2.18([23]).

Let π be a fuzzy subset of set X and let α ∈[0,T]. A mapping $\pi_\alpha^T : \partial \to [0,1]$ is called a α-translation of π if it satisfies: $\pi_\alpha^T(\sigma) = \mu(\sigma)+\alpha$, for all $\sigma \in \partial$.

Definition 2.19([23]).

Let $(\partial; *, 0)$ be a set, a fuzzy subset π of ∂ is called a α-translation fuzzy RG-subalgebra of ∂,(TFRG-SA) if $\forall \sigma, \rho \in \partial$, $\pi_\alpha^T(\sigma * \rho) \geq \min\{\pi_\alpha^T(\sigma), \pi_\alpha^T(\rho)\}$.

Definition 2.20([23]).

For a fuzzy subset π of set ∂, α ∈ [0,T] and $t \in Im(\pi)$ with t ≥ α, then $\mathbf{U}_\alpha(\pi; t) = \{\sigma \in \partial \mid \pi(\sigma) \geq t - \alpha\}$.

Definition 2.21([23]).

Let $(X; *, 0)$ be a set, a α-translation fuzzy subset μ of ∂ is called a α-translation FRG-I (TFRG-I) of ∂ if it satisfies the following conditions: $\forall \sigma, \rho, \varepsilon \in \partial$, ,

(1) $\pi_\alpha^T(0) \geq \pi_\alpha^T(\sigma$,

(2) $\pi_\alpha^T(\rho) \geq \min\{\pi(\sigma * \rho), \pi_\alpha^T(\sigma)\}$.

Definition 2.22([23]).

Let $(\partial; *, 0)$ be a set, a α-translation fuzzy subset π of ∂ is called a α-translation Fq-I (TFq-I) of ∂ if it satisfies the following conditions: $\forall \sigma, \rho, \varepsilon \in \partial$,,

(1) $\pi_\alpha^T(0) \geq \pi_\alpha^T(x\sigma)$,

(2) $\pi_\alpha^T(\sigma * \varepsilon) \geq \min\{\pi_\alpha^T((\sigma * \rho) * \varepsilon), \pi_\alpha^T(\rho)\}$.

Definition 2.23([23]).

Let π be a fuzzy subset of set $(\partial; *, 0)$ and let β ∈(0,1] A mapping $\pi_\beta^M : \partial \to [0,1]$ is called a β-magnified of π if it satisfies: $\pi_\beta^M(\sigma) = \beta . \pi(\sigma)$, for all $\sigma \in \partial$.

Definition 2.24([23]).

Let $(\partial; *, 0)$ be set, a fuzzy subset π of X is called a β-magnified fuzzy subalgebra of X (MFRG-SA) , if $\forall \sigma, \rho \in \partial$,,

$\pi_\beta^M(\sigma * \rho) \geq \min\{\pi_\beta^M(\sigma), \pi_\beta^M, (\rho)\}$.

Definition 2.25([23]).

For a fuzzy subset π of set $(\partial; *, 0)$, β ∈ (0,1] and t∈ Im(μ) with t ≤ β, then $\mathbf{U}_\beta(\pi; t) = \{\sigma \in \partial \mid \pi(\sigma) \geq t/\beta\}$.

Definition 2.26([23]).

Let $(\partial; *, 0)$ be a set, a β-magnified fuzzy subset π of ∂ is called a β-magnified FRG-I of ∂ (M FRG-I) if it satisfies the following conditions: $\forall \sigma, \rho, \varepsilon \in \partial$,

(1) $\pi_\beta^M(0) \geq \pi_\beta^M(\sigma)$,

(2) $\pi_\beta^M(\rho) \geq \min\{\pi_\beta^M(\sigma * \rho), \pi_\beta^M(\sigma)\}$.

Definition 2.27([23]).

Let $(\partial; *, 0)$ be set, a β-magnified fuzzy subset π of ∂ is called a β-magnified Fq-I of ∂ (MFq-I) if it atisfies the following conditions: $\forall \sigma, \rho, \varepsilon \in \partial$,

(1) $\pi_\beta^M(0) \geq \pi_\beta^M(\sigma)$,

(2) $\pi_\beta^M(\sigma * \varepsilon) \geq \min\{\pi_\beta^M((\sigma * \rho) * \varepsilon), \pi_\beta^M(\rho)\}$.

3. (β,α)-magnified Translation of FRG-SAs on RG-A

In this section, we will discuss and investigate new notions called (β,α)-magnified translation FRG-SAs of RG-As and study several basic properties of them.

Definition 3.1.

Let π be a fuzzy subset of an RG-A $(\partial; *, 0)$ and let α ∈[0,T] and β ∈(0,1]. A mapping $\pi_{(\beta,\alpha)}^C : \partial \to [0,1]$ is called a (β,α)-magnified translation of π (MT) if it Satisfies: $\pi_{(\beta,\alpha)}^C(\sigma) = \beta . \pi(\sigma)+\alpha$, for all $\sigma \in \partial$.

Definition 3.2.

Let $(\partial; *, 0)$ be an RG-A, a fuzzy subset π of ∂ is called a (β,α)-magnified translation of FRG-SA of ∂ (MTFRG-SA) if $\forall \sigma, \rho \in \partial, \pi_{(\beta,\alpha)}^c(\sigma * \rho) \geq \min\{\pi_{(\beta,\alpha)}^c(\sigma), \pi_{(\beta,\alpha)}^c(\rho)\}$.

Definition 3.3.

For a fuzzy subset π of an RG-A $(\partial; *, 0)$, α ∈[0,T], β ∈ (0,1] with t ≥ α and t∈ Im(π) with t ≤ β, then $\boldsymbol{U}_{(\beta,\alpha)}(\pi; t) = \{\sigma \in \partial \mid \pi(\sigma) \geq (t - \alpha)/\beta\}$.

Example 3.4.

by Consider an RG-A $\partial = \{0, a, b, c\}$ with the following table:

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Define a fuzzy subsets π and λ of X by:

∂	0	a	b	c
π	0.8	0. 6	0. 5	0.5
λ	0.7	0.6	0.4	0.3

Then π and λ are two FRG-SAs of X. For α = 0.1 , β = 1 and t = 0.5, we obtain $U_{(\beta,\alpha)}$ (λ; t) = {0, a, b} which is a RG-SA of ∂ .

Theorem 3.5.

Let π be a fuzzy subset of an RG-A $(\partial; *, 0)$. $\pi_{(\beta,\alpha)}^c$ is a MTRG-SA of X if and only if, for all α ∈[0,T], t , β ∈ (0,1], $U_{(\beta,\alpha)}(\pi; t)$ is an RG-SA of ∂ .

Proof:

($\Rightarrow$) Suppose that $\pi^c_{(\beta,\alpha)}$ is a MTFRG-SA of ∂ and $U_{(\beta,\alpha)}(\pi; t) \neq \emptyset$, for any $\alpha \in[0,T]$, t, $\beta \in (0,1]$. Then $0 \in U_{(\beta,\alpha)}(\pi; t)$.

Let $\sigma \in U_{(\beta,\alpha)}(\pi; t)$ and $\rho \in U_{(\beta,\alpha)}(\pi; t)$, then $\pi(x) \geq (t-\alpha)/\beta$ and $\pi(y) \geq (t-\alpha)/\beta$. It follows from Def. (3.2)

$\pi^c_{(\beta,\alpha)}(\sigma * \rho) \geq \min\{\pi^c_{(\beta,\alpha)}(\sigma), \pi^c_{(\beta,\alpha)}(\rho)\} \geq (t-\alpha)/\beta$.

Namely, $\pi^c_{(\beta,\alpha)}(x * y) \geq (t-\alpha)/\beta$ and $(\sigma * \rho) \in U_{(\beta,\alpha)}(\mu; t)$. This shows that $U_{(\beta,\alpha)}(\pi; t)$ is a RG-SA of X.

($\Leftarrow$) Conversely, suppose that for each $\alpha \in[0,T]$, t, $\beta \in (0,1]$. $U_{(\beta,\alpha)}(\pi; t)$ is RG-SA of ∂.

Now, we only need to show that $\pi^c_{(\beta,\alpha)}$ satisfies Def. (3.2). Assume $\pi^c_{(\beta,\alpha)}(\sigma * \rho) \geq \min\{\pi^c_{(\beta,\alpha)}(\sigma), \pi^c_{(\beta,\alpha)}(\rho)\}$ is not true, then there exist $\sigma', \sigma' \in \partial$, such that $\pi^c_{(\beta,\alpha)}(\sigma' * \rho') < \max\{\pi^c_{(\beta,\alpha)}(\sigma'), \pi^c_{(\beta,\alpha)}(\rho')\}$.

Putting $t` = \frac{1}{2}(\pi^c_{(\beta,\alpha)}(\sigma` * \rho`) + min\{\pi^c_{(\beta,\alpha)}(\sigma`), \pi^c_{(\beta,\alpha)}(\rho`)\})$, then $\mu^c_{(\beta,\alpha)}(x`) < t`$ and $0 \leq t` < min\{\pi^c_{(\beta,\alpha)}(\sigma`), \pi^c_{(\beta,\alpha)}(\rho`)\} \leq 1$, hence $\pi^c_{(\beta,\alpha)}(\sigma`) > (t-\alpha)/\beta`$ and $\pi^c_{(\beta,\alpha)}(\rho`) > t`$, which imply that $\sigma` \in U_{(\beta,\alpha)}(\pi; t')$ and $\rho` \in U_{(\beta,\alpha)}(\pi; t')$, since $U_{(\beta,\alpha)}(\pi; t')$ is an RG-SA a, it follows that $(\sigma` * \rho`) \in U_{(\beta,\alpha)}(\pi; t')$ and that $\pi(\sigma` * \rho`) \geq t`$, this is also a C!. Therefore, $\forall \sigma, \rho \in \partial$, $\pi^c_{(\beta,\alpha)}(\sigma * \rho) \geq \min\{\pi^c_{(\beta,\alpha)}(\sigma), \pi^c_{(\beta,\alpha)}(\rho)\}$.

Hence $\pi^c_{(\beta,\alpha)}$ is MTFRG-SA of ∂. ⌂

Theorem 3.6.

Let π be a fuzzy subset of RG-A $(\partial; *, 0)$ and $\alpha \in[0,T]$, t, $\beta \in (0,1]$. The (β,α)-magnified translation fuzzy sub set $\pi^c_{(\beta,\alpha)}$ of π is a FRG-SA of ∂, if and only if, π is a FRG-SA of ∂.

Proof:

($\Rightarrow$) Suppose that $\pi^c_{(\beta,\alpha)}$ is a MTFRG-SA of ∂ and $U_{(\beta,\alpha)}(\pi; t) \neq \emptyset$, for any $\alpha \in[0,T]$, t, $\beta \in (0,1]$. Let $\sigma, \rho \in \partial$, then

$$\begin{aligned}
\beta.\pi(\sigma * \rho) + \alpha &= \pi^c_{(\beta,\alpha)}(\sigma * \rho)\\
&\geq min\{\pi^c_{(\beta,\alpha)}(\sigma), \pi^c_{(\beta,\alpha)}(\rho)\}\\
&= min\{\beta.\pi(\sigma) + \alpha, \beta.\pi(\rho) + \alpha\}\\
&= min\{\beta.\pi(\sigma), \beta.\pi(\rho)\} + \alpha\\
&= \beta.min\{\pi(\sigma), \pi(\rho)\} + \alpha \text{ and so}
\end{aligned}$$

$\pi(\sigma * \rho) \geq min\{\pi(\sigma), \pi(\rho)\}$.

Hence π is a FRG-SA of ∂.

($\Leftarrow$) Co., suppose that π is a FRG-SA of ∂ such that $\pi(\sigma * \rho) \geq \min\{\pi(\sigma), \pi(\rho)\}$. If for each $\alpha \in[0,T]$, t, $\beta \in (0,1]$.

$$\begin{aligned}
\pi^c_{(\beta,\alpha)}(\sigma * \rho) &= \beta.\pi(\sigma * \rho) + \alpha\\
&\geq \beta.min\{\pi(\sigma), \pi(\rho)\} + \alpha\\
&= min\{\beta.\pi(\sigma) + \alpha, \beta.\pi(\rho) + \alpha\}\\
&= min\{\pi^c_{(\beta,\alpha)}(\sigma), \pi^c_{(\beta,\alpha)}(\rho)\}
\end{aligned}$$

Hence $\pi^c_{(\beta,\alpha)}$ is a FRG-SA of ∂. ⌂

Proposition 3.7.

Let $\pi^c_{(\beta,\alpha)}$ and $v^c_{(\beta,\alpha)}$ be two MTFRG-SAs of a RG-A $(\partial; *, 0)$. Then $(\pi^c_{(\beta,\alpha)} \cap v^c_{(\beta,\alpha)})$ is also a FRG-SA of ∂.

Proof: $\forall \sigma, \rho \in \partial$,

$(\pi^c_{(\beta,\alpha)} \cap v^c_{(\beta,\alpha)})(\sigma * \rho) = inf(\pi^c_{(\beta,\alpha)}(\sigma * \rho), v^c_{(\beta,\alpha)}(\sigma * \rho))$
$\geq$
$inf(\min\{\pi^c_{(\beta,\alpha)}(\sigma), \pi^c_{(\beta,\alpha)}(\rho)\}, \min\{v^c_{(\beta,\alpha)}(\sigma), v^c_{(\beta,\alpha)}(\rho)\})$,
$= inf(\min\{\pi^c_{(\beta,\alpha)}(\sigma), \pi^c_{(\beta,\alpha)}(\rho), v^c_{(\beta,\alpha)}(\sigma), v^c_{(\beta,\alpha)}(\rho)\})$,
$= inf(\min\{\pi^c_{(\beta,\alpha)}(\sigma), v^c_{(\beta,\alpha)}(\sigma)\}, \min\{\pi^c_{(\beta,\alpha)}(\rho), v^c_{(\beta,\alpha)}(\rho)\})$,
$=$
$\min(inf\{\pi^c_{(\beta,\alpha)}(\sigma), v^c_{(\beta,\alpha)}(\sigma)\}, inf\{\pi^c_{(\beta,\alpha)}(\rho), v^c_{(\beta,\alpha)}(\rho)\})$,
$= \min\{(\pi^c_{(\beta,\alpha)} \cap v^c_{(\beta,\alpha)})(\sigma), (\pi^c_{(\beta,\alpha)} \cap v^c_{(\beta,\alpha)})(\rho)\}$.

Hence $(\pi^c_{(\beta,\alpha)} \cap v^c_{(\beta,\alpha)})$ is a FRG-SA of ∂. ⌂

Theorem 3.8.

A homomorphic pre − image of a MTFRG-SA of RG-A is also a MTFRG-SA of RG-A.

Proof:

Let $f: (\partial; *, 0) \rightarrow (\partial'; *', 0')$ be a homomorphism of RG-As, $\lambda^c_{(\beta,\alpha)}$ the MTFRG-SA of ∂'.

Then λ is a FRG-SA of ∂' by Th. (3.6) and π is the pre − image of λ under f, by Prop. (2.24(1)), π is a FRG-SA of ∂.

Hence $\pi^c_{(\beta,\alpha)}$ the MTFRG-SA of ∂ by Th. (3.6). ⌂

Theorem 3.9.

Let $f: (\partial; *, 0) \rightarrow (\partial'; *`, 0`)$ be an epimorphism between RG-As ∂ and ∂' respectively. For every MTFRG-SA $\pi^c_{(\beta,\alpha)}$ of ∂ with sup property, $f(\pi^c_{(\beta,\alpha)})$ is a MTFRG-SA of ∂'.

Proof:

Let $f: (\partial; *, 0) \rightarrow (\partial'; *`, 0`)$ be a homomorphism of RG-As, $\pi^c_{(\beta,\alpha)}$ the MTRG-SA of ∂.

Then π is a FRG-SA of ∂ by Th. (3.6) and $f(\pi)$ is the image of μ, by Prop.(2.24(2)), $f(\pi)$ is a FRG-SA of ∂'. FRG-SA of ∂'.

Hence $f(\pi^c_{(\beta,\alpha)})$ the MTFRG-SA of ∂' by Th. (3.6). ⌂

4.(β,α)-magnified Translation of Fuzzy Ideals on RG-A

In this section, we will discuss and investigate new notions called (β,α)-magnified translation fuzzy ideals of RG-As and study several basic properties of them.

Definition 4.1.

Let $(\partial; *, 0)$ be an RG-A, a fuzzy subset π of ∂ is called a (β,α)-ma***gnified translation*** of FRG-I of ∂ (MTFRG-I)if $\forall \sigma, \rho \in \partial$,

(1) $\pi^c_{(\beta,\alpha)}(0) \geq \pi^c_{(\beta,\alpha)}(\sigma)$,

(2) $\pi^c_{(\beta,\alpha)}(\rho) \geq \min\{\pi^c_{(\beta,\alpha)}(\sigma * \rho), \pi^c_{(\beta,\alpha)}(\sigma)\}$.

Example 4.2.

by Consider an RG-A $\partial = \{0, a, b, c\}$ with the following table:

X	0	a	b	c
μ	0.8	0. 6	0. 5	0.5
λ	0.7	0.6	0.4	0.3

Define a fuzzy subsets π and λ of X by:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then π and λ are two RG-Is of ∂. For α = 0.1 , β = 1 and t = 0.5, we obtain $U_{(\beta,\alpha)}$ (λ; t) = {0, a, b} which *is an ideal* of ∂ .

Theorem 4.3.

Let π be a fuzzy *subset* of an RG-A $(\partial; *, 0)$. $\pi^c_{(\beta,\alpha)}$ is a MTFRG-I of ∂ *if and only if*, for all α ∈[0,T], t , β ∈ (0,1], $U_{(\beta,\alpha)}$ $(\pi;$ t) is an *ideal* of ∂ .

Proof:

(⇒) *Suppose that* $\pi^c_{(\beta,\alpha)}$ is a MTFRG-I of X and $U_{(\beta,\alpha)}$ $(\pi;$ t) ≠ ∅, *for any* α ∈[0,T], t, β ∈ (0,1]. Then 0 ∈$U_{(\beta,\alpha)}$ $(\pi;$ t).

Since $\pi^c_{(\beta,\alpha)}(0) = \beta.\pi(0) + \alpha \geq \beta.\pi(\sigma) + \alpha = \pi^c_{(\beta,\alpha)}(\sigma)$, then $\pi(0) \geq \pi(\sigma)$ *implies that* $U_{(\beta,\alpha)}(\mu;\ t)(0) \geq U_{(\beta,\alpha)}(\mu;\ t)(x)$.

Let $\sigma * \rho \in U_{(\beta,\alpha)}$ $(\pi;$ t) and $\sigma \in U_{(\beta,\alpha)}$ $(\pi;$ t), then $\pi(\sigma * \rho) \geq (\mathrm{t} - \alpha)/\beta$ and $\pi(\sigma) \geq (\mathrm{t} - \alpha)/\beta$. It follows from Def. (4.1)

$\pi^c_{(\beta,\alpha)}(\rho) \geq \min\{\pi^c_{(\beta,\alpha)}(\sigma * \rho), \pi^c_{(\beta,\alpha)}(\sigma)\} \geq (t - \alpha)/\beta$.

Namely, $\pi^c_{(\beta,\alpha)}(\rho) \geq (\mathrm{t} - \alpha)/\beta$ and $(\rho) \in U_{(\beta,\alpha)}$ $(\pi;$ t). This shows that $U_{(\beta,\alpha)}$ $(\pi;$ t) is an ideal of ∂ .

(⇐) Co., *suppose that for each* α ∈[0,T], t , β ∈ (0,1]. $U_{(\beta,\alpha)}(\pi;\ t)$ is *an ideal* of ∂.

Now, we only need to show that $\mu^c_{(\beta,\alpha)}$ satisfies Def. (4.1).

Since $U_{(\beta,\alpha)}(\pi;\ t)\ (0) \geq U_{(\beta,\alpha)}(\mu;\ t)\ (x)$, then $\mu^c_{(\beta,\alpha)}(0) = \beta.\pi(0) + \alpha \geq \beta.\pi(\sigma) + \alpha = \pi^c_{(\beta,\alpha)}(\sigma)$.

Assume $\pi^c_{(\beta,\alpha)}(\rho) \geq \min\{\pi^c_{(\beta,\alpha)}(\sigma * \rho), \pi^c_{(\beta,\alpha)}(\sigma)\}$ is *not true, then there exist* $\sigma', \rho' \in X$ *, such that* $\pi^c_{(\beta,\alpha)}(\rho') < \max\{\pi^c_{(\beta,\alpha)}(\sigma' * \rho'), \pi^c_{(\beta,\alpha)}(\sigma')\}$.

Putting $t` = \frac{1}{2}(\pi^c_{(\beta,\alpha)}(\rho`) + min\{\pi^c_{(\beta,\alpha)}(\sigma' * \rho'), \pi^c_{(\beta,\alpha)}(\sigma')\})$, *then* $\pi^c_{(\beta,\alpha)}(\rho`) < t`$ and $0 \leq t` < min\{\pi^c_{(\beta,\alpha)}(\sigma' * \rho'), \pi^c_{(\beta,\alpha)}(\sigma')\} \leq 1$, *hence* $\pi^c_{(\beta,\alpha)}(\sigma' * \rho') > (\mathrm{t} - \alpha)/\beta`$ and $\pi^c_{(\beta,\alpha)}(\sigma') > t`$, which imply that $\sigma' * \rho' \in U_{(\beta,\alpha)}(\pi;\ t')$ and $\sigma' \in U_{(\beta,\alpha)}(\pi;\ t')$ , since $U_{(\beta,\alpha)}(\pi;\ t')$ is *an ideal a, it follows* that $(y') \in U_{(\beta,\alpha)}(\pi;\ t')$ and that $\pi(y') \geq t`$, *this is also* a C!. *Therefore*, $\forall \sigma, \rho \in \partial$, $\pi^c_{(\beta,\alpha)}(\rho) \geq \min\{\pi^c_{(\beta,\alpha)}(\sigma * \rho), \pi^c_{(\beta,\alpha)}(\sigma)\}$.

Hence $\pi^c_{(\beta,\alpha)}$ is MTFRG-I of ∂. ⌂

Proposition 4.4.

Let $\pi^c_{(\beta,\alpha)}$ and $v^c_{(\beta,\alpha)}$ be two MTFRG-Is of a RG-A $(X;\ *, 0)$ .Then $(\pi^c_{(\beta,\alpha)} \cap v^c_{(\beta,\alpha)})$ is also a FRG-I of X.

Proof: For all $x, y \in X$,

1. $(\pi^c_{(\beta,\alpha)} \cap v^c_{(\beta,\alpha)})(0) = inf\{\pi^c_{(\beta,\alpha)}(0), v^c_{(\beta,\alpha)}(0)\}$,
$\geq \inf\{\pi^c_{(\beta,\alpha)}(\sigma), v^c_{(\beta,\alpha)}(\sigma)\} = (\pi^c_{(\beta,\alpha)} \cap v^c_{(\beta,\alpha)})(\sigma)$.

2. $= min\{\pi^c_{(\beta,\alpha)}(y), v^c_{(\beta,\alpha)}(y)\}$
$\geq min\{\min\{\pi^c_{(\beta,\alpha)}(\sigma * \rho), \pi^c_{(\beta,\alpha)}(\sigma)\}, \min\{v^c_{(\beta,\alpha)}(\sigma * \rho), v^c_{(\beta,\alpha)}(\sigma)\}\}$,
$= min\{\min\{\pi^c_{(\beta,\alpha)}(\sigma * \rho), \pi^c_{(\beta,\alpha)}(\sigma), v^c_{(\beta,\alpha)}(\sigma * \rho), v^c_{(\beta,\alpha)}(\sigma)\}\}$,
$= min\{\min\{\mu^c_{(\beta,\alpha)}(x * y), v^c_{(\beta,\alpha)}(x * y)\}, \min\{\mu^c_{(\beta,\alpha)}(x), v^c_{(\beta,\alpha)}(x)\}\}$,
$= \min\{min\{\mu^c_{(\beta,\alpha)}(\sigma * \rho), v^c_{(\beta,\alpha)}(\sigma * \rho)\}, min\{\mu^c_{(\beta,\alpha)}(\sigma), v^c_{(\beta,\alpha)}(\sigma)\}\}$,
$= \min\{(\pi^c_{(\beta,\alpha)} \cap v^c_{(\beta,\alpha)})(\sigma * \rho), (\pi^c_{(\beta,\alpha)} \cap v^c_{(\beta,\alpha)})(\sigma)\}$.

Hence $(\pi^c_{(\beta,\alpha)} \cap v^c_{(\beta,\alpha)})$ is a FRG-I of ∂. ⌂

Proposition 4.5.

Let μ be *a fuzzy subset* of RG-A $(\partial; *, 0)$ and α ∈[0,T], t , β ∈ (0,1]. The (β,α)-*magnified translation fuzzy subset* $\mu^c_{(\beta,\alpha)}$ of π is a FRG-I of ∂, *if and only if*, π is a FRG-I of ∂ .

Proof:

(⇒) *Suppose that* $\pi^c_{(\beta,\alpha)}$ is MTFRG-I of ∂ and $U_{(\beta,\alpha)}$ (μ; t) ≠ ∅, for any α∈[0,T], t, β ∈ (0,1].

Since $\pi^c_{(\beta,\alpha)}(0) = \beta.\pi(0) + \alpha \geq \beta.\pi(\sigma) + \alpha = \pi^c_{(\beta,\alpha)}(\sigma)$, then $\pi(0) \geq \pi(\sigma), \sigma \in \partial$. Let $\forall \sigma, \rho \in \partial$,, then

$\beta.\pi(\rho) + \alpha = \pi^c_{(\beta,\alpha)}(\rho)$
$\geq min\{\pi^c_{(\beta,\alpha)}(\sigma * \rho), \pi^c_{(\beta,\alpha)}(\sigma)\}$
$= min\{\beta.\pi(\sigma * \rho) + \alpha, \beta.\mu(\sigma) + \alpha\}$
$= min\{\beta.\pi(\sigma * \rho), \beta.\mu(\sigma)\} + \alpha$
$= \beta.min\{\pi(\sigma * \rho), \pi(\sigma)\} + \alpha$ and so
$\pi(\rho) \geq min\{\pi(\sigma * \rho), \pi(\sigma)\}$.

Hence π is a FRG-I of ∂ .

($\Leftarrow$) Co., suppose that π is a FRG-I of ∂ such that Since $\pi(0) \geq \pi(\sigma)$, then
$\pi^c_{(\beta,\alpha)}(0) = \beta.\pi(0) + \alpha \geq \beta.\pi(\sigma) + \alpha = \pi^c_{(\beta,\alpha)}(\sigma)$.
$\forall \sigma, \rho \in \partial, \pi(\rho) \geq min\{\pi(\sigma * \rho), \pi(\sigma)\}$. If for each $\alpha \in [0,T]$, t , $\beta \in (0,1]$.
$\pi^c_{(\beta,\alpha)}(\rho) = \beta.\pi(\rho) + \alpha$
$\geq \beta.min\{\pi(\sigma * \rho), \pi(\sigma)\} + \alpha$
$= min\{\beta.\pi(\sigma * \rho) + \alpha, \beta.\pi(\sigma) + \alpha\}$
$= min\{\pi^c_{(\beta,\alpha)}(\sigma * \rho), \pi^c_{(\beta,\alpha)}(\sigma)\}$

Hence $\pi^c_{(\beta,\alpha)}$ is a FRG-I of ∂ . ⌂

Proposition 4.6.

A homomorphic pre − image of MTFRG-I of RG-A is also MTFRG-I of RG-A.

Proof:

Let $f: (\partial;\ *, 0) \rightarrow (\partial';\ *`, 0`)$ be a homomorphism of RG-As, $\lambda^c_{(\beta,\alpha)}$ the MTFRG-I of ∂' .

Then λ is a FRG-I of '∂' by Th. (4.5) and μ is the pre − image of λ under f, by Prop. (2.24(3)), μ is a FRG-I of ∂.

Hence $\pi^c_{(\beta,\alpha)}$ the MTFRG-I of X by Th. (4.5). ⌂

Proposition 4.7.

Let $f: (\partial;\ *, 0) \rightarrow (\partial';\ *`, 0`)$ be an epimorphism between RG-As ∂ and ∂' respectively. For every MTFRG-I $\pi^c_{(\beta,\alpha)}$ of ∂ with sup property, $f(\pi^c_{(\beta,\alpha)})$ is MTFRG-I of ∂'.

Proof:

Let $f: (\partial;\ *, 0) \rightarrow (\partial';\ *`, 0`)$ be a homomorphism of RG-As, $\pi^c_{(\beta,\alpha)}$ the MTFRG-I of ∂ . Then π is a FRG-I of ∂ by Th. (4.5) and $f(\pi)$ is the image of π, by Prop. (2.24(4)), $f(\pi)$ is a FRG-I of ∂'.

Hence $f(\pi^c_{(\beta,\alpha)})$ is MTFRG-I of ∂' by Th. (4.5) . ⌂

Proposition 4.8.

If MTFRG-I of an RG-A ∂, then π is a FRG-SA.

Proof:

Since $\pi^c_{(\beta,\alpha)}$ be MTFRG-I of ∂, then by Prop. (2.21), $\pi^c_{(\beta,\alpha)}$ be a FRG-SA of ∂. ⌂

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