SEE Transform Technique in Control Theory

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DOI: http://dx.doi.org/10.31642/JoKMC/2018/110218
Received Apr. 10, 2024. Accepted for publication Aug. 17, 2024

Abstract—*As in the Laplace transform technique, a new integral technique called SEE (Sadiq-Emad-Eman) transform is submitted. Showed propositions and important properties of SEE integral technique for differentiate of functions and showed shifting property for SEE technique. Got transfer function of dynamic system in control theory applying SEE integral technique. Solved some important problems in control theory via SEE technique. Showed that the Laplace integral technique and every integral transform technique like the Laplace transform available in the literature are particular cases of the SEE technique.*

Keywords— Integral transforms, Laplace transform, dynamic systems, control theory, Transfer functions, Ordinary differential equations.

I. Introduction

Styling a control system is obligatory to understand that how the required system carries with respect to another controller models. We designed by as a system of these operators to reach dynamic response. There are three distinction domains in the dynamic answer of the system is discussed for making control model. These three domains are Laplace domains, frequency domain and the state space, [2,5].

In this work, we are representing a novel technique called SEE transform [1]. The beauty of this technology is the Laplace transform is a particular condition of SEE technique generate v-domain in state s-domain the aim of the SEE transform technique is to find the solution of all dynamical system in control theory in the possible simplest approach .The main goal of the paper is to use the SEE transform in control theory .Although the Sadik transform is more general than the SEE transform, generality does not mean that it is easier to find solutions. Due to the importance of control theory in the engineering and physics fields ,especially in medical device engineering and medical physics .Certainly , different methods must be found in finding exact solutions and simple , easy techniques in solving differential equations related to control theory.

II. Preliminaries

In this part, we start with definition of the Laplace transformation.

Definition(2.1), [1]:

If $g(t)$ is a piecewise continuous on $[0, a]$ for $a > 0$, and it is an exponential order of a then the Laplace transformation of $g(t)$ is symbolized by $G(v)$ and is known:

$$G(v) = L\{g(t)\} = \int_0^\infty e^{-vt}\, g(t)dt\ .$$

If the integral exist.
Wherever, V is a complex parameter.

Definition (2,2), [1] If :

1. g(t) is a piecewise continuous on the closed interval $[0, A]$ for $A > 0$.
2. $|g(t)| \le k\, e^{at}$, when $t \ge M$, where a is a real constant and positive constants K and M

Then SEE integral transform technique of g(t) is known:

$$S\{g(t)\} = \frac{1}{v^n}\int_0^\infty e^{-vt}\, g(t)dt = T(v).$$

Where

v Is a complex parameter, n is any integer number.

A. ***Properties of SEE Tranform [1]:***

1- Linearity of SEE Transform:

$S\{Ag_1(t) \mp Bg_2(t)\} = AS\{g_1(t)\} \mp BS\{g_2(t)\}$

Where A and B are constants, g_1 and g_2 are functions of variable t.

2- Shifting of SEE Transform :

If SEE integral transform of $g(t)$ is $T(v)$ then SEE transform of the function $e^{at}\, g(t)$ is given by $\frac{(v-a)^{(n)}}{v^n}.T(v-a)$.

That is $S\{g^{(at)}(t)\} = \frac{(v-a)^{(n)}}{v^n}.T(v-a)$, where a is a constant.

3- Change Scale of SEE Transform:

If $S\{g(t)\} = T(v) = \frac{1}{v^n}\int_0^\infty e^{-vt}\, g(t)dt$.

Then, $S\{g(at)\} = \frac{1}{a^{n+1}}T(\frac{v}{a})$, $a > 0$ is a constant number.

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B. ***The Application of the SEE Integral Transform to Some Well-Known Functions [1]***

1- If $g(t) = 1$ then $S\{1\} = \frac{1}{v^{n+1}}$, $v > 0$.
2- If $g(t) = t$ then $S\{t\} = \frac{1}{v^{n+2}}$.
3- If $g(t) = t^2$ then $S\{t^2\} = \frac{1}{v^{n+2}}$.
4- If $g(t) = t^m$ then $S\{t^m\} = \frac{m!}{v^{n+m+1}}$, $m \in \mathbb{N}$.
5- If $g(t) = t^m$ then $S\{t^m\} = \frac{\gamma\,(m+1)}{v^{n+m+1}}$, $m > -1$ *and where* γ *is a gamma function*.
6- If $g(t) = e^{at}$ then $S\{e^{at}\} = \frac{1}{v^n(v-a)}$.
7- If $g(t) = sin(at)$ then $S\{\sin(at)\} = \frac{a}{v^n(v^2+a^2)}$.
8- If $g(t) = cos(at)$ then $S\{\cos(at)\} = \frac{a}{v^n(v^2+a^2)}$.
9- If $g(t) = sinh(at)$ then $S\{\sinh(at)\} = \frac{a}{v^n(v^2-a^2)}$.
10- If $g(t) = cosh(at)$ then $S\{\cosh(at)\} = \frac{a}{v^n(v^2-a^2)}$.

If $g(t) = t\, g(t)$then $S\{t\, g(t)\} = \frac{-n}{v}\, T(v) - \frac{dT(v)}{dv}$.

III. INVERSE OF SEE INTEGRAL TRANSFORM OF FAMOUS FUNCTION[1]:

Definition(3-1):

If $S[g(t)] = T(v)$ is the SEE integral transform, then $g(t) = S^{-1}[T(v)] = \frac{1}{2\pi j}\int_{v=\delta-j\infty}^{\delta+j\infty} v^n\, F(v)e^{vt}\, dv$, is called an inverse of the SEE integral transform.

$S^{-1}[G(v)] = \frac{1}{2\pi j}\sum_{v=\delta-j\infty}^{\delta+j\infty} v_k^n\, F(v_k)e^{v_k t}\,\Delta v$,

$v > 0$, $\delta > 0$, j is a complex number.

Property (3-1):

The inverse of $T(v)$ is linear that is:

$S^{-1}[C_1T_1(v) \pm C_2T_2(v)] = C_1S^{-1}[T_1(v)] \pm C_2S^{-1}[T_2(v)]$.

When C_1 and C_2 arbitrary constants.

TABLE I. INVERSE SEE TRANSFORM OF SOME FREQUENTLY ENCOUNTERED FUNCTIONS [1]:

S.N	$T(v)$	$G(t) = S^{-1}[T(v)]$
1	$\frac{1}{v^{n+1}}$	1
2	$\frac{m!}{v^{n+m+1}}$	t^m m:+ive integer number
3	$\frac{1}{v^n(v-a)}$	e^{at} , a is constant.
4	$\frac{a}{v^n(v^2+a^2)}$	$\sin(at)$, where a is a real constant.
5	$\frac{a}{v^n(v^2+a^2)}$	$\cos(at)$, where a is a real constant.
6	$\frac{a}{v^n(v^2-a^2)}$	$\sinh(at)$, where a is a real constant.
7	$\frac{a}{v^n(v^2-a^2)}$	$\cosh(at)$, where a is a real constant.
8	$\frac{-n}{v}\,T(v) - \frac{dT(v)}{dv}$	$t\,G(t)$.

IV. CONTROL PROBLEMS

The following problem in mechanical modeling elaborates the effectiveness of a novel technique (SEE Transformation Technique).

Problem(4.1)

The following linear homogeneous second order ordinary differential equation with constant coefficients is the differential operator equation of motion for an ideal spring – mass system with damping, [4]

$x''(t) + 3x'(t) + 2x = 0$

Subject to initial conditions:

$x(0) = 1,\ x'(0) = 0$.

Solution:

Now, we take SEE transform of the above equation and applying given initial conditions, we have:

$$v^2X(v) - v^{1-n}x(0) - v^{-n}x'(0) + 3vX(v) - 3v^{-n}x(0) + 2X(v) = 0,$$

Therefore,

$$X(v) = \frac{v^{1-v}+3v^{-n}}{(v^2+3v+2)},$$

$$X(v) = \frac{v^{-n}(v+3)}{(v+2)(v+1)}. \dots \quad (4.1)$$

For the simplicity, we take n=1 (Aboodh transform [1]).
Hence,

$$X(v) = \frac{1}{v}\left[\frac{2}{v+1} - \frac{1}{v+2}\right] = \frac{2v^{-1}}{v+1} - \frac{v^{-1}}{v+2}.$$

Taking the inverse of Aboodh transform, we obtain
$X(t) = 2e^{-t} - e^{-2t}$.

IF we choose n=2 (Emad- Sara Transform,[6]).
Hence

$$X(v) = \frac{v^{-2}(v+3)}{(v+2)(v+1)} = \frac{1}{v^2}\left[\frac{-1}{v+2} + \frac{2}{v+1}\right].$$

Taking the inverse of Emad-Sara transform, we get
$X(t) = 2e^{-t} - e^{-2t}$

If we choose $n = -2$ (Mohand Transform, [3]).
Hence

$$X(v) = v^2\left[\frac{-1}{v+2} + \frac{2}{v+1}\right].$$

Now, taking the inverse of Mohand transform, we obtain $x(t) = 2e^{-t} - e^{-2t}$.

(You may choose another value of n).

Now, carefully taking inverse SEE transform for equation (4.1), we get the exact solution $x(t) = 2e^{-t} - e^{-2t}$

It is a required solution in time domain t.

And so on:

If n= -1 (Mahgoub transform [7]).
If n=3 (Gupta transform [8]).
If n= -3 (Rohit transform [8]).

In each of the above transforms, the value of n differs , and the all give an exact solution , but they differ in the algebraic calculations , some of which are simple and some of which difficult .

Problem (4.2) A shaft of inertia I is rotated for the angle $\boldsymbol{\varphi}$ due to used torque τ against a bearing friction. Evaluate the transmit function of the operators.

$$\tau(\varphi) = I\varphi'' + g\varphi'.$$

With initial conditions:

$$\varphi(0) = \varphi'(0) = 0.$$

solution: (By the SEE integral transform technique) the differential problem from the above statement is $\tau(t) = I\varphi'' + g\varphi'$.

Using the SEE integral transform, we obtain:

$$T(v) = I[v^2\Psi(v) - v^{1-v}\varphi(0) - v^{-n}\varphi'(0)] + g[v\Psi(v) - v^{-n}\varphi(0)],$$

we get

$$T(v) = \Psi(v)[I v^2 + gv]$$

Therefore

$$\frac{\Psi(v)}{T(v)} = \frac{1}{Iv^2 + gv}$$

This is a required transmit function in v-domain and this function can be changed via choosing value of n, it is an advantage of SEE technique.

V. CONCLUSION

A novel integral technique called SEE integral technique works effectively to find the exact solution of dynamic systems in control Theory. Instead of the Laplace integral technique we may apply SEE integral technique for best realizing the problems in control theory. As well as, more details of SEE method will be discuss in further study work. We also conclude that there is no preference between one transform and another .Each transform has a specific advantage in a special application that does not depend on generality.

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