Applications of Wavelets for BVPs and Signal Processing

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Abstract *The transfer of information using the signal needs speed, which leads to the compression of the information. It is only possible through the process of using a mathematical technique at work.*
To demonstrate an increase in theory efficiency, it was used in signal processing, compression, and good results. In section 4 Matrix (6×6) *was used because M=3 was taken, where six functions were obtained, when these functions were integrated, the operations matrix of integration* (6×6) *was added, which was added to solve Boundary Value Problems (BVP$_s$) numerically. In addition to solving problems numerically, using the proposed technique, which is signal processing, to demonstrate the efficiency of the proposed theory as indicated in section 2, wavelets are built on the dependence of the four effects*(n, m, k, r)*. In addition, the number of equations obtained is calculated based on the value of M where* $m = 0,1,2,3,\dots,M-1$ *six functions are obtained and the greater value of M is obtained More functions, leading to greater accuracy in obtaining the best results.*

Keywords ***Chebyshev wavelets; operation matrix of integration OMI; Spectral method; Boundary Value Problems BVP; signal processing; compression signal.***

I. INTRODUCTION

In many applications, the role of wavelets is evident in finding solutions to many mathematical and engineering issues, and these wavelets are derived from the parent function [1,8,9].

Today, there are many works on wavelets methods for approximating the solution of the problems, such as Hermite wavelets method [2], third kind Chebyshev wavelets [3], Haar wavelets method [4], and Sin and Cos wavelets method [5].

Several numerical methods have been proposed in the last years to solve (BVP$_s$) which are based on orthogonal polynomials, also wavelets approach was used in several papers to solve (BVP$_s$) [6]. In addition, the efficiency of the technology proposed in this work will be enhanced. An example illustrates this efficiency in signal processing and compression.

III. CHEBYSHEV WAVELETS (CW)

The new wavelengths are created from the mother wavelet, based on two factors, the coefficient of contraction and expansion d and f [6].

$$\mu_{d,f}(r) = |d|^{\frac{1}{2}}\mu\left(\frac{r-f}{d}\right) \quad d, f \in R, \quad d \neq 0 \qquad (1)$$

where $\mu(r) = [\mu_0(r), \mu_1(r), \dots, \mu_{M-1}(r)]^T$

The elements $\mu_0(t), \mu_1(t), \dots, \mu_{M-1}(r)$ are the basis functions, they are orthogonal on [0,1] [7].

II. SECOND CHEBYSHEV WAVELETS (SCW)

SCW $\mu^2(r), \mu^2_{nm}(r) = \mu^2(r, n, m, k)$;
Where r is the normalized time, $n = 1,2,\dots,2^{k-1}$,m is the degree of SCW polynomials, $k = 1,2,\dots$. They are defined on [0,1) as

$$\mu^2_{nm}(r) = \begin{cases} 2^{\frac{k}{2}}\widetilde{U}_m(2^k r - 2n + 1) & \frac{n-1}{2^{k-1}} \le r < \frac{n}{2^{k-1}} \\ 0 & \text{o.w} \end{cases} \qquad (2)$$

$\widetilde{U}_m(r) = \sqrt{\frac{2}{\pi}}U_m(r)$, $m = 0,1,2,\dots,M-1$. In equation (2) we know that SCW polynomials is orthogonal with weight function $w(r) = \sqrt{1-r^2}$ on [-1,1], we should note that in dealing with SCW the weight function $\overline{w}(r) = w(2^k r - 2n + 1)$.

A function h(r) in [0, 1)

$$h(r) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} C_{nm}\mu^2_{nm}(r) \qquad (3)$$

$$C_{nm} = (f(t), \mu^2_{nm}(r)) \qquad (4)$$

$$h(r) = \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} C_{nm}\mu^2_{nm}(r) = C^T\mu^2(r) \qquad (5)$$

$$C = [C_{10}, C_{11}, \dots, C_{1(M-1)}, C_{20}, \dots, C_{2(M-1)}, \dots, C_{2^{k-1}}, \dots, C_{2^{k-1}M-1}]^T \qquad (6)$$

$$\mu^2(t) = \begin{bmatrix} \mu^2_{10}(t), \mu^2_{11}(t), \dots, \mu^2_{1M-1}(t), \mu^2_{20}(t), \dots, \mu^2_{2^{k-1}M-1}(t), \\ \dots \mu^2_{2^{k-1}0}(t), \dots \mu^2_{2^{k-1}M-1}(t) \end{bmatrix}^T \qquad (7)$$

IV. OMI FOR SCW

In this section, the OMI for FCW P_{μ^1}. First, we find 6×6 matrix P_{μ^2} .The basis six functions are given by [5].

From equation (2) if k=2 and M=3 we obtain the following six equations

$$\left.\begin{aligned} \mu^2_{1,0}(r) &= \frac{2\sqrt{2}}{\sqrt{\pi}} \\ \mu^2_{1,1}(r) &= 2\sqrt{\frac{2}{\pi}}(8r-2) \\ \mu^2_{1,2}(r) &= 2\sqrt{\frac{2}{\pi}}\,(4(4r-1)^2-1) \end{aligned}\right\} \quad 0 \le t < \frac{1}{2} \qquad (8)$$

$$\left.\begin{aligned} \mu^2_{2,0}(r) &= \frac{2\sqrt{2}}{\sqrt{\pi}} \\ \mu^2_{2,1}(r) &= 2\sqrt{\frac{2}{\pi}}(8r-6) \\ \mu^2_{2,2}(r) &= 2\sqrt{\frac{2}{\pi}}\,(4(4r-3)^2-1) \end{aligned}\right\} \quad \frac{1}{2} \le t < 1 \qquad (9)$$

Then by integrating these six equations from 0 to r and using equation (5) we obtain the OMI P_{μ^2} is:

$$P_{\mu^2_{6\times 6}} = \begin{bmatrix} \frac{1}{4} & \frac{1}{8} & 0 & \vdots & \frac{1}{2} & 0 & 0 \\ -\frac{3}{16} & 0 & \frac{1}{16} & \vdots & 0 & 0 & 0 \\ -\frac{1}{12} & -\frac{1}{24} & 0 & \vdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \vdots & \frac{1}{4} & \frac{1}{8} & 0 \\ 0 & 0 & 0 & \vdots & -\frac{1}{16} & 0 & \frac{1}{16} \\ 0 & 0 & 0 & \vdots & -\frac{1}{12} & -\frac{1}{24} & 0 \end{bmatrix}$$

The matrix P_{μ^2} can be written as:

$$P_{\mu^2_{6\times 6}} = \begin{bmatrix} L_{3\times 3} & S_{3\times 3} \\ O_{3\times 3} & L_{3\times 3} \end{bmatrix}$$

Where $\mu^2(r)$ has been given in equation (7) and P_{μ^2} is a $(2^k M) \times (2^k M)$ matrix given by:

$$P_{\mu^2_{M\times M}} = \frac{1}{2^k}\begin{bmatrix} L & S & S & \dots & S \\ 0 & L & S & \dots & S \\ 0 & 0 & L & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & S \\ 0 & 0 & 0 & \cdots & L \end{bmatrix}$$

Where S, L are $M \times M$ matrices as follows:

$$S = \begin{bmatrix} 2 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$L = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \dots & 0 \\ -\frac{3}{4} & 0 & 0 & \dots & 0 \\ \frac{1}{3} & 0 & 0 & \dots & 0 \\ -\frac{1}{4} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1^{M-2}\frac{1}{M-1} & 0 & 0 & \dots & \frac{1}{2(M-1)} \\ -1^{M-1}\frac{1}{M} & 0 & 0 & \dots & 0 \end{bmatrix}$$

IV.I. Orthogonal of second Chebyshev wavelet

$U_m(r)$ are orthogonal with respect to $w(r) = (1-r^2)^{\frac{1}{2}}$. Through trigonometric relationships then the (SCW) is orthogonal with respect to the weight function

$$w(r) = (1-(2^k r - 2n + 1)^2)^{\frac{1}{2}}$$

Theorem 1:

The orthogonal of second of Chebyshev wavelet if

$$\int_{\frac{n-1}{2^{k-1}}}^{\frac{n}{2^{k-1}}} \mu^2_{nm}(2^k r - 2n + 1)\mu^2_{n',m'}(2^k r - 2n' + 1)w(2^k r - 2n + 1) = \begin{cases} \mathbf{0} & \boldsymbol{if} \quad \boldsymbol{m \neq m'} \\ 2^k \frac{\pi}{2} & \boldsymbol{if} \quad \boldsymbol{m = m'} \end{cases}$$

See [10]

V. APPLICATIONS OF MATRICES P_{μ^1} AND P_{μ^2} FOR SOLVING BVPS

In order to solve linear or nonlinear differential equation by using the OMI P_{μ^1} and P_{μ^2}, some numerical examples illustrate the procedure which will be given.

Through the higher derivative hypothesis is equal *i*, and the variable is *r* then equation (10) is equal the operation matrix of integration then the integration of the two sides in the equation (10) begins more than once with the number *i* to be obtained $y(r)$ in equation (11)

$$y^i(r) = C^T \mu_{n,m}(r) \qquad (10)$$

$$y^{i-1}(r) = C^T \int_0^r \mu_{n,m}(r)dr + y^{i-1}(0) = C^T P\mu_{n,m}(r) + y^{i-1}(0) \qquad (11)$$

$$\vdots$$

$$\vdots$$

$$y(r) = C^T P^i \mu_{n,m}(r) + y^{i-1}(0)r^{i-2} + y^{i-2}(0)r^{i-3} + \cdots + y(0)$$

Example V.I

Consider the following BVP $z'' = -z$ with the boundary condition z(0)=0, z(1)=1, the exact solution for this problem is:

$z(r) = \frac{\sin r}{\sin 1}$

To solve this problem using an algorithm of SCW, assume that

$$z''(r) = C^T \mu_{n,m}^2 \tag{12}$$

Find

$$z'(r) = C^T \int_0^r \mu_{n,m}^2(r)dr + z'(0) \tag{13}$$

$$z'(r) = C^T P_{\mu^2} \mu_{n,m}^2(r) + z'(0) \tag{14}$$

$$z(r) = C^T P_{\mu^2} \int_0^r \mu_{n,m}^2(r)dr + z'(0)\, r + z(0) \tag{15}$$

$$z(r) = C^T P_{\mu^2}^2 \mu_{n,m}^2 + z'(0) r + z(0) \tag{16}$$

By using the above equations we get

$$C^T \mu_{n,m}^2 + C^T P_{\mu^2}^2 \mu_{n,m}^2 + z'(0) r + z(0) = 0 \tag{17}$$

$$C^T \left(1 + P_{\mu^2}^2\right) \mu_{n,m}^2 + r = 0 \tag{18}$$

r in equation (18) can be expressed in SCW as:

$r = d^T \mu_{n,m}^2$

Then equation (18) can be written:

$$C^T \left(1 + P_{\mu^2}^2\right) \mu_{n,m}^2 + d^T = 0 \tag{19}$$

$for\ \ M = 3\ and\ \ k = 2$, we obtain

$$d = [0.18617905\ 0.093089524\ \ 0\ 0.55853715\ 0.093089524\ 0]^T \tag{20}$$

After substituting d in equations (17) and (18) we get the following numerical results that shown in Table1.

Table1. Show numerical results.

t	Exact solution	SCW	Absolute Error (Exact-SCW)
0	0	-0.0001491	0.0001491
0.1	0.11864154	0.11890673	0.00026519
0.2	0.23609766	0.2363801	0.00028244
0.3	0.35119477	0.35092851	0.00026626
0.4	0.46278285	0.46255195	0.0002309
0.5	0.56974637	0.57125042	0.00150405
0.6	0.67101835	0.67269758	0.00167923
0.7	0.76558515	0.76797951	0.0016128
0.8	0.85250247	0.85362615	0.00112368
0.9	0.93090187	0.93198217	0.0010803
1	1	1.002266022	0.00054097

Chart1 of Table1 shows the comparison of results that we obtained

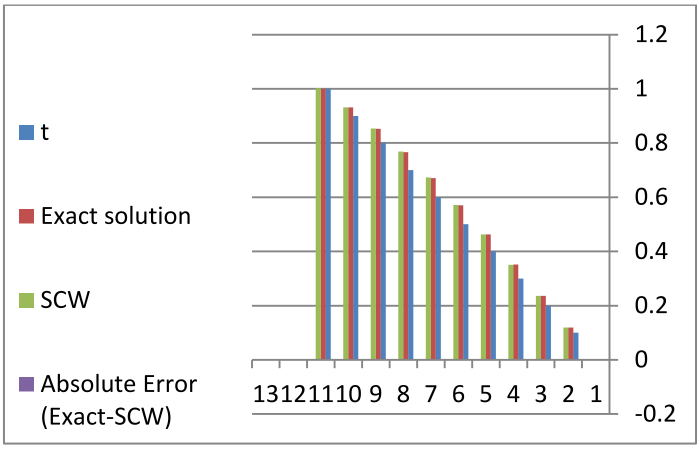

Chart1

The comparison between the solutions obtained by using SCW and exact solution is made as shown in Fig.1, and the 3D of the operation matrix of integration shown in Fig.2

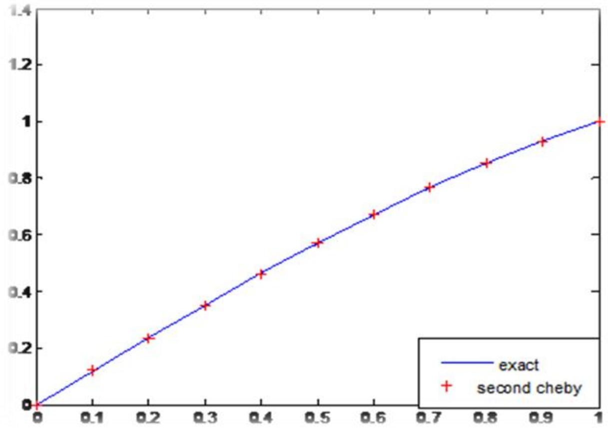

Fig.1. Shows the Comparison results with exact

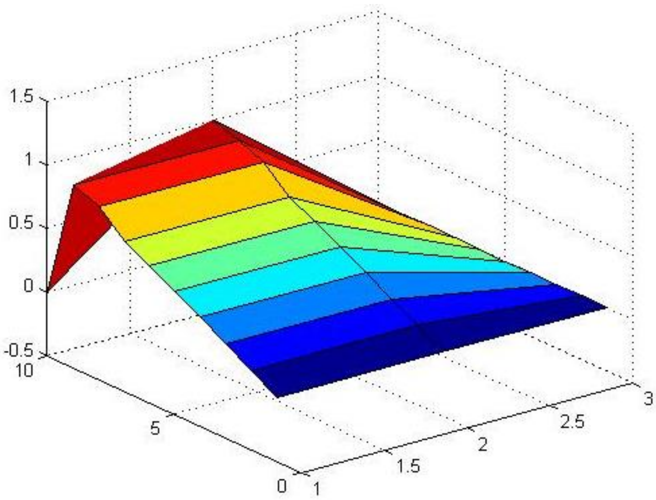

Fig.2. The 3D of the operation matrix of integration

t	Exact solution	SCW	Absolute Error (Exact-SCW)
0	1	0.99165117	0.00834883
0.1	1.1868265	1.18746056	0.00063406
0.2	1.36079545	1.36143651	0.00064106
0.3	1.51916945	1.51857901	0.00059044
0.4	1.65936691	1.65888805	0.00047886
0.5	1.77898785	1.78236365	0.00337585
0.6	1.87583791	1.86162549	0.01421242
0.7	1.94795021	1.93318897	0.01476124
0.8	1.99360506	1.97769611	0.01590895
0.9	2.01134714	1.9951469	0.01620024
1	2	1.98554136	0.01445864

ExampleV.II.

Consider the following boundary value problem $y'' = y$ with the boundary conditions: $y(0) = 0, y(1) = 1$, the exact solution is: $y(t) = \frac{e^{t} - e^{-t}}{e^{1} - e^{-1}}$

Similarity example (V.I) and by using our algorithm we obtained the following results in Table2

Table2. Show numerical results.

Chart2 of Table2 shows the comparison of results that we obtained

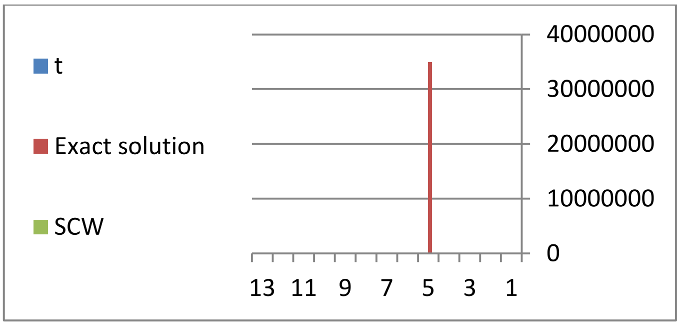

Chart2

The comparison between the solutions obtained by using SCW is made as shown in Fig.3

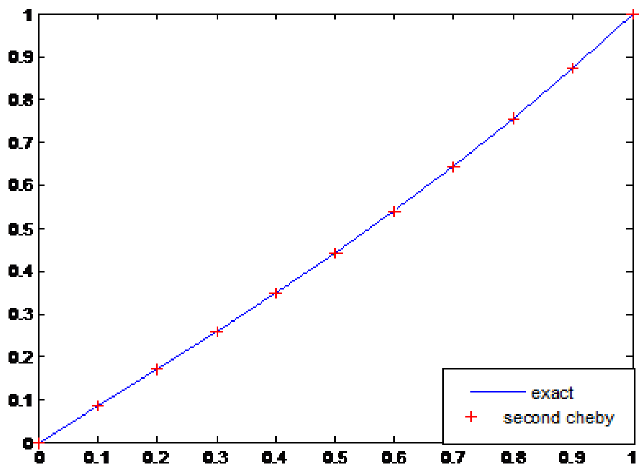

Fig.3. Shows the Comparison results with exact

ExampleV.III.

Consider the following boundary value problem $g'' + g = -r$, with the boundary conditions: $g(0) = 1$, $g(1) = 2$, the exact solution is: $g(r) = \cos r \frac{3 - \cos 1}{\sin 1} \sin r - r$

Similarity example (V.I) and by using our algorithm we obtained the following results shown in Table3

Table3. Show numerical results.

t	Exact solution	SCW	Absolute Error (Exact-SCW)
0	0	0.00114883	0.00114883
0.1	0.0852337	0.08505153	0.00018217
0.2	0.17132045	0.17111507	0.00020538
0.3	0.25912184	0.25933944	0.0002176
0.4	34951660	0.34972464	0.00020804
0.5	0.44340944	0.44276674	0.0006427
0.6	0.54174007	0.54035565	0.00138443
0.7	0.64549262	0.64402083	0.00147179
0.8	0.75570548	0.75470746	0.0009985
0.9	0.87348169	0.87241554	0.00106615
1	1	0.99714506	0.00285494

Chart3 of Table3 shows the comparison of results that we obtained

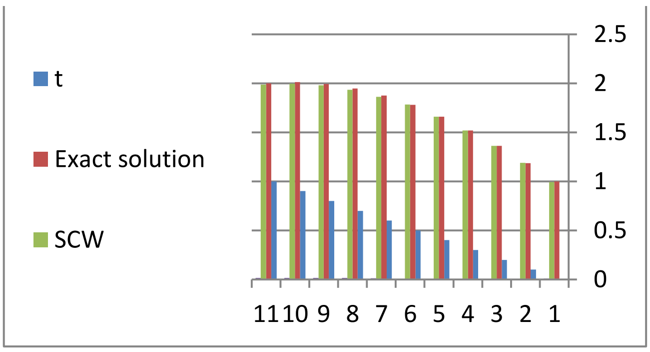

Chart3

The comparison between the solutions obtained by using SCW is made as shown in Fig.4

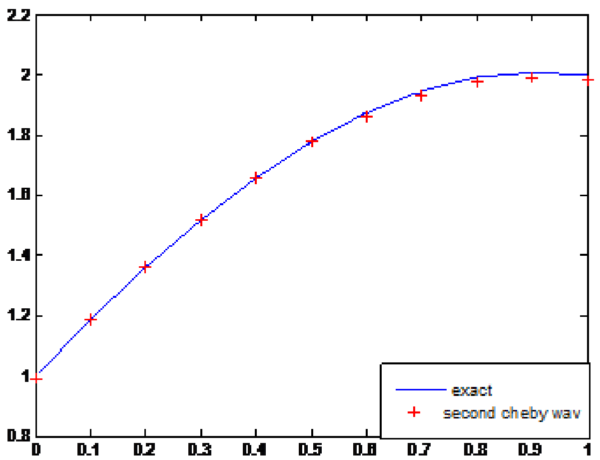

Fig.4. Shows the Comparison results with exact

VI. Signal processing

To transfer information and data need large areas and a large time in addition to a loss of information during transportation to avoid these problems recently discovered discontinuous waves because they have a smooth behavior through contraction and expansion, which leads to the discovery of a typical or good filter for use in the process of signal compression where the signal is divided into rounding and detail parameters, which reduces the area the signal takes without losing signal information.

VI.I Compression Signal by wavelets filter

In this section the intermittent wavelet filter will be applied in the field of signal compression using the Matlab program. An example illustrates this technique, Table(4) shows the most important results obtained by Global thresholding method with two methods connected with main method there are Balance Sparsity and Remove near zero. Through the terms that are relied upon to demonstrate the efficiency of the proposed method is norm threshold, retained energy, number of zero, L^1 Norm and L^2 Norm.

The proposed algorithm in signal processing is that the integral matrix is entered into the input signal it is compressed to reduce the area and speed of information transfer. As for when referring to the original signal without loss the inverse of the matrix is used the signal is obtained without losses. Fig.5 shows the original signal with wavelet, and Fig.6 shows the compression of signal by wavelet with Balance Sparsity – norm, Fig.7 shows the compression of signal by wavelet with Remove near zero and Table4 shows the most important results obtained. The algorithm 1 shows the stages of the process of applying the proposed method of signal processing.

Algorithm 1: The signal processed using (OMI) for (SCW).

Input: original signal

Output: compression signal

Step 1: the operation matrix of integration for second Chebyshev wavelet is constructed

Step 2: download OMI in MATLAB program by design program in command window

Step 3: download original signal and cross with OMI the coefficients are divided into approximate coefficients and details coefficients, where the coefficients of approximate represent the signal, either zero and negative numbers represent the details, the signal will be compressed

Step 4: After passing the inverse OMI, the original signal will be referenced without losing the original information of the signal after it is compressed

End Algorithm

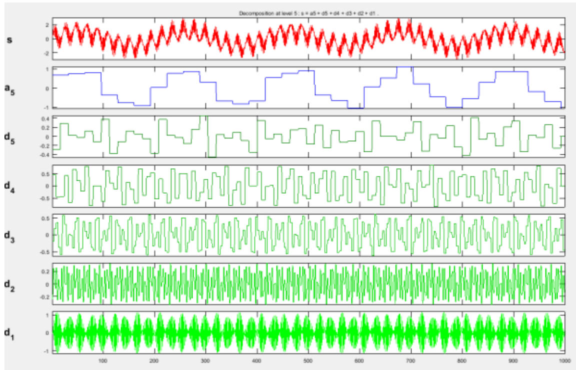

Fig.5. The original signal with wavelet

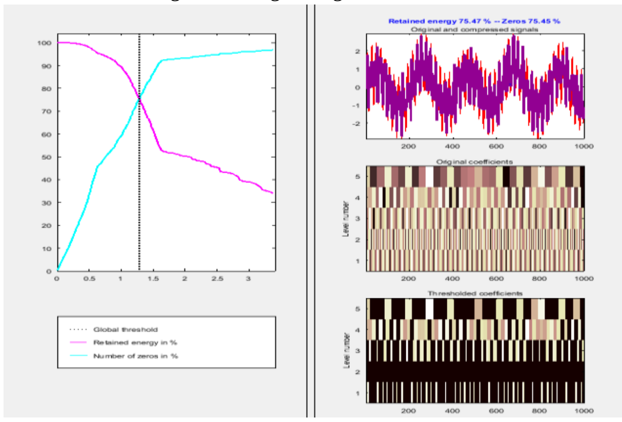

Fig.7. The compression of signal by wavelet with Remove near zero

Table4. Shows the most important results obtained

Method of threshold	**Norm threshold**	**Retained energy**	**Number of zero**	**L^1Norm**	**L^2Norm**
Balance Sparsity -norm	1.28	75.47%	75.45%	320.5	12.69
Remove near zero	0.9907	89.58%	58.88%	320.5	12.69

VII. Conclusion

In this paper presented for SCW and its OMI, the numerical results show an algorithm very efficient for the numerical solution of BVP_s and we obtained a good approximate solution for these problems.

In section IV Matrix (6×6) was used because *M=3* was taken, where six functions were obtained, and when these functions were integrated, the operations matrix of integration (6×6) was added, which was added to solve problems Boundary Value Problems (BVP_s) numerically. In addition to solving problems numerically, using the proposed technique, which is signal processing, to demonstrate the efficiency of the proposed theory.

As indicated in section II, wavelets are built on the dependence of the four effects(n, m, k, r). In addition, the number of equations obtained is calculated based on the value of $M$where $m = 0,1,2,3, \dots, M-1$ six functions are obtained and the greater the value of M is obtained more functions, leading to greater accuracy in obtaining the best results.

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