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Contra T^* - Continuous Functions

By

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Abstract

In this paper, we introduce and study contra T^* - continuous functions these are functions $f: X \to Y$ such that the inverse image of every open set in Y is T^*-closed in X. Several properties of this type of functions are proved.

1-Introduction:

Let (X,τ) be a topological space and let T: $P(X)\rightarrow P(X)$ be a function such that $W\subseteq T(W)$, for each $W\in\tau$. Then, T is called an operator associated with topology τ defined on X, and the triple $(X,\tau,T)$is called an operator topological space (O.T.S),[3]. Let $A\subseteq X$, if $A\subseteq T(A)$,then A is called a T^*-open subset of X ,[5]. The complement of T^*- open set is called a T^*- closed set.

In this work, we introduce and study contra T^*- continuous functions, these are the function f: $(X,\tau, T)\rightarrow(Y,\sigma)$ from an operator topological space (X,τ,T) into a topological space (Y,σ) such that the inverse image of every open set in Y is a T^*-closed set in X.

2-Preliminaries:

In this section, we introduce and recall the basic definitions and examples we need in this work.

2.1 Definition [1]:

Let (X,τ) be a topological space and let A be a subset of X, then the set $\cap\{U\in \tau: A\subseteq U\}$ is called the kernel of A and is denoted by $ker(A)$.

2.2 Example:

Let $X=\{1,2,3,4\}$ and $\tau=\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$. Suppose that $A=\{2, 3\}$, then $ker(A)=\{2,3\}$.

2.3 Lemma[4]:

Let (X,τ) be a topological space. Then, the following properties hold for subsets A and B of X:

i. $x\in ker(A)$ if and only if, $A\cap F\neq\phi$, for each closed set F in X and $x\in F$.
ii. if A is open in X , then $A= ker(A)$.
iii. if $A\subseteq B$, then $ker(A)\subseteq ker(B)$.

2.4 Definition [1]:

Let (X,τ, T) be any operator topological space and let $A\subseteq X$, then T^*- closure of A is defined as the intersection of all T^*- closed set which contains A and denoted by T^*-$cl(A)$.

2.5 Definition:

Let (X,τ, T) be any operator topological space, we say that :

i. X is T^*- compact, if every T^*- open cover of X has a finite sub cover,[5].
ii. X is T^*- connected, if it is not the union of two disjoint non empty T^*- open subsets of X, [5].
iii. X is T^*- strongly S-closed, if every T^*- closed cover of X has a finite sub cover,[1].

Next, we introduce a new definition:

2.6 Definition [2]:

Let (X,τ,T) be any operator topological space and let $A\subseteq X$, then A is called an IT^*- open set , if $A\subseteq int\ T(A)$. The complement of every IT^*-open set is an IT^*-closed set.

2.7 Example:

i. Let (X ,τ) be any topological space and let T: $P(X)\rightarrow P(X)$ such that: $T(A) = int\ [cl\ (A)]$, where $A\subseteq X$. The IT^*-open set in this example is exactly the pre-open set.

ii. Let (X,τ) be any topological space and let T: $P(X)\to P(X)$ such that ; $T(B) = cl\ [int\ (B)]$, where $B\subseteq X$. In this example, T^*- open set is exactly the semi- open set.

2.8 Remark:

i. Every open set is an IT^*- open set.

ii. Every IT^*-open set is a T^*-open set.

So, we have the following implication:

open $\to$ IT^*- open $\to$ T^*- open

Now, we can introduce the following definitions:

2.9 Definition:

Let (X,τ,T) be any operator topological space, we say that :

i. X is IT^*- compact, if every IT^*-open cover of X has a finite sub cover.

ii. X is IT^*- connected, if it is not the union of two disjoint non empty IT^*-open subsets of X.

iii. X is IT^*- strongly S-closed, if every IT^*-closed cover of X has a finite sub cover.

2.10 Definition:

Let (X,τ,T) and (Y,σ,L) be operator topological spaces and let f: $(X,\tau,T)\to(Y,\sigma,L)$ be a function from an operator topological space (X,τ,T) into an operator topological space (Y,σ,L). We say that f is a contra - (T^*, L^*) - continuous (resp., contra - (IT^*, IL^*) - continuous) function, if the inverse of every L^*-open (resp., IL^*-open) set in Y is a T^*-closed (resp., IL^*-closed) set in X.

2.11 Remark:

If L is the identity operator, then f will be a contra T^*- continuous function.

3-Main Results:

3.1 Theorem:

Let (X,τ,T) be an operator topological space and let (Y,σ) be a topological space. For a function $f : (X,\tau,T)\to(Y,\sigma)$ the following conditions are equivalent :

i. f is a contra T^* - continuous function.

ii. the inverse image of each closed set in Y is T^*-open in X.

iii. for each $x\in X$ and each F is closed in Y, such that $f(x)\in F$, there exists a T^*-open set U in X such that $x\in U$ and $f(U)\subseteq F$.

iv. $f\,[T^*\text{-}cl(A)]\subseteq ker\,[f(A)]$, for each $A\subseteq X$.

v. $[T^*\text{-}cl[f^{-1}(B)]]\subseteq f^{-1}[ker(B)]$, for each $B\subseteq Y$.

Proof: (i) →(ii)

Let F be any closed set in Y, then F^c is an open set in Y. Since f is a contra T^*- continuous function, then $f^{-1}(F^c)$ is a T^*- closed set in X. Then, $[f^{-1}(F^c)]^c= f^{-1}(F)$ is a T^*- open set in X.

(ii) →(iii)

Let $x\in X$ and let F be a closed set in Y such that $f(x)\in F$. From (ii) $f^{-1}(F)$ is a T^*- open set in X and $x\in f^{-1}(F)$. So, there exists a T^*- open set U in X such that $x\in U$. Thus, $x\in U\subseteq f^{-1}(F)$, for each $x\in f^{-1}(F)$. So, $f(U)\subseteq F$.

(iii) →(iv)

Let $A\subseteq X$. Suppose that $y\notin ker\,[f(A)]$. Then, by lemma (2.3) part (i) , there exists a closed set F in Y such that $y\in F$ and $f(A)\cap F=\phi$. Thus, $A\cap f^{-1}(A)=\phi$ and $T^*\text{-}cl(A)\cap f^{-1}(A)=\phi$.

Then, $f\,[T^*\text{-}\,cl(A)] \cap F=\phi$ and $y \notin f\,[T^*\text{-}\,cl(A)]$. This implies to $f\,[T^*\text{-}\,cl(A)] \subseteq ker\,[f(A)]$.

(iv) →(v)

Let $B \subseteq Y$, then $f^{-1}((B) \subseteq X$. By (iv), we have $f\,[T^*\text{-}\,cl(f^{-1}(B))] \subseteq ker\,(B)$ and $T^*\text{-}\,cl[f^{-1}(B))] \subseteq f^{-1}[\,ker\,(B)]$.

(v) →(i)

Let V be any open set in Y. By (v), we have $T^*\text{-}\,cl(f^{-1}(V)) \subseteq f^{-1}[\,ker\,(V)] = f^{-1}(V)$. But, $f^{-1}(V) \subseteq T^*\text{-}\,cl(f^{-1}(V))$. So, $T^*\text{-}\,cl(f^{-1}(V)) = f^{-1}(V)$. This means $f^{-1}(V)$ is a T^*- closed set in X. Thus, f is a contra T^*- continuous function.

<u>3.2 Remark</u>:

The equivalents of contra - (T^*, L^*) - continuous functions and contra - (IT^*, IL^*) - continuous functions are similar to theorem (3.1).

<u>3.3 Theorem</u>:

Let (X,τ,T) be an operator topological space, (Y,σ) be a topological space, $f:(X,\tau,T)\to(Y,\sigma)$ be a contra T^*- continuous surjection function and let X be a T^*- connected space, then Y is a connected space.

Proof:

Suppose that Y is disconnected. Then, there are two open sets W_1 and W_2 in Y such that $W_1\neq\phi$, $W_2\neq\phi$, $Y= W_1\cup W_2$, and $W_1\cap W_2=\phi$. Since f is contra T^*- continuous, then $f^{-1}(W_1)$ and $f^{-1}(W_2)$ are T^*- closed sets in X. Thus, $[f^{-1}(W_1)]^c$ and $[f^{-1}(W_2)]^c$ are T^*- open sets in X, also $[f^{-1}(W_1)]^c \neq\phi$, $[f^{-1}(W_2)]^c \neq\phi$, and $[f^{-1}(W_1)]^c \cap [f^{-1}(W_2)]^c =\phi$. Since f is a surjection function, $X= [f^{-1}(W_1)]^c \cup [f^{-1}(W_2)]^c$, which is a contradiction, then Y is a connected space.

<u>3.4 Corollary</u>:

Let (X,τ,T) and (Y,σ,L) be operator topological spaces, $f:(X,\tau,T)\to(Y,\sigma,L)$ be a contra - (T^*, L^*) - continuous surjection function and let X be a T^*- connected space, then Y is a L^*- connected space.

<u>3.5 Corollary</u>:

Let (X,τ,T) and (Y,σ,L) be operator topological spaces, $f:(X,\tau,T)\to(Y,\sigma,L)$ be a contra - (IT^*, IL^*) - continuous surjection function and let X be an IT^*- connected space, then Y is an IL^*- connected space.

<u>3.6 Theorem</u>:

Let (X,τ,T) be an operator topological space, (Y,σ) be a topological space, $f:(X,\tau,T)\to(Y,\sigma)$ be a contra T^*- continuous surjection function and let X be a T^*- strongly S-closed space, then Y is a compact space .

Proof:

Let $\{A_i : i\in I\}$ be any open cover of Y , then $Y = \bigcup_{i\in I} A_i$. Since f is a contra T^*- continuous surjection function, then $\{f^{-1}(A_i) : i\in I\}$ is a T^*- closed cover of X and so $X = \bigcup_{i\in I} f^{-1}(A_i)$. Since X is a T^*- strongly S-closed space, then there exists a finite subset M of I such that $X = \bigcup_{i\in M} f^{-1}(A_i)$.

Hence, $f(X) = Y = f[\bigcup_{i\in M} f^{-1}(A_i)] = \bigcup_{i\in M} A_i$, therefore Y is a compact space.

<u>3.7 Corollary</u>:

Let (X,τ,T) and (Y,σ,L) be operator topological spaces, $f:(X,\tau,T)\to(Y,\sigma,L)$ be a contra - (T^*,L^*) - continuous surjection function and let X be a T^*- strongly S-closed space, then Y is an L^*- compact space .

3.8 Corollary:

Let (X,τ,T) and (Y,σ,L) be operator topological spaces, $f:(X,\tau,T)\to(Y,\sigma,L)$ be a contra - (IT^*,IL^*) - continuous surjection function and let X be an IT^*- strongly S-closed space, then Y is a compact space .

3.9 Corollary:

Let (X,τ,T) and (Y,σ,L) be operator topological spaces, $f:(X,\tau,T)\to(Y,\sigma,L)$ be a contra - (IT^*,IL^*) - continuous surjection function and let X be an IT^*- strongly S-closed space , then Y is an IL^*- compact space .

3.10 Lemma :

If (X,τ,T) and (Y,σ,L) are two operator topological spaces , then $(X\times Y,\tau\times\sigma,T\times L)$ is also an operator topological space (where , $T\times L$: $P(X)\times P(Y)\to P(X)\times P(Y)$).

Proof:

Suppose that G be any open se in $\tau\times\sigma$. Then, $G=\bigcup_{i\in I}(G_{1i}\times G_{2i})$, we must prove that $G\subseteq T\times L(G)$. Since (X,τ,T) is (O.T.S), then $G_{1i}\subseteq T(G_{1i}),\forall i\in I$. Since (Y,σ,L) is (O.T.S), then $G_{2i}\subseteq T(G_{2i}),\forall i\in I$. So, $G_{1i}\times G_{2i}\subseteq T(G_{1i}\times G_{2i})$, $\forall i\in I$. Then, $\bigcup_{i\in I}[G_{1i}\times G_{2i}]\subseteq\bigcup_{i\in I}[T(G_{1i})\times T(G_{2i})]=\bigcup_{i\in I}[T\times L(G_{1i}\times G_{2i})]=T\times L[\bigcup_{i\in I}(G_{1i}\times G_{2i})]$. Thus, $G\subseteq T\times L(G)$. So, $(X\times Y,\tau\times\sigma,T\times L)$ is an operator topological space .

3.11 Theorem :

Let (X_1,τ_1,T_1) and (X_2,τ_2,T_2) be two operator topological spaces, (Y_1,σ_1) and (Y_2,σ_2) be two topological spaces, and let $f_1:(X_1,\tau_1,T_1)\to(Y_1,\sigma_1)$ and $f_2:(X_2,\tau_2,T_2)\to(Y_2,\sigma_2)$ be two surjection functions. If $f_1\times f_2:(X_1\times X_2,\tau_1\times\tau_2,T_1\times T_2)\to(Y_1\times Y_2,\sigma_1\times\sigma_2)$ is a contra $(T_1^*\times T_2^*)$ - continuous function, then both $f_1$and f_2 are contra T_1^*- continuous function and contra T_2^*- continuous function, respectively.

Proof:

Let G_1 be an open set in Y_1. Then $G_1\times Y_2$ is an open set in $Y_1\times Y_2$. Since $f_1\times f_2$ is a contra $T_1^*\times T_2^*$ - continuous function, then $(f_1\times f_2)^{-1}(G_1\times Y_2)$ is a $T_1^*\times T_2^*$- closed set in $X_1\times X_2$. But $(f_1\times f_2)^{-1}(G_1\times Y_2)=[f_1^{-1}(G_1)]\times[f_2^{-1}(Y_2)]=[f_1^{-1}(G_1)]\times X_2$. And since $(f_1\times f_2)^{-1}(G_1\times Y_2)$ is a $T_1^*\times T_2^*$- closed set in $X_1\times X_2$. Then, $(f_1^{-1}(G_1)\times X_2)^c=(f_1^{-1}(G_1))^c\times X_2$ is a $T_1^*\times T_2^*$- open set in $X_1\times X_2$. Then, $f_1^{-1}(G_1)$ is an T_1^*-closed set in X_1. This mean, $f_1:(X_1,\tau_1,T_1)\to(Y_1,\sigma_1)$ is a contra T_1^*- continuous function.

In similarly fashion, we can prove that $f_2:(X_2,\tau_2,T_2)\to(Y_2,\sigma_2)$ is a contra T_2^*- continuous function.

3.12 Corollary :

Let (X_1,τ_1,T_1), (X_2,τ_2,T_2), (Y_1,σ_1,L_1) and (Y_2,σ_2,L_2) be any operator topological spaces, and let f_1: $(X_1,\tau_1,T_1) \to (Y_1,\sigma_1,L_1)$ and f_2: $(X_2,\tau_2,T_2) \to (Y_2,\sigma_2,L_2)$ be any surjection functions. Now, if $f_1\times f_2$: $(X_1\times X_2,\tau_1\times\tau_2,T_1\times T_2) \to (Y_1\times Y_2,\sigma_1\times\sigma_2,L_1\times L_2)$ is a contra - $(T_1^*\times T_2^*, L_1^*\times L_2^*)$ - continuous function, then both $f_1$and f_2 are contra - (T_1^*,L_1^*)- continuous function and contra - (T_2^*,L_2^*) - continuous function, respectively.

3.13 Corollary :

Let (X_1,τ_1,T_1), (X_2,τ_2,T_2), (Y_1,σ_1,L_1) and (Y_2,σ_2,L_2) be any operator topological spaces, and let f_1: $(X_1,\tau_1,T_1) \to (Y_1,\sigma_1,L_1)$ and f_2: $(X_2,\tau_2,T_2) \to (Y_2,\sigma_2,L_2)$ be any surjection functions. Now, if $f_1\times f_2$: $(X_1\times X_2,\tau_1\times\tau_2,T_1\times T_2) \to (Y_1\times Y_2,\sigma_1\times\sigma_2,L_1\times L_2)$ is a contra - $(IT_1^*\times IT_2^*, IL_1^*\times IL_2^*)$ - continuous function, then both $f_1$and f_2 are contra - (IT_1^*,IL_1^*) - continuous function and contra - (IT_2^*,IL_2^*) - continuous function, respectively.

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الدوال المستمرة T^* - المضادة

من قبل

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الملخص

في هذا البحث ، قدمنا ودرسنا الدوال المستمرة T^* - المضادة، الدالة $f: X\to Y$ تسمى دالة مستمرة -T^* مضادة إذا كانت الصورة العكسية لكل مجموعة مفتوحة في Y تكون مغلقة-T^* في X. برهنا العديد من خصائص هذا النوع من الدوال.