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ON GENERALIZED ERIVATIONS OF SEMIPRIME NEAR-RINGS

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Abstract

The main purpose of this paper is to study and investigate concerning a generalized derivations on semiprime near- rings,we give some results when N admsit to satisfy some conditions on 3-semiprime near- rings and 3-prime near-ring N ,we give some results about that.

Keywords and phrases:near-ring,prime near-ring,semiprime near-ring, symmetric bi -(σ,τ)-derivation derivation, generalized derivation.

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1.Introduction

Near-ring are one of the generalized structures of rings.The study and research on near-rings is very systematic and continuous.Near-ring are being used since the development of Calculus,but the key idea important to near-rings were formalized in 1905 by Dickson who defined the near fields.Veblin and Wedderburn used Dickson's near field to give examples of Nondesarguesian planes.In late 1930s Wieland studied near-rings,which were not near fields. Further rich text material about the subject can be found from two famous books on the near-rings[1and 2].Near-rings abound in all directions of mathematics and continuous research iis being conducted,which shows that their structure has power and beauty all its own.Near-ring have applications in various fields such as group theory ,geometry and its other branches ,combinations,design of statistical experiments ,coding theory and cryptography.

The concept of a symmetric bi-derivation has been introduced by Maksa in [4].Some recent results on properties of prime rings, semiprime rings and near-rings with derivations has been investigated in several ways (5,6,7,8,9,10,11,12).Ozturk and Jun [5]have introduced the concept of a symmetric bi- derivation of a near-ring and studied some properties.Ceven and Ozturk [5] have introduced the concept of symmetric bi-(σ,τ)-derivation of near-ring and gave some properties,where a symmetric bi-additive mapping $D:N_{\times}N\rightarrow N$ such that $D(xy,z)=D(x,z)\sigma(y)+\tau(x)D(y,z)$ for all $x,y,z \in N$. Note that if $\sigma =1$ and $\tau =1$, then D is a symmetric bi-derivation.Golbasi [15] proved that, let (f,d) be a non-zero generalized derivation of N. If f acts as a homomorphism on N, then f is the identity map, where N is near-ring and (f,d) denote a generalized derivation $f:N\rightarrow N$ determined by a derivation d of N,d is a non-zero derivation of N and according to Golbasi [15] let N be a near-ring and d a derivation of N, an additive mapping $f:N \rightarrow N$ is said to be a right generalized derivation of N associated with d if .

(i) $f(xy)=f(x)y+xd(y)$ for all $x,y \in R$.And f is said to be a left generalized derivation of N associated with d if.

(ii) $f(xy)=d(x)y+x f(y)$ for all $x,y\in N$.Finally, f is said to be a generalized derivation of N associated with d if it is both a left and right generalized derivation of N associated with d.S.J.Abbasi and K.Iqbal[4]proved that,the right cancellation law holds in a near-ring R if and only if R is near integral domain.The main purpose of this paper is to study and investigate concerning a generalized derivations on semiprime near- rings,we give some results about that.

2. Preliminaries

Throughout this paper,according to[13]near-ring is a triple (N,+,.) satisfying the condition :

(i) (N,+)is a group which may not be a belian .

(ii) (N,.) is a semigroup .

(iii) For all $x,y,x\in N$, $x(y+z)=xy + xz$ .In fact, condition (iii). makes N a left near-ring .

If we replace (iii) by(iv) for all $x,y,z \in N$, $(x+y)z =xz+yz$, then we obtain a right near-ring N. A ring N is said to be n-torsion free, where $n\neq o$ is an integer,if whenever $nx=0$,with $x\in N$,then x=.Recall that a near ring N is 3-prime if $aNb=\{0\}$ implies that $a=0$ or $b=0$,and 3-semiprime if $aNa=\{0\}$ implies that $a=0$. A prime ring is semiprime but the converse is not true in general. σ and τ will be two near-ring automorphisms of N, for all $x,y \in N$,$[x,y]$and $[x,y]\sigma,\tau$ will denote the commutator $xy-yx$ and $\sigma(x)y-\tau(y)x$ respectively.A mapping $D:N\times N\rightarrow N$ is said to be symmetric if $D(x,y)=D(y,x)$ for all $x,y \in N$.A mapping $d;N\rightarrow N$ defined by $d(x)=D(x,x)$ is called the trace of D where $D:N\times N\rightarrow N$ is symmetric mapping.It is obvious that,if $D:N\times N\rightarrow N$ is symmetric mapping which also bi-additive (i.e.additive in both arguments),then the trace of D satisfies the relation $d(x+y)=d(x)+2D(x,y)+d(y)$ for all $x,y \in N$.Aymmetric bi-additive mapping $D:N\times N\rightarrow N$is called a symmetric bi-derivation if if $D(xy,z)=D(x,z)y+xD(y,z)$ for all $x,y,z \in N$**.**For the terminology used vin near-rings,see[2] **.**A derivation on N is an additive endomorphism D of N such that $D(xy)=xD(y)+D(x)y$ for all $x,y \in N$ .Ageneralized derivation f with associated derivation D is an additive endomorphism $f:N\rightarrow N$ such that $f(xy)=f(x)y+xD(y)$ for all $x,y \in N$ .According to Ceven and Ozturk [14] have introduced the concept of symmetric bi-(σ,τ)-derivation of near-ring and gave some properties,wherea symmetric bi-additive mapping $D:N_{\times}N\rightarrow N$ such that $D(xy,z)=D(x,z)\sigma(y)+\tau(x)D(y,z)$ for all $x,y,z \in N$. Note that if $\sigma =1$ and $\tau =1$, then D is a symmetric bi-derivation.

The following Lemmas are necessary for the paper

Lemma 1 [14 :Lemma 5]

Let N be a near-ring, D a symmetric bi -(σ,τ)-derivation of N and d the trace of D. T hen, for all x ,y,z,w $\in$N.Then $(D(x,z)\sigma (y)+\tau(x)D(y,z))w = D(n,z)\sigma (y)w +\tau (x)D(y,z)w$.

Lemma 2 [7:Lemma 3]

Let N be a 3-prime near-ring, Z(N)center of N.

(i) If z $\in$Z(N)-{0},then z is not a zero divisor.

(ii) If Z(N)-{0} contain an element z for which z+z $\in$Z(N), then (N,+)is abelian .

Lemma 3 [9:Lemma 3.1]

Let N be a 2- torsion free near-ring, D a symmetric bi-additive mapping on N and d the trace of D. If d (x)=0 for all x $\in$N, then D=0.

Lemma 4 [9:Lemma 3]

Let N be a 2-torsion free 3-prime near-ring,D a symmetric bi-(σ,τ)- derivation of N and d the trace of D.If xd(N)=0 for all x $\in$N,then x=o or D=0.

3.The main results

Lemma 3.1

Let N be a 3-semiprime near-ring, D be a symmetric bi-(σ,τ)-derivation of N. Then D(N,N)x=0 for all x$\in$N implies D=0 and x D(N,N)=0 implies D=0 .

Proof: Suppose D(y,z) z=0 for all x,y, z $\in$N.Then taking yw instead of y with using Lemma1,we obtain $D(yw,z)x=(D(y,z)\sigma(w) + \tau(y)D(w,z))x=0$ for all x,y,z,w $\in$N. According to our suppose, above relation become $D(y,z)\sigma(w)x=0$ for all x,y,z,w $\in$N.

Since σ is autorphism, we obtain D(y,z)wx=0 for all n,y,z,w $\in$R.

Replacing x by D (y,z) , then D(y,z)ND(y,z)=0 .Since N is 3-semiprime , we obtain D=0.

Theorem 3.2

Let N be a 3-prime near–ring with cancellation property and D a non-zero symmetric bi-(σ,τ)–derivation of N such that D(N,N) $\subset$Z(N), then σ-τ $\in$Z(N).

Proof: We have D(N,N)$\subset$Z(N) and D is a non-zero,there exists non-zero elements x,y $\in$N such that D(x,y)$\in$Z(N)-{0}.Then d(x,y-y)=D(x,y)+D(x,y) $\in$Z(N) for all x,y $\in$N, and hence (N,+) is a belian by Lemma2,D(x,y) $\in$Z(N) for all x,y $\in$N implies zD(x,y)=D(x,y)z for all z $\in$N.

Replacing x by xw, we obtain $z(D(x,y)\sigma(w)+\tau(x)D(w,y))=(D(x,y)\sigma(w)+\tau(x)D(w,y))z$ for all x,y,w$\in$N.

By Lemma 1,and $D(N,N) \in Z(N)$ we obtain.

$D(x,y)z\sigma(w)+D(w,y)z\tau(x)=D(x,y)\sigma(w)z+\tau(x)D(w,y)z$ for all $x,y,z,w \in N$,

or (N,+) is a belian, then we have for all $x,y,z,w \in N$

$D(x,y)[z,\sigma(w)]=D(w,y)[z,\tau(x)]$. (1)

Taking $D(u,r)$ instead of $\sigma(w)$ for all $u,r \in N$ and since $D(u,r)\in Z(N)$,we obtain $D(u,v),y)\ [z,\tau(x)]=0$ for all $u,v,x,y,z\in N$. (2)

Again in (1) taking $D(u,v)$ instead of $\tau(x)$ for all u,v ЄN with using $D(u,r) \in Z(N)$.

$D(D(u,v),y)[z,\sigma(w)]=0$ for all $u,v,y,z,w \in N$. (3)

Then from (2) and (3), we obtain

$D(D(u,v),y)[z,\tau(x)]=D(D(u,v),y)[z,\sigma(w)]$ for all $u,v,x,y,z\ w \in N$.By using the cancellation property in a bove relation $D(D(u,v),y)$, we get

$[z,\tau(x)]=[z,\sigma\ (w)]$for all $x,z,w \in N$. Then

$[z,\tau(x)-\sigma(w)]=0$ for all $x,z,w \in N$.Thus,we obtain $\tau-\sigma \in Z(N)$.Then the proof is completed.

Theorem 3.3

Let N be a 2-torsion free 3-prime near-ring,with cancellation property,D a symmetric bi-(σ,τ)-derivation of N and d the trace of D.If $[d(x),x]\sigma,\tau =0$ for all $x \in N$,then D=0 or x=0.

Proof: We have D is a symmetric bi-(σ,τ)-derivation, we get

$D(x(x+y),x)=(D(x,x)\sigma(x+y)+\tau(x)D(x+y),x)$ for all $x,y \in N$. Then

$D(x(x+y),x)=d(x)\sigma(x)+d(x)\sigma(y)+\tau(x)d(x)+\tau(x)D(y,x)$ for all $x,y \in N$. (4)

Also, we have

$D(x(x+y),x)=D(x^2+xy,x)=D(x^2,x)+D(xy,x)$ for all $x,y \in N$.Then from above relation we obtain

$D(x(x+y),x)=d(x)\sigma(x)+\tau(x)d(x)+d(x)\sigma(y)+\tau(x)D(y,x)$ for all $x,y \in N$. (5)

From (5) and (4), we get

$d(x)\sigma(y)+\tau(x)d(x)=\tau(x)d(x)+d(x)\sigma(y)$ for all $x,y \in N$. (6)

Since $[d(x),x]\sigma,\tau=0$,then the relation(6) reduce to $d(x)(\sigma(y)+\sigma(x)-\sigma(y)-\sigma(x))=0$ for all $x,y \in N$.

Then $d(x)(\sigma(x),\sigma(y))=0$ for all $x,y \in N$. (7)

By using the cancellation property in (7) on $(\sigma(x),\sigma(y))$, we obtain $d(x)=0$ for all $x \in N$.

Left –multiplying by σ with the using Lemma 4, we get x=0 or D=0.Then the proof is completed.

Remark3.4

In Theorem, when $\sigma=1$ and $\tau=1$ with Lemma 3,we obtain D=0.

Theorem 3.5

Let N be a 3-semiprime ring and f be a generalized derivation on N with associated derivation D. If $f^2=0$, then $D^3=0$.

Proof: We have the following relation $f^2(xy)=f(f(x)y+xD(y))=0$ for all $x,y \in N$. Then

$f(x)D(y)+f(x)D(y)+xD^2(y)=0$ for all $x,y \in N$.

(8)

Applying f to (8) with using the relation $f^2=0$, gives

$f(x)D^2(y)+f(x)D^2(y)+f(x)D^2(y)+xd^3(y)=0$ for all $x,y \in N$.

(9)

Again in (8) replacing y by D(y), we obtain

$f(x)D^2(y)+f(x)D^2(y)+xD^3(y)=0$ for all $x,y \in N$.

(10)

Substituting (10) in (9), we get

$f(x)D^2(y)=0$ for all $x,y \in N$.

(11)

Applying (11) in (10), we obtain

$xD^3(y)=0$ for all $x,y \in N$.

(12)

Left-multiplying (12) by $D^3(y)$,we get $D^3(y)ND^3(y)=0$.Since N is semiprime near-ring,we obtain, $D^3(y)=0$ for all $y \in N$.Then the proof is completed.

Theorem 3.6

Let N be a 2-torsion free 3-semiprime near- ring with 1.If f is a generalized derivation on N with associated derivation D such that $f^2=0$ and $f(1) \in Z(N)$, then f =0.

Proof: Note that f(x)=f(1)x+1D(x) for all $x \in N$,so

$f(x)=cx+D(x),\ c \in Z(N)$.

(13)

If c=0,then f=D and $D^2(x)=0$ for all $x \in N$.

Replacing x by xy,we obtain

$D^2(xy)=0$ for all $x,y \in N$.Then

$D(D(xy))=0$ for all $x,y \in N$.

$D(D(x)y+xD(y))=0$ for all $x,y \in N$.Then by using $D^2(x)=0$,we obtain

$2D(x)D(y)=0$ for all $x,y \in N$.Since is 2-torsion free,we obtain

$D(x)D(y)=0$ for all $x,y \in N$.Replacing x by xr,we get

$D(r)xD(y)=0$ for all $x,y \in N$.Then $D(r)ND(y)=0$. Since N is 3-semiprime near-ring,we obtain

$D(y)=0$ for all $y \in N$.By using this result in our relation $f=D$,we obtain $f=0$.

If $c\neq 0$,then c is not a zero divisor,hence by (13),we obtain

$D^2=0$,by same method a bove ,,we obtain

$D=0$.But then $f(x)=cx$ and $f^2 = c^2x$ for all $x \in N$.According to our assumption, $f^2 = 0$,we get

$c^2x=0$ for all $x \in N$.Then

$c^2Nc^2=0$.Since N is 3-semiprime near –ring,we get

$c^2=0$.Then

$c^2r=0$ for all $r \in N$.Since $c \in Z(N)$,then $cNc=0$ .Since N is 3-semiprime near –ring ,then we must be get

$c=0$.By using this result in our relation $f(x)=cx$,we completed our proof.

Theorem 3.7

Let N be a 3-semiprime ring and f be a generalized derivation on N associated with a non-zero derivation D.

If $D(f(N))=0$, then $f(D(N))=0$,where D acts as automorphism .

Proof: We are assuming that $D(f(x))=0$ for all $x\in N$ .It follows that by replacing x by xy,we obtain

$D(f(xy))=0$ for all $x,y\in N$.Then

$D(f(x)y+xD(y))=0$ for all $x,y\in N$.

$f(x)D(y)+D(x)D(y)+xD^2(y)=0$ for all $x,y\in N$. (14)

Applying D again in (14),we obtain

$f(x)D^2(y)+D^2(x)D(y)+D(x)D^2(y)+D(x)D^2(y)+xD^3(y)=0$ for all $x,y\in N$. (15)

Taking D(y) instead of y in(14),we obtain

$f(x)D^2(y)+D(x)D^2(y)+xD^3(y)=0$ for all $x,y\in N$. (16)

Hence(15),reduces to

$D^2(x)D(y)+D(x)D^2(y)=0$ for all $x,y\in N$. (17)

Substitute D(x) for x in (14),we obtain

$f(D(x))D(y)+D^2(x)D(y)+D(x)D^2(y)=0$ for all $x,y\in N$.

According to (17) above relation become

$f(D(x))D(y)=0$ for all $x,y\in N$. (18)

Replacing y by yr in(18),we get

$f(D(x))(D(y)r+rD(y))=0$ for all $x,y,r\in N$.

According to (18) with using that f acts as automorphism,we get

$f(D(x))rf(D(y))=0$ for all $x,y,r\in N$.Replacing y by x ,we get

$f(D(x))Nf(D(x))= 0$.Since N is 3-semiprime ring,we get

$f(D(x))= 0$ for all $x\in N$. We completed our proof.

References

[1] JDP.Meldrum **(1985)**, Near rings and their links with groups,*Research Notes in Mathematics* Series ISBN 0-273-08701-0,Pitmann,London,3-10.

[2]Piz.G**.(1983)**,Near-rings,2nd ed.,ISBN 0720405661,*North Holland/American Elsevier,Amsterdam*,5-15.

[3]S.J.Abbasi and K.Iqbal **(2007)**,On near-integral domain,*TechnologyForces*,ISSN1994-862x,Vol.01,99-100.

[4]M.Ashraf,A.Ali and S.Ali**(2004)**,(σ,τ)-derivations on prime rings,*Archivum Mathematicum(BRNO)*,Tomus40,281-286.

[5]H.E.Bell and G.Mason**(1987)**,On derivations in near-ring,near-rings and near-fields,*North-Holland Mathematics Studies* 137,31-35.

[6]O.Golbasi**(2005)**,Some properties of prime near-rings with (σ,τ)-derivations,*Siberian Mathematical Journal* ,no.2,46,270-275.

[7]G.Maksa**(1987),**On the trace of symmetric bi-derivations,*Comptes rendus mathématiques Mathematical Reports* Canada 9,303-307.

[8] M.A.Ozturk and Y.B.Jun**(1999)**,On generalized symmetric bi-derivations in prime rings, *East Asian Mathematical Journal*,no.2,15,165-176.

[9] M.A.Ozturk and Y.B.Jun**(2004)**,On trace of symmetric bi-derivations in near-rings, *International Journal of Pure and Applied Mathematics*,no.1,17,95-102.

[10]M.Sapanci,M.A.Ozturk and Y.B.Jun**(1999)**,Symmetric bi-derivations on prime rings, *East Asian Mathematical Journal*,no.1,15,105-109.

[11]J.Vukman**(1989)**,Symmetric bi-derivations on prime and semiprime rings,*Aequationes Mathematicae*,38,245-254.

[12]X.K.Wang**(1994),**Derivations in prime rings,*Proceedings of the American Mathematical Society.*,no.2,121,361-366.

[13]Fong,Y.,Ke,W.F and Wang,G.S.,**(1998)**,Syntactic nearrings,*Kluwer Academic Publishers*,Printed in the Netherlands,133-139.

[14]Y.Ceven and M.A.Ozturk**(2007)**,Some properties of symmetric bi-(σ,τ)-derivations in near-rings, *Communications of the Korean Mathematical Society*,no.4,22,487-491.

[15]O.Golbasi**(2006)**,On generalized derivations of prime near-rings,*Hacettepe Journal of Mathematics and Statistics*,Vol.35(2),173-180.

حول الاشتقاقات العامة على الحلقات المقتربة شبه الاولية

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الملخص:الغرض الرئيسي من هذا البحث دراسة وتحري بخصوص الاشتقاقات العامة على الحلقات المقتربة شبه الاولية N سوف نعطي بعض النتائج عندما تكون N تسمح بتحقيق بعض الشروط على3- الحلقات المقتربة شبه الاولية N و3- الحلقات المقتربة الاولية N.