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Applying Modren Fuzzy Partial Transform in Solving A Heat Equations

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Abstract— Fuzzy partial integro-differential equations are used extensively in the scientific and engineering domains. This work presents a new method for using the fuzzy Ro-transform to solve fuzzy partial differential equations. We use highly expanded notions of H-differentiability to establish formulas for fuzzy partial derivatives of first and second order. Fuzzy partial Ro-transforms are introduced at the beginning of the work, and fuzzy partial differential equations are then developed using these novel findings. Lastly, an application shows the potential of the suggested approach.

Keywords— partial Ro-transform, fuzzy partial Ro- transform, heat equation

I. INTRODUCTION

Differential equations are fundamental for modeling physical phenomena. However, constructing precise differential equation models can be a laborious and impractical process. This often makes a fuzzy math approach a more suitable alternative. Over the past few decades, fuzzy differential equations (FDEs) have seen extensive use across science and engineering. The foundational concepts of fuzzy sets, including their basic notions and arithmetic, were introduced by L. A. Zadeh in [17]. Subsequently, the ideas of fuzzy derivatives and fuzzy integration were explored in [3,10, 13], then further developed in [2,3,1,8, 14, 18], and [3,10, 19, 20], respectively.Partial differential equations (PDEs) are mathematical tools widely used in various fields like physics, engineering, chemistry, and biology. They are mathematical formulas that involve multiple variables and their derivatives. Ordinary differential equations often fall short in solving real-world problems because observing events frequently requires considering several factors simultaneously. For example, simulating heat transport through a wire necessitates accounting for both space and time. Numerous researchers [4, 6] have demonstrated analytical and numerical solutions for PDEs. Notably, fuzzy integrals can simplify the original function, making them useful for solving linear partial differential equations [1,9].

The format of this document is as follows: Along with general ideas of fuzzy numbers and fuzzy functions, it provides findings for fuzzy partial derivatives of the first and second orders using the partition Ro-transform. Systems of partial equations can be solved using these formulas.

2. BASIC CONCEPTS

Definition 1 [7] A fuzzy number expressed in parametric form ω is a pair $(\underline{\omega},\overline{\omega})$ of functions. $\underline{\omega}(\vartheta), \overline{\omega}(\vartheta)$, $0 \le \vartheta \le 1$, which satisfies the following requirements:

1. $\underline{\omega}(\vartheta)$ is a bounded left continuous that doesn't decrease. function in $(0,1]$, and continuing right at 0 .

2. $\overline{\omega}(\vartheta)$ is a right continuous at and a bounded non-increasing continuous left function in $(0,1]$ at 0.

3. $\underline{\omega}(\vartheta) \le \overline{\omega}(\vartheta)$, $0 \le \vartheta \le 1$.

Theorem 1. [15] Let $f : R \to E$ (E: all sets of ambiguous numbers) and it is portrayed by $\left[\underline{f}(\varpi;\vartheta), \overline{f}(\varpi;\vartheta)\right]$

. For any definite $\vartheta \in (0,1]$ suppose that $\underline{f}(\varpi;\vartheta)$ and $\overline{f}(\varpi;\vartheta)$ are Riemann – integrable functions

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every $c \geq d$, and consider two positive functions. $\underline{\Im}_{\vartheta}$ and $\overline{\Im}_{\vartheta}$ such that $\int_d^c \left|\underline{f}(\varpi;\vartheta)\right| d\varpi \leq \underline{\Im}_{\vartheta}$ and $\int_d^c \left|\overline{f}(\varpi;\vartheta)\right| d\varpi \leq \overline{\Im}_{\vartheta}$

for every $c \geq d$ .Then, $f(\varpi)$ is in adequate fuzzy Riemann − integrableon $[a,\infty)$ Additionally, we have: $\int_d^\infty f(\varpi)\, d\varpi = \left[\int_d^\infty \underline{f}(\varpi;\vartheta)\, d\varpi, \int_d^\infty \overline{f}(\varpi;\vartheta)\, d\varpi\right]$..

Definition2. [11] Suppose that $\omega, \nu \in \mathrm{E}$. Assuming there is $\rho \in \mathrm{E}$ such that $\omega + \nu = \rho$ then ρ referred to such as the Hukuhara distinction. of ω and ν and it is identify by $\omega \ominus \nu$.

Definition 3. [12] Let $u:(0,\infty)\times(0,\infty)\to \mathrm{E}$, is side to be first-order H-differentiable at $\delta_0 \in (d,c)$, with respect to δ, If such a thing exists $\frac{\partial}{\partial \delta} u(\delta_0,t)$ belongto E such that:

in $\forall h > 0$ those who are small enough $\exists u(\delta_0 + h,t)\ominus u(\delta_0,t), \exists u(\delta_0,t)\ominus u(\delta_0 - h,t)$,

then the following limit hold

$$\lim_{h\to 0^+} \frac{u(\delta_0 + h,t)\ominus u(\delta_0,t)}{h} = \lim_{h\to 0^+} \frac{u(\delta_0,t)\ominus u(\delta_0 - h,t)}{h} = \frac{\partial}{\partial \delta} u(\delta_0,t)$$

Or

in $\forall h > 0$ those who are small enough

$$\exists u(\delta_0,t)\ominus u(\delta_0 + h,t), \exists u(\delta_0 - h,t)\ominus u(\delta_0,t)$$

then the following limit hold

$$\lim_{h\to 0^+} \frac{u(\delta_0,t)\ominus u(\delta_0 + h,t)}{-h} = \lim_{h\to 0^+} \frac{u(\delta_0 - h,t)\ominus u(\delta_0,t)}{-h} = \frac{\partial}{\partial \delta} u(\delta_0,t)$$

Definition 4. [12] Let $u:(d,c)\times(d,c)\to \mathrm{E}$,is side to be H-differentiable of the first-order at $t_0 \in (d,c)$, with respect to t, if $\exists \frac{\partial}{\partial t} u(\delta,t_0)$ belongto E such that:

1. in $\forall h > 0$ sufficiently small

$$\exists u(\delta,t_0 + h)\ominus u(\delta,t_0), \exists u(\delta,t_0)\ominus u(\delta,t_0 - h)$$

then the following limit hold

$$\lim_{h\to 0^+} \frac{u(\delta,t_0 + h)\ominus u(\delta,t_0)}{h} = \lim_{h\to 0^+} \frac{u(\delta,t_0)\ominus u(\delta,t_0 - h)}{h} = \frac{\partial}{\partial t} u(\delta,t_0) .$$

Or in $\forall h > 0$ sufficiently small

$$\exists u(\delta,t_0)\ominus u(\delta,t_0 + h), \exists u(\delta,t_0 - h)\ominus u(\delta,t_0)$$

then the following limit hold

$$\lim_{h\to 0^+} \frac{u(\delta,t_0)\ominus u(\delta,t_0 + h)}{-h} = \lim_{h\to 0^+} \frac{u(\delta,t_0 - h)\ominus u(\delta,t_0)}{-h} = \frac{\partial}{\partial t} u(\delta,t_0) .$$

Definition 5. Let $u = u(\varpi,t)$ be a continuous If t is a real parameter and fuzzy-valued function, the function's fuzzy Ro-transform follows u. Denote by $\Re_t(\varpi,\upsilon)$ is defined as follows:

$$\Re_t(\varpi,\upsilon) = R_t\left[u(\varpi,t)\right] = \upsilon^2 \int_0^\infty e^{-\left(i\sqrt[\mathcal{Q}]{\upsilon}\right)t} u(\varpi,t)\, dt$$

$$= \lim_{\tau\to\infty} \upsilon^2 \int_0^\tau e^{-\left(i\sqrt[\mathcal{Q}]{\upsilon}\right)t} u(\varpi,t)\, dt,$$

$$\Re_t(\varpi,\upsilon) = \begin{bmatrix} \lim_{\tau\to\infty} \upsilon^2 \int_0^\tau e^{-\left(i\sqrt[\mathcal{Q}]{\upsilon}\right)t} \underline{u}(\varpi,t)\, dt, \\ \lim_{\tau\to\infty} \upsilon^2 \int_0^\tau e^{-\left(i\sqrt[\mathcal{Q}]{\upsilon}\right)t} \overline{u}(\varpi,t)\, dt \end{bmatrix}.$$

whenever there are limits. The $\vartheta-$cut representation of $\Re_t(\varpi,\upsilon)$ is given as:

$$\Re_t(\varpi,\upsilon;\vartheta) = R_t\left[u(\varpi,t;\vartheta)\right] = \left[\gamma\left(\underline{u}(\varpi,t)\right), \gamma\left(\overline{u}(\varpi,t)\right)\right],$$

Theorem 2. [5] Let $u:(0,\infty)\times(0,\infty)\to E$ be a function and denote by $\left[u(\varpi,t)\right]^{\vartheta} = \left[\underline{u}(\varpi,t;\vartheta), \overline{u}(\varpi,t;\vartheta)\right]$. Such that $\underline{u}(\varpi,t;\vartheta)$ and $\overline{u}(\varpi,t;\vartheta)$ are first-order differentiable functions with regard to to t, then:

1. If $u(\varpi,t)$ is the first form differentiable function, then:

$$\left[\frac{\partial}{\partial t} u(\varpi,t)\right]^{\vartheta} = \left[\frac{\partial}{\partial t} \underline{u}(\varpi,t;\vartheta), \frac{\partial}{\partial t} \overline{u}(\varpi,t;\vartheta)\right],$$

2. If $u(\varpi,t)$ is the second form differentiable function, then

$$\left[\frac{\partial}{\partial t} u(\varpi,t)\right]^{\vartheta} = \left[\frac{\partial}{\partial t} \overline{u}(\varpi,t;\vartheta), \frac{\partial}{\partial t} \underline{u}(\varpi,t;\vartheta)\right].$$

3. For the first and second orders, fuzzy partial derivative formulas.

Theorem 3. Let $u:(0,\infty)\times(0,\infty)\rightarrow E$ continuous function with fuzzy values, and u_t a portion of u its derivative with regard to t. suppose that $\upsilon^2 e^{-(i\upsilon)t}u(\varpi,t)$ and $\upsilon^2 e^{-(i\upsilon)t}u_t(\varpi,t)$ are inadequate Uncertain Riemann-integrable on $[0,\infty]$, then:

1. If u is the first form differentiable function with respect to t.

$$R_t\left[u_t(\varpi,t)\right]=(i\upsilon)R_t\left[u(\varpi,t)\right]\ominus\upsilon^2u(\varpi,0),$$

2. If u is the second form differentiable function with respect to t.

$$R_t\left[u_t(\varpi,t)\right]=-\upsilon^2u(\varpi,0)\ominus(-i\upsilon)R_t\left[u(\varpi,t)\right].$$

proof: Since $u_t(\varpi,t)$ is a continuous fuzzy-valued function, then the following two situations apply:

Case 1. Since u is the first arbitrary form differentiable function $\vartheta\in[0,1]$, From Theorem 4/1:

$$R_t\left[u_t(\varpi)\right]=\gamma_t\left[\underline{u}_t(\varpi,t;\vartheta)\right],\gamma_t\left[\overline{u}_t(\varpi,t;\vartheta)\right] \quad (1)$$

From Theorem 3/1:

$$\begin{aligned}\gamma_t\left[\underline{u}_t(\varpi,\vartheta)\right]&=(i\upsilon)\gamma_t\left[\underline{u}(\varpi,t;\vartheta)\right]-\upsilon^2\underline{u}(\varpi,0;\vartheta),\\ \gamma_t\left[\overline{u}_t(\varpi,\vartheta)\right]&=(i\upsilon)\gamma_t\left[\overline{u}(\varpi,t;\vartheta)\right]-\upsilon^2\overline{u}(\varpi,0;\vartheta).\end{aligned} \quad (2)$$

Substitute (2) in (1), then:

$$\begin{aligned}R_t\left[u_t(\varpi)\right]&=(i\upsilon)\gamma_t\left[\underline{u}(\varpi,t;\vartheta)\right]-\upsilon^2\underline{u}(\varpi,0;\vartheta),\\ &\quad(i\upsilon)\gamma_t\left[\overline{u}(\varpi,t;\vartheta)\right]-\upsilon^2\overline{u}(\varpi,0;\vartheta).\end{aligned}$$

By Theorem 1:

$$R_t\left[u_t(\varpi,t)\right]=(i\upsilon)R_t\left[u(\varpi,t)\right]\ominus\upsilon^2u(\varpi,0).$$

Case 2. Since u is a differentiable function in its second form for any random $\vartheta\in[0,1]$ From Theorem 4/2:

$$R_t\left[u_t(\varpi)\right]=\gamma_t\left[\overline{u}_t(\varpi,t;\vartheta)\right],\gamma_t\left[\underline{u}_t(\varpi,t;\vartheta)\right].$$

By same way, then:

$$R_t\left[u_t(\varpi,t)\right]=-\upsilon^2u(\varpi,0)\ominus(-i\upsilon)R_t\left[u(\varpi,t)\right].$$

Theorem 4. $u:(0,\infty)\times(0,\infty)\rightarrow E$ be continuous fuzzy-valued function Assume that, u,u_t are fuzzy partial derivatives of fuzzy Ro-transforms, which are continuous fuzzy-valued functions. with respect to t. about second order will be:

1. If u and u_t are the first form differentiable functions with respect to t then:

$$R_t\left[u_{tt}(\varpi,t)\right]=(i\upsilon)^2R_t\left[u(\varpi,t)\right]\ominus\upsilon^2(i\upsilon)u(\varpi,0)\ominus\upsilon^2u_t(\varpi,0),$$

2. If u is the first form differentiable function with respect to t and u_t second form differentiable function with respect to t, then:

$$R_t\left[u_{tt}(\varpi,t)\right]=-\upsilon^2(i\upsilon)u(\varpi,0)\ominus(-i\upsilon)^2R_t\left[u(\varpi,t)\right]\ominus\upsilon^2u_t(\varpi,0),$$

3. If u is the second form differentiable function with respect to t and u_t first form differentiable function with respect to t, then:

$$R_t\left[u_{tt}(\varpi,t)\right]=-\upsilon^2(i\upsilon)u(\varpi,0)\ominus(-i\upsilon)^2R_t\left[u(\varpi,t)\right]-\upsilon^2u_t(\varpi,0),$$

4. If u and u_t are the second form differentiable functions with respect to t, then:

$$R_t\left[u_{tt}(\varpi,t)\right]=(i\upsilon)^2R_t\left[u(\varpi,t)\right]\ominus\upsilon^2(i\upsilon)u(\varpi,0)-\upsilon^2u_t(\varpi,0).$$

proof: We will proof two cases as following:

Case 1. Since u and u_t are the first form differentiable functions with respect to t, for any arbitrary $\vartheta\in[0,1]$, From Theorem 4/1:

$$R_t\left[u_{tt}(\varpi)\right]=\gamma_t\left[\underline{u}_{tt}(\varpi,t;\vartheta)\right],\gamma_t\left[\overline{u}_{tt}(\varpi,t;\vartheta)\right].$$

$$\begin{aligned}R\left[u_{tt}(\varpi)\right]&=(i\upsilon)^2\gamma_t\left[\underline{u}(\varpi,t;\vartheta)\right]-\upsilon^2(i\upsilon)\underline{u}(\varpi,0;\vartheta)\\ &\quad-\upsilon^2\underline{u}_t(\varpi,0;\vartheta),(i\upsilon)^2\gamma_t\left[\overline{u}(\varpi,t;\vartheta)\right]\\ &\quad-\upsilon^2(i\upsilon)\overline{u}(\varpi,0;\vartheta)-\upsilon^2\overline{u}_t(\varpi,0;\vartheta).\end{aligned}$$

By Theorem 1:

$$R_t\left[u_{tt}(\varpi,t)\right]=(i\upsilon)^2R_t\left[u(\varpi,t)\right]\ominus\upsilon^2(i\upsilon)u(\varpi,0)\ominus\upsilon^2u_t(\varpi,0)$$

Case 2. Since u is the first form differentiable function with respect to t and u_t is a differentiable function in its second form for any random t, for any arbitrary $\vartheta\in[0,1]$, From Theorem 4/2:

$$R_t\left[u_{tt}(\varpi)\right]=\gamma_t\left[\overline{u}_{tt}(\varpi,t;\vartheta)\right],\gamma_t\left[\underline{u}_{tt}(\varpi,t;\vartheta)\right].$$

By Theorem 1:

$$R_t\left[u_{tt}(\varpi,t)\right]=-\upsilon^2(i\upsilon)u(\varpi,0)\ominus(-i\upsilon)^2 R_t\left[u(\varpi,t)\right]$$
$$\ominus\upsilon^2 u_t(\varpi,0)$$

To solve fuzzy partial differential equation about first order by use Ro-transform, we have:

Case 1: Let's think about u is the first form differentiable functions or u is the second form differentiable functions, then we get the subsequent:

$$R\left(\frac{\partial u(\varpi,t)}{\partial\varpi}\right)=\left[\frac{\partial \underline{u}(\varpi,t;\vartheta)}{\partial\varpi},\frac{\partial \bar{u}(\varpi,t;\vartheta)}{\partial\varpi}\right],\ 0\le\vartheta\le1.$$

Case 2: Let's think about u is the second form differentiable function, then we get the following:

$$R\left(\frac{\partial u(\varpi,t)}{\partial\varpi}\right)=\left[\frac{\partial \bar{u}(\varpi,t;\vartheta)}{\partial\varpi},\frac{\partial \underline{u}(\varpi,t;\vartheta)}{\partial\varpi}\right],\ 0\le\vartheta\le1.$$

Case 3: Let's think about u is the first form differentiable functions, then we get the subsequent:

$$R_t\left[u_t(\varpi,t)\right]=(i\upsilon)R_t\left[u(\varpi,t)\right]\ominus\upsilon^2 u(\varpi,0).$$

Case 4: Let's think about u is the first form differentiable functions, then we get the following:

$$R_t\left[u_t(\varpi,t)\right]=-\upsilon^2 u(\varpi,0)\ominus(-i\upsilon)R_t\left[u(\varpi,t)\right].$$

In this part, we'll look at example of lateral type H-differentiability with fuzzy beginning and boundary conditions.

Example: Consider the following fuzzy partial differential equation:

$$u_\varpi+u_t=0\ ;\ \text{I.C.}\quad (i)\ \ u(\varpi,0)=(\vartheta+1,3-\vartheta),$$
$$\text{B.C.}\quad (ii)\ \ u(0,t)=(2+\vartheta,4-\vartheta).$$

Applying partial Ro-transform for both sides of the original equation to get the following:

$$R_t\left[u_\varpi(\varpi,t)\right]+R_t\left[u_t(\varpi,t)\right]=0$$

1. (A_1& A_3) or (A_2& A_4).

$$\frac{\partial}{\partial\varpi}R_t\left[u(\varpi;t)\right]+(i\upsilon)R_t\left[u(\varpi;t)\right]\ominus\upsilon^2 u(\varpi;0)=0$$

Using initial conditions, and by taking the inverse of Ro-transform on both side of above equation, to get the solutions as follows.

$$\underline{u}(\varpi,t;\vartheta)=(\vartheta+1)+(2+\vartheta)u(t-\varpi)-(\vartheta+1)u(t-\varpi),$$
$$\bar{u}(\varpi,t;\vartheta)=(3-\vartheta)+(4-\vartheta)u(t-\varpi)-(3-\vartheta)u(t-\varpi).$$

2. (A_1& A_4) or (A_2& A_3).

$$\frac{\partial}{\partial\varpi}R_t\left[u(\varpi,t)\right]-\upsilon^2 u(\varpi,0)\ominus\left(-\frac{1}{i\sqrt[\Omega]{\upsilon}}\right)R_t\left[u(\varpi,t)\right]=0.$$

Using initial conditions:

$$\frac{\partial}{\partial\varpi}\gamma_t\left[\underline{u}(\varpi,t;\vartheta)\right]+\left(\frac{1}{i\sqrt[\Omega]{\upsilon}}\right)\gamma_t\left[\bar{u}(\varpi,t;\vartheta)\right]=\upsilon^2(3-\vartheta),$$
$$\frac{\partial}{\partial\varpi}\gamma_t\left[\bar{u}(\varpi,t;\vartheta)\right]+\left(\frac{1}{i\sqrt[\Omega]{\upsilon}}\right)\gamma_t\left[\underline{u}(\varpi,t;\vartheta)\right]=\upsilon^2(\vartheta+1).$$

By taking the inverse of Ro-transform on both side of above equation, to get the solutions as follows:

$$\underline{u}(\varpi,t;\vartheta)=2+(2+\vartheta)u(t-\varpi)-2u(t-\varpi),$$
$$\bar{u}(\varpi,t;\vartheta)=-(1-\vartheta)+(4-\vartheta)u(t-\varpi)-(1-\vartheta)u(t-\varpi).$$

4.Discussion

The findings of this research align with and extend existing work in the field of fuzzy differential equations. For instance, while previous studies have utilized methods like the fuzzy Laplace transform or differential transform to tackle similar problems, our approach introduces a novel fuzzy partial transform that simplifies the solution process for heat equations with complex initial or boundary conditions. This is a significant contribution, as it offers a more direct and computationally efficient tool for researchers and engineers.

5. CONCLUSION

In this study, we establish various fuzzy partial Ro-transform properties, discover partial fuzzy Ro-transform derivatives for the first and second orders, and apply these formulas to resolve fuzzy partial differential equation**.**

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