A New Approach for Solving Multi Products Transportation Problem

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Abstract— *The problem of transportation is studied in many areas, most importantly in the field of logistics and operations management. The distribution problem of goods and commodities from sources to destinations is an important problem where many methods have been used to obtain its optimum solution, which represents the minimum cost of distribution the goods from sources to destinations. Generally, the transportation classical cost of one unit of a good is depending on the source and the destination. In this paper, we suggest an approach to obtain a solution to the transportation problem consisting of two products or more and then by using the modified Kruskal's algorithm we find the minimum feasible solution.*

The proposed approach may represent a solution for a wide range of applications in general and products transportation in specific, where the cost of transportation, reliability and feasibility are of great importance.

Keywords— Transportation Problem, Multi Products Algorithms, Kruskal's Algorithm, Graph Theory.

I. INTRODUCTION

transportation theory is the name granted to the study of ideal transportation and distribution of resources. This problem has been occurred in many areas such as Optimization Techniques, Management, Economics, and Mathematics [1]. Transportation problem is one of the primary problems of network flow problem that used to lower the transportation cost for goods while satisfying the supply and demand requirement [2].

The problem was initially formalized in 1781 by French mathematician G. Monge. During World War II Major advances were made in this field by Leonid Kantorovich Economist and Mathematician [1]. During 1939 Kantorovich arrive up with the mathematical technique known as linear programming [3]. In 1941 F. L. Hitchcock presented study entitled (the distribution of a product from several sources to numerous localities), this considered as the first contribution to solve the Transportation problems [1].
During 1947 T. C. Koopmans presented an independent study entitled (optimum utilization of the transportation system). These two contributions helped introductions by F. L. Hitchcock and T. C. Koopmans in the development of transportation methods which involve a number of shipping sources and a number of destinations [1,4]. In 1973 Lee and Moore have examined the multi-target development of transportation issues [5].
During 1979 Iserman introduced a calculation for tackling direct multi-target transportation problems. In 1987 Ringuest and Rinks suggested two methodologies for getting the arrangement of direct multi-target transportation problems. In 1999 Das et al. used a programming way to solve the multi-target interim transportation problem [5]. Transportation Problem with single objective to minimize the cost of transportation has been studying by much researchers such as Tikekar and Seshan, Swarup and Sharma, Lee and Wahead. et all [2].
Kadhim B. S. Aljanabi and Anwar Nsaif Jasim proposed an approach for solving transportation problem using modified Kruskal's Algorithm [6]. Jose E. Florez et al. proposed a new model for planning Multi-Modal transportation problems [7].

II. THE COMPONENT OF TRANSPORTATION MODAL

A. The amount yield (supply) at each origin and the amount of, requirement (demand) at each destination.

B. The transportation cost from (supply) to (demand).

Since there is only one product, a goal can receive its requirement from more than one source. The Transportation problem is interested with finding the lower limit cost of transporting the wares from a given number of sources to destinations [8,9]. The objective is to minimize the amount shipping cost through to decide how much should be shipped from any source to any destination so as to minimize the overall transportation cost [1].

III. TRANSPORTATION MODEL WITH ONE PRODUCT

A transportation model content (n) sources and (m) destinations. Every source is represented by a vertex and destination is represented by another vertex. The link among sources and destinations are represented by edges. The edge is connected between two vertices. The value of supply available at source (i) is (ai), the demand required at destination (j) is (bj).

Let (Cij) denote the cost of transfer one unit between source (i) and destination(j). Let (Xij) denotes the amount transfer from source (i) to destination (j) [10]. The cost related with this: Cost × quantity =CijXij

Minimize $$Z=\sum_{i=1}^{m}\ \sum_{j=1}^{n} c_{ij}x_{ij} \tag{1}$$

Subject to $$\sum_{j=1}^{n} x_{ij}=a_i,\quad i=1,2,3,\dots,m \tag{2}$$

and $$\sum_{i=1}^{m} x_{ij}=b_j,\quad j=1,2,3,\dots,n \tag{3}$$

where $$x_{ij}\ge 0 \tag{4}$$

$$\sum_{i=1}^{m} a_i=\sum_{j=1}^{n} b_j \tag{5}$$

$$\sum_{i=1}^{m} a_i\ne\sum_{j=1}^{n} b_j \tag{6}$$

i.e. the two sets of constraints will be consistent. The formulation in above equations 1-5 is called a balanced transportation model, equation (6) is unbalanced transportation model.

IV. PROPOSED APPROACH AND FORMULATION

An illustrative example to explain how the proposed algorithm works: This example contains two sources S1 and S2 each may supply two different products P1 and P2 to two different demands D1 and D2. Suppose that Cij and Xij represent the cost and the quantity supplied from source Si to demand Dj. The graph of such example is shown in Figure (1).

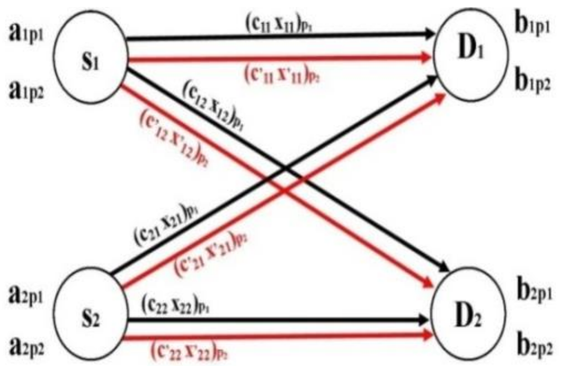

Fig. 1. general transportation model with two products

The Cij (i=1, 2,..,n and j=1,2,..,m) represents the cost of transporting a unit of product (P1) from (Si) to (Dj) and (C'ij) represents the cost of transporting a unit of product(P2) from (Si) to (Dj). Let Xij (i=1,2,…n, j=1,2,…m) represents the quantity of product (P1) supplied from (Si) to (Dj) and (X'ij) represents the quantity of product (P2) supplied from (Si) to (Dj).

Let (a1p1) and (a1p2) be total amount of product (P1) and (P2) available in (S1) respectively, (a2p1) and (a2p2) be total amount of product (P1) and(P2) available in (S2), (b1p1) and (b1p2) be total amount of product (P1) and (P2) respectively required by demand (D1).

Let (b2p1) and (b2p2) be total amount of product (P1) and (P2) required by (D2). The cost related with this:

Cost × quantity of product Pt= (cij xij) Pt.

Minimize

$$z=\sum_{t=1}^{k}\sum_{i=1}^{n}\sum_{j=1}^{m}(c_{ij}x_{ij})_{pt}$$

where (t=1,2,3,..,k , i=1,2,3,..,n and j=1,2,3,..,m).

Several methods were used to solve transportation problems to transport a specific product from the sources to the destinations. This work proposes the transport of two products or more from the sources to the destinations at same time.

V. NUMERICAL EXAMPLE OF TRANSPORTATION MODEL WITH TWO PRODUCTS

The following example is a numerical (1) with two sources, two demands and two products. Solve this transportation problem using modified Kruskal's Algorithm [6], then minimize the total transportation cost is 3610.

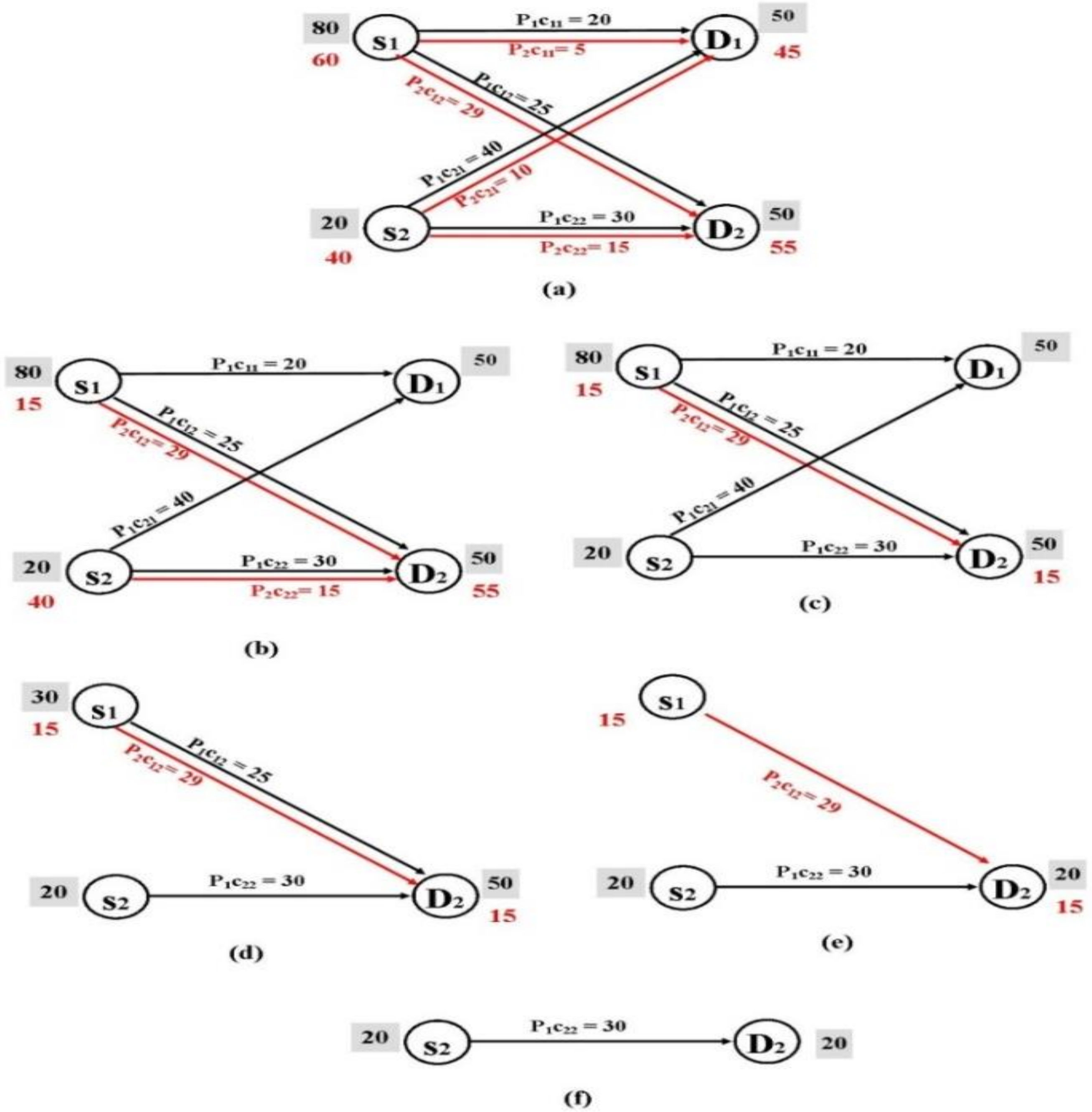

Fig. 2. Example of transportation model with two products.

The solution steps of the problem in the first example given in the figure 2 from (a) to (f) and these details shown in table I.

VI. Numerical Example of Transportation Model with Three products

The following example is a numerical (2) with two sources, three demands and three products. Solve this transportation problem using modified Kruskal's Algorithm (as shown in figure 3), then minimize the total transportation cost is 6745.

The solution steps of the problem in the second example given in the figure 3 from (a) to (k) and these details shown in table II.

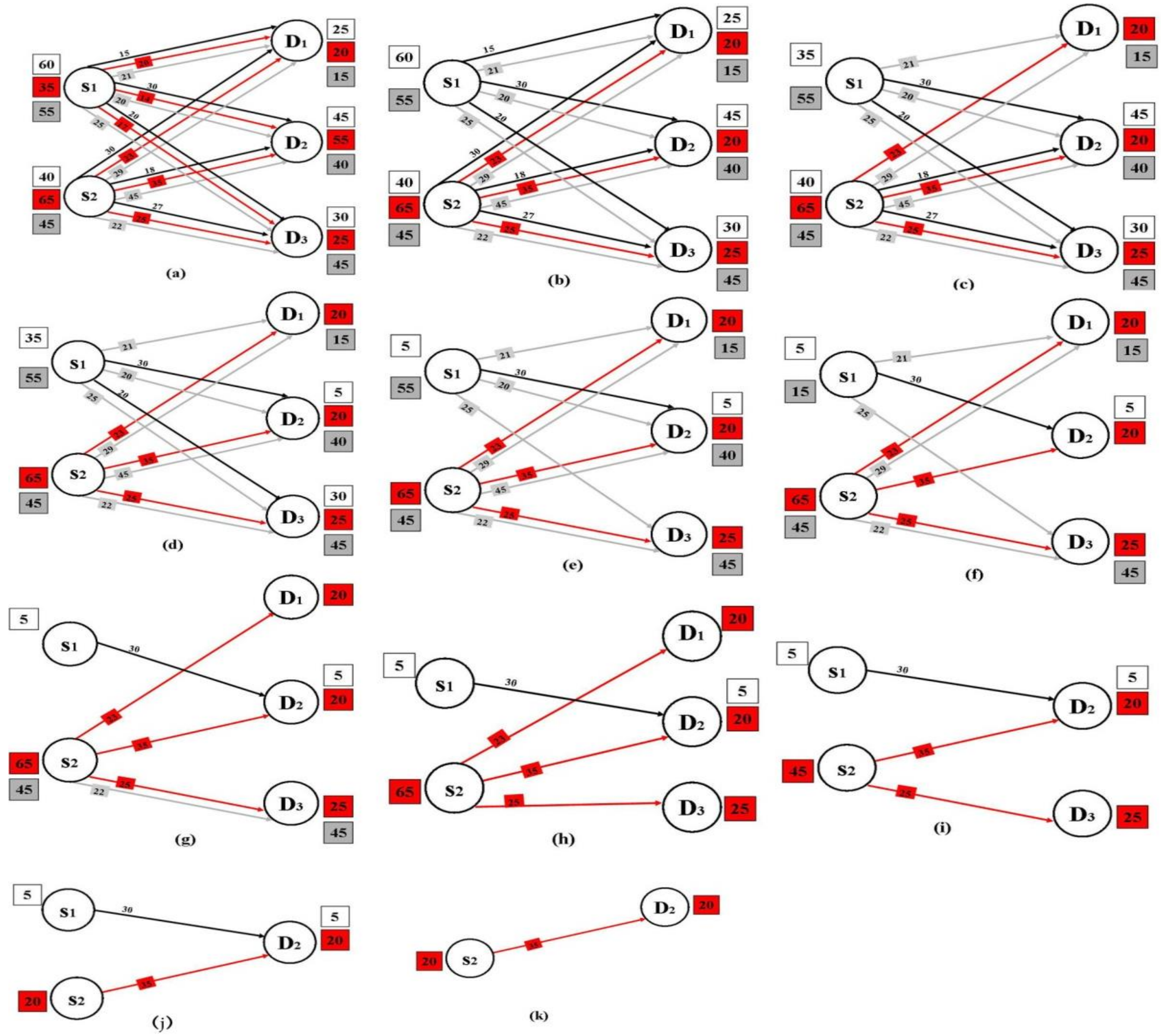

Figure 2. Example of transportation model with three products

TABLE I. TABLE 1. NUMERICAL REPRESENTATION TO THE EXAMPLE WITH TWO PRODUCTS

No.	edge	cost	Kruskal step (a) Cost =5*45 =225		Kruskal step (b) Cost =225+ 15*40 =825		Kruskal step (c) Cost =825+20*50 =1825		Kruskal step (d) Cost =1825+25*30 =2575		Kruskal step (e) Cost =2575+29*15 =3010		Kruskal step (f) Cost =3010+30*20 =3610
1.	P_1c_{11}	20	P_1c_{11}	20	P_1c_{11}	20	×	×	×	×	×	×	×
2.	P_2c_{11}	5	×	×	×	×	×	×	×	×	×	×	×
3.	P_1c_{12}	25	P_1c_{12}	25	P_1c_{12}	25	P_1c_{12}	25	×	×	×	×	×
4.	P_2c_{12}	29	P_2c_{12}	29	P_2c_{12}	29	P_2c_{12}	29	P_2c_{12}	29	×	×	×
5.	P_1c_{21}	40	P_1c_{21}	40	P_1c_{21}	40	×	×	×	×	×	×	×
6.	P_2c_{21}	10	×	×	×	×	×	×	×	×	×	×	×
7.	P_1c_{22}	30	P_1c_{22}	30	P_1c_{22}	30	P_1c_{22}	30	P_1c_{22}	30	P_1c_{22}	30	×
8.	P_2c_{22}	15	P_2c_{22}	15	×	×	×	×	×	×	×	×	×

TABLE II. TABLE 2. NUMERICAL REPRESENTATION TO THE EXAMPLE WITH THREE PRODUCTS.

No.	edge	cost	Kruskal step (a) Cost =20*35 =700		Kruskal step (b) Cost =700+ 15*25 = 1075		Kruskal step (c) Cost =1075+18*40 = 1795		Kruskal step (d) Cost =1795 +20*30 =2395		Kruskal step (e) Cost= 2395+20*40 = 3195		Kruskal step (f) Cost= 3195+21*15 =3510		Kruskal step (g) Cost = 3510+22*45 = 4500		Kruskal step (h) Cost = 4500+23*20 =4960		Kruskal step (i) Cost = 4960+25*25 =5585		Kruskal step (j) Cost = 5585+23*20 = 6045		Kruskal step (k) Cost = 6045+35*20 =6745
9.	P_1c_{11}	15	P_1c_{11}	15	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
10.	P_2c_{11}	20	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
11.	P_3c_{11}	21	P_3c_{11}	21	P_3c_{11}	21	P_3c_{11}	21	P_3c_{11}	21	P_3c_{11}	21	×	×	×	×	×	×	×	×	×	×	×
12.	P_1c_{12}	30	P_1c_{12}	30	P_1c_{12}	30	P_1c_{12}	30	P_1c_{12}	30	P_1c_{12}	30	P_1c_{12}	30	P_1c_{12}	30	P_1c_{12}	30	P_1c_{12}	30	×	×	×
13.	P_2c_{12}	14	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
14.	P_3c_{12}	20	P_3c_{12}	20	P_3c_{12}	20	P_3c_{12}	20	P_3c_{12}	20	×	×	×	×	×	×	×	×	×	×	×	×	×
15.	P_1c_{13}	20	P_1c_{13}	20	P_1c_{13}	20	P_1c_{13}	20	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
16.	P_2c_{13}	17	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
17.	P_3c_{13}	25	P_3c_{13}	25	P_3c_{13}	25	P_3c_{13}	25	P_3c_{13}	25	P_3c_{13}	25	P_3c_{13}	25	×	×	×	×	×	×	×	×	×
18.	P_1c_{21}	30	P_1c_{21}	30	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
19.	P_2c_{21}	23	P_2c_{21}	23	P_2c_{21}	23	P_2c_{21}	23	P_2c_{21}	23	P_2c_{21}	23	P_2c_{21}	23	P_2c_{21}	23	×	×	×	×	×	×	×
20.	P_3c_{21}	29	P_3c_{21}	29	P_3c_{21}	29	P_3c_{21}	29	P_3c_{21}	29	P_3c_{21}	29	×	×	×	×	×	×	×	×	×	×	×
21.	P_1c_{22}	18	P_1c_{22}	18	P_1c_{22}	18	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
22.	P_2c_{22}	35	P_2c_{22}	35	P_2c_{22}	35	P_2c_{22}	35	P_2c_{22}	35	P_2c_{22}	35	P_2c_{22}	35	P_2c_{22}	35	P_2c_{22}	35	P_2c_{22}	35	P_2c_{22}	35	×
23.	P_3c_{22}	45	P_3c_{22}	45	P_3c_{22}	45	P_3c_{22}	45	P_3c_{22}	45	×	×	×	×	×	×	×	×	×	×	×	×	×
24.	P_1c_{23}	27	P_1c_{23}	27	P_1c_{23}	27	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
25.	P_2c_{23}	25	P_2c_{23}	25	P_2c_{23}	25	P_2c_{23}	25	P_2c_{23}	25	P_2c_{23}	25	P_2c_{23}	25	P_2c_{23}	25	P_2c_{23}	25	×	×	×	×	×
26.	P_3c_{23}	22	P_3c_{23}	22	P_3c_{23}	22	P_3c_{23}	22	P_3c_{23}	22	P_3c_{23}	22	P_3c_{23}	22	×	×	×	×	×	×	×	×	×

VII. CONCLUSION

The proposed method considered is close to application in real life problems to transport products from the sources such as (factories) to the destinations such as (warehouses or supermarkets) where the whole goal is to minimize the overall transportation cost. In this work we proposed transport of two products or more from the sources to the destinations at same time. The proposed approach relies primarily convert the transport model into graph model with default vertices and edges, then the minimum feasible solution is found using modified Kruskal's algorithm. The accuracy the proposed approach was studied using two different numerical examples as shown in figures (2) and (3).

Time complexity of the proposed approach is highly dependent on number of sources, number of destinations and the number of products (i.e. S, D and P).

One of the significant advantage of the proposed approach over the other approaches is that it takes into account a real world problem in which multi sources, multi destinations and multi products are studied together in one environment.

VIII. FUTURE WORK

- Multi weights transportation can be studied.
- Cyclic supplying process is an emerging transportation field.

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