Applications of Novel Integral transform: 'Kuffi-Abbas-Jawad' (KAJ) Transform to Damped Mechanical and Electrical Oscillators

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Abstract— *This study utilizes the 'Kuffi-Abbas-Jawad' (KAJ) transform to determine the responses of mechanical and electrical oscillators. The paper introduces the KAJ transform as a novel approach for analyzing simple harmonic oscillators, as well as damped mechanical and electrical oscillators. Similar to other integral transform techniques, the 'Kuffi-Abbas-Jawad' (KAJ) transform serves as a practical and effective mathematical tool for obtaining these responses.*

Keywords— 'Kuffi-Abbas-Jawad' (KAJ) transform, and 'Kuffi-Abbas-Jawad' (KAJ) transform inverse, Damped Mechanical and Electrical Oscillators, Differential Equation Responses.

I. INTRODUCTION

All the applied fields like chemistry, physics, and many other fields depend prominently on differential equations. Integral transform methods provide effective ways for solving a variety of problems arising in applied sciences and engineering. Differential equations (DEs) play a crucial role across various scientific and engineering disciplines, as they mathematically represent changes occurring in complex problems. These equations incorporate derivatives and can be solved by expressing relationships between constants and interdependent variables. By doing so, DEs serve as predictive models for underlying scientific and engineering phenomena [1,2]. Specified the significance of differential equations, numerous methods have been developed to solve them, including mathematical transforms [3-7] and the exploration and introduction of new approaches for solving these equations continue to evolve, ensuring ongoing advancements in the field. Typically, the analysis of damped mechanical and electrical oscillators utilizes various integral transforms and approaches, such as the Convolution theorem method, Laplace Transform, Convolution Method, Matrix Method, Residue Theorem Approach, and Gupta Transform [8-20], etc. Gupta et al. studied RL-series and RC networks with steady excitation sources using the Gupta Transform [21-22]. Integral transformations have shown to be a highly effective tool in applied mathematics and engineering science over the past 200 years. Due to their popularity, integral transformations are widely recognized for their part in the solution of integral, difference equations, and other scientific issues [23-29]. The common Laplace integral is where the KAJ-transformation is derived from [30]. We discusses the theory of mechanical and electrical oscillators to obtain their responses through the application of a new integral transform called the 'Kuffi-Abbas-Jawad' (KAJ) transform. Kuffi et.al.[31-33], used New integral 'Kuffi-Abbas-Jawad' (KAJ) transform, which is a modified version of the Sadiq-Emad-Eman (SEE) integral transform proposed in 2021, and obtained the solution of the applications containing ordinary differential equations. This study introduces a novel transform known as Kuffi-Abbas-Jawad' (KAJ) transform for analyzing the responses of simple harmonic oscillator, damped mechanical and electrical oscillators. Unlike the SEE transform, which relies solely on the variable , the 'Kuffi-Abbas-Jawad' (KAJ) transform incorporates two variables, and . This enhancement significantly improves the effectiveness of determining the responses of, mechanical and electrical oscillators.

II. BASIC EQUATIONS

The thorough description and formulation for the SEE Integral Transform is redefined or derived by using Ja'far transformation method and its inverse transformation, and simplify the foundational principles.

A. Integral Transforms

Dignified operator that converts a function into another function with the help of integration in contradiction of a kernel known as an integral transform and it is expressed as:

$$I\{f(t)\} = \int_a^b f(t)\, K(s,t) dt = F(s) \qquad (1)$$

where: $f(t)$ be the original function, $K(s,t)$be the kernel of the transform, $F(s)$The transformed function, a and b limits depend on the type of transform (Fourier, Laplace, etc.).

B. Ja'far Transformation Method:

The Unifying approach that provides a generalized kernel framework for defining new transforms which is known as Ja'far Transformation Method. It forms upon the following structure:

$$J\{f(t)\} = \int_0^\infty f(t)\,\Phi(s,t)dt = F(s) \quad (2)$$

where: $f(t)$ be the original function, $\Phi(s,t)$ be the generalized kernel transform and it derive various transforms like (Laplace, Fourier, KAJ, SEE, etc., $F(s)$ be the transformed value.

C. Deriving SEE Transform Using Ja'far Method:

SEE Transform in terms of Ja'far transformation by choosing the appropriate kernel as $\Phi(s,t) = e^{-\eta t}$, therefore, SEE transform is a special case of the Ja'far transform with the exponential- kernel.

D. SEE Integral Transform Definition [31-32]

In 2021, Eman A Mansour *et al.* introduced the 'SEE Integral Transform,' which is applied to the functions $f(t)$ for solving differential equations and other mathematical problems for $t \geq 0$, the SEE transform in exponential form is defined as:

$$S\{f(t)\} = \frac{1}{\eta^n}\int_0^\infty e^{-\eta t} f(t)dt = T(\eta), t \geq 0, \eta \neq 0, n \in Z, l_1 \leq \eta \leq l_2 and l_1, l_2 > 0. \quad (3)$$

where, the operator S is called SEE transform operator.

E. Tabulated values of SEE Integral Transform for Frequently Encountered Functions:

Function $f(t)$	β	t^m	$e^{\alpha t}$	$\sin at$	$\sinh at$	$\cos at$	$\cosh at$
$T(\eta) = S[f(t)]$	$\frac{\beta}{\eta^{n+1}}$	$\frac{m!}{\eta^{n+m+1}}$ $m \in N$	$\frac{1}{\eta^n(\eta - a)}$ $a \in R$	$\frac{\alpha}{\eta^n(\eta^2+\alpha^2)}$	$\frac{\alpha}{\eta^{n1}(\eta^2-\alpha^2)}$	$\frac{\eta}{\eta^n(\eta^2+\alpha^2)}$	$\frac{\eta}{\eta^n(\eta^2-\alpha^2)}$

F. Derivative of SEE Integral Transform[31-32]:

mth derivatives of SEE integral transform is as:

$$S[f^m(t)] = -\frac{f^{(m-1)}(0)}{\eta^n} - \frac{f^{(m-2)}(0)}{\eta^{n-1}} - \dots\dots\dots - \frac{f(0)}{\eta^{n-m+1}} + \eta T(\eta) \quad (4)$$

G. Definition of 'Kuffi-Abbas-Jawad' (KAJ) Transform [33]:

Kuffi-Abbas-Jawad' (KAJ) Integral transform, which is a modification of the Sadiq-Emad-Eman (SEE) integral transform proposed in the year, 2021, KAJ transform is defined below as:

$$K_J[f(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt = K(\eta), t \geq 0, \eta \neq 0, n \in Z, 0 < n_1 \leq \eta < n_2, \quad (5)$$

where n_1 and n_2 are either finite or infinite.

H. Inverse of 'Kuffi-Abbas-Jawad' (KAJ) Transform [33]:

Inverse of 'Kuffi-Abbas-Jawad' (KAJ) transform from equation (6), defined as below:

$$f(t) = K_J^{-1}\left\{\frac{1}{\eta}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt\right\} = K_J^{-1}\{K(\eta)\}, t \geq 0, \eta \neq 0 \quad (6)$$

with the help of 'Kuffi-Abbas-Jawad' (KAJ) transform, we can easily solve the application of simple harmonic oscillators, as well as damped mechanical and electrical oscillators., containing ordinary differential equations.

I. Theoretical Foundation: Riemann Integral Framework:

On the way to build and confirm the strength of the KAJ Transform, we depend on on the Riemann integral, which requires, Riemann integrability conditions as:
Let $f: [0,\infty) \to R$ be a function, then f is Riemann integrable on any finite interval $[0,a]$, if:
(i) F is bounded on $[0,a]$
(ii) F has at most a finite number of discontinuities on $[0,a]$.

KAJ transform contains integration over $[0,\infty)$, which is define as an improper Riemann integral.

Theorem1. Existence of the KAJ Transform: Let $f: [0,\infty) \to R(or C)$ be a piecewise continuous and of exponential order α , then there exist constants $M, \alpha, T > 0$ such that
$|f(t)| \leq Me^{\alpha t}$, for all $t > T$. Prove that KAJ transform:

$$K_J(f(t)) = \int_0^\infty f(t)\, t^n e^{-st}dt, \text{ exist for all } s > \alpha. \quad (7)$$

Proof of Existence: I am using comparison test for improper Riemann integrals. Let $f(t)$ be of exponential function of order α , then $|f(t)| \leq Me^{\alpha t}$ then for $s > \alpha$, we get

$$|f(t)t^n e^{-st}| \leq Mt^n e^{\alpha t} e^{-st} = Mt^n e^{(\alpha - s)t} \quad (8)$$

Now study the integral: $\int_0^\infty t^n\, e^{(\alpha-s)}dt$, which is known as a convergent gamma-type integral.

Therefore, the integrand is absolutely integrable, and by the comparison test, the KAJ transform converges. Hence, $K_J(f(t)) = \int_0^\infty f(t)\, t^n e^{-st} dt$, exist for all $s > \alpha$.

Theorem 2. Uniqueness Theorem for the KAJ Transform: Let $K_J[f(t)] = K_J[g(t)]$ for all $s > \alpha$ and both $f(t)$ and $g(t)$ are the functions of exponential order and piecewise continuous. Then $f(t) = g(t)$almost everywhere on $[0, \infty)$.

Proof: Let$h(t) = f(t)$. Then $K_J[h(t)] = \int_0^\infty h(t)\, t^n e^{-st}$ for all $s > \alpha$. This suggests the moment transform of $h(t)$vanishes; therefore, $K_J[h(t)] = \int_0^\infty h(t)\, t^n e^{-st} = 0$. It is an integral of the form of the Laplace-Stieltjes transform of t^n. Allowing to the uniqueness theorem for Laplace-like transforms, if the integral vanishes for all $s > \alpha$, then virtually everywhere.

Therefore, $f(t) = g(t)$ virtually everywhere

f(t)=g(t)almost everywheref(t) = g(t) \quad \text{almost everywhere}f(t)=g(t)almost everywhere

Hence, Uniqueness is established**.**

1.1. KAJ Integral transform for Some Basic Functions with Proof::

(i) When $f(t) = 1$, then KAJ Transform of $f(t)$is

$$K_J[f(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt = \frac{1}{\eta^n}\int_0^\infty e^{-t}\left(\frac{1}{\eta}\right) dt = \frac{1}{\eta^{n+1}}\int_0^\infty e^{-t}\, dt = \frac{1}{\eta^{n+1}}$$

(ii) When $f(t) = t$, then KAJ Transform of $f(t)$is

$$K_J[f(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt = \frac{1}{\eta^n}\int_0^\infty e^{-t}.\left(\frac{t}{\eta}\right) dt = \frac{1}{\eta^{n+1}}\int_0^\infty e^{-t}.tdt = \frac{1}{\eta^{n+1}}$$

(iii) When $f(t) = t^m$, then KAJ Transform of $f(t)$is

$$K_J[f(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt = \frac{1}{\eta^n}\int_0^\infty e^{-\eta t}.\left(\frac{t}{\eta}\right)^m dt = \frac{1}{\eta^{n+m}}\int_0^\infty e^{-t}.t^m dt = \frac{m!}{\eta^{n+m}}; m \in N.$$

(iv) When $f(t) = e^{at}$, then KAJ Transform of $f(t)$is

$$K_J[f(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt = \frac{1}{\eta^n}\int_0^\infty e^{-t}.e^{\frac{at}{\eta}} dt = \frac{1}{\eta^n}\int_0^\infty e^{-t+\frac{at}{\eta}}.dt = \frac{\eta}{\eta^n(\eta - a)}; m \in N.$$

(v) When $f(t) = sin(at)$, then KAJ Transform of $f(t)$is

$$K_J[f(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt = \frac{1}{\eta^n}\int_0^\infty e^{-t}.sin\left(\frac{at}{\eta}\right) dt = \frac{a\eta}{\eta^n(\eta^2 + a^2)} = \frac{a}{\eta^{n-1}(\eta^2 + a^2)}$$

(vi) When $f(t) = \sinh(at)$, then KAJ Transform of $f(t)$ is

$$K_J[f(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt = \frac{1}{\eta^n}\int_0^\infty e^{-t}.sinh\left(\frac{at}{\eta}\right) dt = \frac{a\eta}{\eta^n(\eta^2 - a^2)} = \frac{a}{\eta^{n-1}(\eta^2 - a^2)}$$

(vii) When $f(t) = cos(at)$, then KAJ Transform of $f(t)$is

$$K_J[f(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt = \frac{1}{\eta^n}\int_0^\infty e^{-t}.cos\left(\frac{at}{\eta}\right) dt = \frac{\eta^2}{\eta^n(\eta^2 + a^2)} = \frac{1}{\eta^{n-2}(\eta^2 + a^2)}$$

(viii) When $f(t) = cos\,h\,(at)$, then KAJ Transform of $f(t)$is

$$K_J[f(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt = \frac{1}{\eta^n}\int_0^\infty e^{-t}.cosh\left(\frac{at}{\eta}\right) dt = \frac{\eta^2}{\eta^n(\eta^2 - a^2)} = \frac{1}{\eta^{n-2}(\eta^2 - a^2)}$$

1.2. Linearity Property of 'Kuffi-Abbas-Jawad' (KAJ) Transform [33]:

If $K_1(\eta)$ and $K_2(\eta)$ respectively, are the 'Kuffi-Abbas-Jawad' (KAJ) transform of functions $f_1(t)$ and $f_2(t)$,

Therefore, $\Rightarrow K_J[Ef_1(t) + Ff_2(t)] = EK_1(\eta) + FK_2(\eta)$ (9)

1.3. Derivatives of 'Kuffi-Abbas-Jawad' (KAJ) Transform [33]:

$$\left.\begin{aligned} &\text{First derivative:} K_J[f'(t)] = \eta K(\eta) - \frac{\eta}{\eta^n} f(0) = K_J\left[\frac{df(t)}{dt}\right] \\ &\text{Second derivative:} K_J[f''(t)] = \eta^2 K(v) - \frac{1}{\eta^n}[\eta^2 f(0) + \eta f'(0)] \\ &\qquad = K_J\left[\frac{d^2 f(t)}{dt^2}\right] \end{aligned}\right\} \quad (10)$$

Theorem 2.7(A). First derivative:$K_J[f'(t)] = \eta K(\eta) - \frac{\eta}{\eta^n} f(0) = K_J\left[\frac{df(t)}{dt}\right]$;

Where, $K(\eta) = K_J[f(t)]$

Proof: We known from the definition of KAJ transform

$$K_J[f(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt = K(\eta) t \geq 0, \eta \neq 0 \quad (11)$$

Taking first derivative of equation (11), we get

$$K_J[f'(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f'\left(\frac{t}{\eta}\right) dt \quad (12)$$

Using integration by parts in equation (12)

Assume that$u = e^{-t}, du = -e^{-t}$,

$$d\eta = f'\left(\frac{t}{v}\right), \eta = \eta f\left(\frac{t}{\eta}\right)$$

Therefore,

$$K_J[f'(t)] = \frac{1}{\eta^n}\left[\eta e^{-t} f\left(\frac{t}{\eta}\right)\Big|_0^\infty + \int_0^\infty \eta f\left(\frac{t}{\eta}\right) e^{-t} dt\right]$$

$$K_J[f'(t)] = \frac{1}{\eta^n}[0 - \eta f(0) + \eta K(\eta)] = \eta K(\eta) - \frac{\eta}{\eta^n} f(0)$$

$$K_J[f'(t)] = \eta K(\eta) - \frac{\eta}{\eta^n} f(0) \quad (13)$$

Theorem 2.7(B): Second derivative: $K_J[f''(t)] = \eta^2 K(v) - \frac{1}{\eta^n}[\eta^2 f(0) + \eta f'(0)] = K_J\left[\frac{d^2 f(t)}{dt^2}\right]$

where, $K(\eta) = K_J[f(t)]$

Proof: We known from the definition of KAJ transform

$$K_J[f(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt = K(\eta) t \geq 0, \eta \neq 0 \quad (14)$$

Taking second derivative of equation (14), we get

$$K_J[f''(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f''\left(\frac{t}{\eta}\right) dt \quad (15)$$

Using integration by parts in equation (15)

Assume that $u = e^{-t}$, $du = -e^{-t}$,

$$d\eta = f'\left(\frac{t}{v}\right), \eta = \eta f\left(\frac{t}{\eta}\right)$$

Therefore, from equation (15), we get

$$K_J[f''(t)] = \frac{1}{\eta^n}\left[\eta e^{-t} f'\left(\frac{t}{\eta}\right)\right\}_0^\infty + \int_0^\infty e^{-t} f'\left(\frac{t}{\eta}\right) dt\right]$$

$$K_J[f''(t)] = \frac{1}{\eta^n}\left[\eta e^{-t} f'\left(\frac{t}{\eta}\right)\right\}_0^\infty + \left[e^{-t} f\left(\frac{t}{\eta}\right)\right\}_0^\infty + \int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt\right]\right]$$

$$K_J[f'(t)] = \frac{1}{\eta^n}[-\eta^2 f(0)| - \eta f'(0)] + \eta^2 K(\eta).$$

$$K_J[f'(t)] = \frac{1}{\eta^n}[0 - \eta f(0) + \eta K(\eta)] = \eta K(\eta) - \frac{\eta}{\eta^n} f(0) \quad (16)$$

J. Tabulated Values of 'Kuffi-Abbas-Jawad' (KAJ) Transform [33]:

The KAJ transform denoted by $K_J(\eta)$, be the modification of the SEE integral transform, that is given by $K_J[f(t)] = \frac{1}{\eta^n}\int_0^\infty e^{-t} f\left(\frac{t}{\eta}\right) dt = K(\eta), t \geq 0, \eta \neq 0, n \in Z, 0 < n_1 \leq \eta < n_2$,

where n_1 and n_2 are either finite or infinite.

Kuffi-Abbas-Jawad(KAJ) transform and its Inverse used for frequently encountered mathematical functions are given in Table -2.8.1. and Table -2.8.2.

Function $f(t)$	β	t^m	$e^{\alpha t}$	$\sin at$	$\sinh at$	$\cos at$	$\cosh at$
$K(\eta) = K_J[f(t)]$	$\frac{\beta}{\eta^n}$, $\beta \in R$	$\frac{m!}{\eta^{n+m}}$ $m \in N$	$\frac{1}{\eta^{n-1}(\eta - \alpha)}$ $\alpha \in R$	$\frac{\alpha}{\eta^{n-1}(\eta^2 + \alpha^2)}$	$\frac{\alpha}{\eta^{n-1}(\eta^2 - \alpha^2)}$	$\frac{1}{\eta^{n-2}(\eta^2 + \alpha^2)}$	$\frac{1}{\eta^{n-2}(\eta^2 - \alpha^2)}$

Table -2.8.1

'Kuffi-Abbas-Jawad' (KAJ) transform inverse of function $N(t)$ defined as $N(t) = K_J^{-1}\{K(\eta)\}$ and Some Standard results of 'Kuffi-Abbas-Jawad' (KAJ) transform nverse of the functions are listed in Table 2.8.2.

$K(\eta) = K_J[f(t)]$	$\frac{\beta}{\eta^n}$, $\beta \in R$	$\frac{m!}{\eta^{n+m}}$ $m \in N$	$\frac{1}{\eta^{n-1}(\eta - \alpha)}$ $\alpha \in R$	$\frac{\alpha}{\eta^{n-1}(\eta^2 + \alpha^2)}$	$\frac{\alpha}{\eta^{n-1}(\eta^2 - \alpha^2)}$	$\frac{1}{\eta^{n-2}(\eta^2 + \alpha^2)}$	$\frac{1}{\eta^{n-2}(\eta^2 - \alpha^2)}$
$f(t) = K_J^{-1}\{K(\eta)\}$	β	t^m	$e^{\alpha t}$	$\sin at$	$\sinh at$	$\cos at$	$\cosh at$

Table – 2.8.2

III. MATERIAL AND TECHNIQUE:

Integral transform techniques are usually used for obtaining the solutions of initial value problems stated in terms of differential equations. In this paper, the 'Kuffi-Abbas-Jawad' (KAJ) transform employing for solving the damped mechanical oscillator and damped electrical oscillator equations which are expressed in terms of differential equations, coming to light in the field of engineering, basic sciences and material sciences.

A. Application. **Applications of 'Kuffi-Abbas-Jawad' (KAJ) transform in damped mechanical oscillator:**

Consider the application of damped mechanical oscillator which is in the form of differential equation [13, 15] as given below:

$$\frac{d^2u(t)}{dt^2} + 2\lambda\frac{du(t)}{dt} + \omega^2 u(t) = 0 \quad (17)$$

where, For a large damping $\lambda < \omega$ and $\omega = \sqrt{\frac{\kappa}{m}}$, be the natural frequency of the oscillator,

$2\lambda = \frac{r}{m}$, be the damping constant per unit mass.

Assuming the boundary condition as:[13],[15]

(i) At the time $t = 0$, displacement of the damped mechanical oscillator be $u(0) = 0$

(ii) At the time $t = 0$, velocity of the damped mechanical oscillator be $\frac{du(0)}{dt} = \eta_0$.

Relating the 'Kuffi-Abbas-Jawad' (KAJ) transform to the equation (17), both the sides, we have

$$\boldsymbol{K_J\left(\frac{d^2u(t)}{dt^2}\right) + 2\lambda K_J\left(\frac{du(t)}{dt}\right) + \omega^2 K_J(u(t)) = 0} \quad (18)$$

Let$K_J[u(t)] = K(\eta)$; using derivative property results of 'Kuffi-Abbas-Jawad' (KAJ) transform from equation (10), in equation (18), we get

$$\eta^2K(\eta) + \frac{1}{\eta^n}\left(-\eta^2u(0) - \eta u'(0)\right) + 2\lambda\left(\eta K(\eta) - \frac{\eta u(0)}{\eta^n}\right) + \omega^2K(\eta) = 0 \quad (19)$$

With the condition that at time $t = 0$, the displacement and velocity are as :

$u(0) = 0, at t = 0,$and$\frac{du(0)}{dt} = \eta_0$, using these value in equation (19), we acquire:

$$\eta^2K(\eta) + \frac{1}{\eta^n}(-\eta^2(0) - \eta\eta_0) + 2\lambda\left(\eta K(\eta) - \frac{\eta(0)}{\eta^n}\right) + \omega^2K(\eta) = 0$$

$$\eta^2K(\eta) + \frac{1}{\eta^n}(-\eta\eta_0) + 2\lambda(\eta K(\eta)) + \omega^2K(\eta) = 0$$

$$K(\eta) = \frac{\eta_0}{\eta^{n-1}}\left(\frac{1}{\eta^2 + 2\lambda\eta + \omega^2}\right) \quad (20)$$

$$K(\eta) = \frac{\eta_0}{\eta^{n-1}(\eta+\eta_1)(\eta+\eta_2)} \quad (21)$$

where, $\eta_1 = \lambda - i\sqrt{\omega^2 - \lambda^2}and\eta_2 = \lambda + i\sqrt{\omega^2 - \lambda^2}$

Therefore, $\eta_2 - \eta_1 = 2i\sqrt{\omega^2 - \lambda^2}$.

After simplification from equation (21), we obtain:

$$K(\eta) = \frac{\eta_0}{(\eta_2-\eta_1)}\left(\frac{1}{\eta^{n-1}(\eta+\eta_2)}\right) - \frac{\eta_0}{(\eta_2-\eta_1)}\left(\frac{1}{\eta^{n-1}(\eta+\eta_1)}\right) \quad (22)$$

Relating the inverse 'Kuffi-Abbas-Jawad' (KAJ) transform to both the side of equation (22), we have

$$K_J^{-1}(K(\eta)) = \frac{\eta_0}{(\eta_2 - \eta_1)}\left\{K_J^{-1}\left(\frac{1}{\eta^2(\eta + \eta_2)}\right) - K_J^{-1}\left(\frac{1}{\eta^2(\eta + \eta_1)}\right)\right\}$$

$$u(t) = \frac{\eta_0}{(\eta_2-\eta_1)}\{e^{-\eta_2 t} - e^{-\eta_1 t}\} \quad (23)$$

Substituting the values of η_1 *and* η_2 and the value of$\eta_2 - \eta_1 = 2i\sqrt{\omega^2 - \lambda^2}$ in equation (23), we get

$$u(t) = \eta_0 e^{-\lambda t}\left\{\frac{e^{i\left(\sqrt{\omega^2-\lambda^2}\right)t} - e^{-i\left(\sqrt{\omega^2-\lambda^2}\right)t}}{2i\sqrt{\omega^2 - \lambda^2}}\right\}$$

$$u(t) = \frac{\eta_0 e^{-\lambda t}}{\sqrt{\omega^2 - \lambda^2}}\left\{\frac{e^{i\left(\sqrt{\omega^2-\lambda^2}\right)t} - e^{-i\left(\sqrt{\omega^2-\lambda^2}\right)t}}{2i}\right\}$$

$$u(t) = \frac{\eta_0 e^{-\lambda t}}{\sqrt{\omega^2-\lambda^2}} sin\left(\sqrt{\omega^2 - \lambda^2}\right) \quad (24)$$

Equation (24) gives us the displacement oscillatory of the motion of a lightly damped oscillator with constant amplitude. For an over damped oscillator [3], $\lambda > \omega$, therefore replacing $\sqrt{\omega^2 - \lambda^2}$ by $i\sqrt{\omega^2 - \lambda^2}$in equation(24), we obtain the displacement of an overdamped oscillator as:

$$u(t) = \frac{\eta_0 e^{-\lambda t}}{i\sqrt{\omega^2 - \lambda^2}} sin\left(i\sqrt{\omega^2 - \lambda^2}\right)$$

$$u(t) = \frac{\eta_0 e^{-\lambda t}}{\sqrt{\lambda^2-\omega^2}} sinh\left(\sqrt{\omega^2 - \lambda^2}\right) \quad (25)$$

The equation (25) provides the response of heavily damped oscillator and reveals that the motion of heavily damped oscillator is non-oscillatory.

*B. Application (*Application of 'Kuffi-Abbas-Jawad' (KAJ) transform in damped electrical oscillator):

Consider the application of Damped Electrical Oscillator which is in the form of differential equation [12, 13, 17], as given below:

$$\boldsymbol{\frac{d^2Q(t)}{dt^2} + 2\lambda\frac{dQ(t)}{dt} + \omega^2Q(t) = 0} \quad (26)$$

where, $\omega = \sqrt{\frac{1}{LC}}$, be the angular frequency of the electrical oscillator, $2\lambda = \frac{R}{L}$, be the damping coefficient, $Q(t)$be the charge at any instant. For a large damping $\lambda < \omega$.

Assuming the boundary condition as:[17]

(i) At the time $t = 0$, charge of the damped electrical oscillator be $Q(0) = 0$

(ii) At the time $t = 0$, current in the circuit $\frac{dQ(0)}{dt} = i_0$.

Relating the 'Kuffi-Abbas-Jawad' (KAJ) transform to the equation (26), both the sides, we have

$$\boldsymbol{K_J^{-1}\left(\frac{d^2Q(t)}{dt^2}\right) + 2\lambda K_J^{-1}\left(\frac{dQ(t)}{dt}\right) + \omega^2 K_J^{-1}(Q(t)) = 0} \quad (27)$$

Let$K_J[Q(t)] = K(\eta)$;Using the derivative property results of 'Kuffi-Abbas-Jawad' (KAJ) transform from equation (10), in equation (27), we obtained

$$\eta^2K(\eta) + \frac{1}{\eta^n}\left(-\eta^2Q(0) - \eta Q'(0)\right) + 2\lambda\left(\eta K(\eta) - \frac{\eta Q(0)}{\eta^n}\right) + \omega^2K(\eta) = 0 \quad (28)$$

With the condition that at time $t = 0$, the displacement and velocity are as :

$Q(0) = 0, at t = 0$,and $\frac{dQ(0)}{dt} = i_0$, using these value in equation (28), we acquire:

$$\eta^2 K(\eta) + \frac{1}{\eta^n}(-\eta^2(0) - \eta i_0) + 2\lambda\left(\eta K(\eta) - \frac{\eta(0)}{\eta^n}\right) + \omega^2 K(\eta) = 0$$

$$\eta^2 K(\eta) - \frac{\eta i_0}{\eta^n} + 2\lambda\eta K(\eta) + \omega^2 K(\eta) = 0 \quad (29)$$

After oversimplification from equation (29), we have

$$K(\eta) = \frac{i_0}{\eta^{n-1}(\eta^2 + 2\lambda\eta + \omega^2)}$$

$$K(\eta) = \frac{i_0}{\eta^{n-1}(\eta^2 + \eta_1)(\eta^2 + \eta_2)} \quad (30)$$

where, $\eta_1 = \lambda - i\sqrt{\omega^2 - \lambda^2}$ and $\eta_2 = \lambda + i\sqrt{\omega^2 - \lambda^2}$

Therefore, $\eta_2 - \eta_1 = 2i\sqrt{\omega^2 - \lambda^2}$.

After simplification from equation (30), we obtain:

$$K(\eta) = \frac{\eta_0}{(\eta_2 - \eta_1)}\left\{\left(\frac{1}{\eta^{n-1}(\eta + \eta_2)}\right) - \left(\frac{1}{\eta^{n-1}(\eta + \eta_1)}\right)\right\} \quad (31)$$

Relating the inverse 'Kuffi-Abbas-Jawad' (KAJ) transform to equation (31), we have

$$K_J^{-1}\{K(\eta)\} = \frac{\eta_0}{(\eta_2 - \eta_1)}\left\{K_J^{-1}\left(\frac{1}{\eta^{n-1}(\eta + \eta_2)}\right) - K_J^{-1}\left(\frac{1}{\eta^{n-1}(\eta + \eta_1)}\right)\right\}$$

$$Q(t) = \left(\frac{i_0}{\beta_2 - \beta_1}\right)\left\{e^{-\beta_2 t} - e^{-\beta_1 t}\right\} \quad (32)$$

Substituting the values of β_1 *and* β_2 and the value of $\beta_2 - \beta_1 = 2i\sqrt{\omega^2 - \lambda^2}$ in equation (32), we get

$$Q(t) = i_0 e^{-\lambda t}\left\{\frac{e^{i\left(\sqrt{\omega^2 - \lambda^2}\right)t} - e^{-i\left(\sqrt{\omega^2 - \lambda^2}\right)t}}{2i\sqrt{\omega^2 - \lambda^2}}\right\}$$

$$Q(t) = \frac{i_0 e^{-\lambda t}}{\sqrt{\omega^2 - \lambda^2}}\left\{\frac{e^{i\left(\sqrt{\omega^2 - \lambda^2}\right)t} - e^{-i\left(\sqrt{\omega^2 - \lambda^2}\right)t}}{2i}\right\}$$

$$Q(t) = \frac{i_0 e^{-\lambda t}}{\sqrt{\omega^2 - \lambda^2}} sin\left(\sqrt{\omega^2 - \lambda^2}\right) t \quad (33)$$

When $R = 0$, then $\lambda = \frac{R}{2L} = 0$, therefore, equation (34), reduces to $Q(t) = \frac{i_0}{\omega} sin\,\omega\, t$ (34)

This equation (34) describes the response of a damped electrical oscillator. The oscillator exhibits oscillatory behavior, with its charge oscillating while the amplitude gradually decreases over time in an exponential manner. This reduction in amplitude, known as damping, is influenced by the resistance R present in the circuit—a phenomenon referred to as resistance damping [12]. If $R = 0$, the amplitude remains constant, indicating no energy dissipation. In an LRC circuit, resistance serves as the sole dissipative element.

IV. DISCUSSION:

The **KAJ Transform** is the innovative mathematical tool, posing a **rich structure, wide-range applicability,** and **hard hypothetical basis**. Its overview is appropriate for modern scientific fields dealing with complex dynamics, memory effects, and anomalous processes. The KAJ transform definitely used for all piecewise continuous functions of exponential order. The KAJ transform of a function exclusively determines that function virtually everywhere. The KAJ transform preserves ordinary linear and differential properties and making it manageable algebraically. The KAJ transform is well-suited for problems in the field of Fractional differential equations, Control theory, and Heat transfer with memory, Anomalous diffusion, **Electrical and mechanical systems** with time-varying coefficients.

V. CONCLUSION:

This paper explores the application of the 'Kuffi-Abbas-Jawad' **(**KAJ) transform, presenting an effective approach for solving commonly known differential equations that describe the responses of simple harmonic oscillator, damped mechanical and electrical oscillators. The proposed 'Kuffi-Abbas-Jawad' **(**KAJ) transform provides a theoretical framework for analyzing these oscillators. Additionally, an innovative method has been developed to obtain their responses in a novel and structured manner.

VI. CONFLICT OF INTEREST

The authors declare no conflict of interest.

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