Anti Fuzzy k-Ideal of Ternary Semiring

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Abstract

In this paper, we introduce the notion of anti fuzzy k-ideal of ternary semi ring and study some properties of it.

Introduction

The notion of fuzzy subset of a set was introduced by Zadeh in 1965 [4].The notion of fuzzy subgroup was made by Rosenfeld in 1971[1].Fuzzy ideal in a ring were introduced by W.Liu in 1982 [9].In 1996, Kim and Park studied fuzzy ideal in semirings[2].The notion of ternary semi rings introduced by Dutta and Kar in 2003[8].In 2007, J.kavikumar and Azme Bin khamis introduced The notion of fuzzy ideals in ternary semirings[3] . R. Biswas given the notion of anti fuzzy in 1999 [5]. The notion of fuzzy k-ideals in ternary semirings was introduced by Sathinee Malee and Ronnason Chinram in 2010[7].

The main purpose of the paper is to introduce the notion of anti fuzzy k-ideal of ternary semiring and study some properties of it.

1- Preliminaries

In this section we review some basic definition which will be used in this paper.

Definition (1.1) [6] A non empty set R together with a binary operation, called addition and ternary multiplication, is said to be a ternary semi ring if R is an additive commutative semigroup satisfying the following conditions

(i) (abc)de=a(bcd)e=ab(cde),
(ii) (a+b)cd=acd+bcd,
(iii) a(b+c)d=abd+acd,
(iv) ab(c+d)=abc+abd, for all a,b,c,d,e$\in$R

Definition (1.2) [6] Let R be a ternary semiring .If there exists an element $0 \in R$ such that

$0 + x = x \quad and \quad 0xy = x0y = xy0, \; forall \quad x, y \in R$

then “0” is called the zero element or simply the zero of ternary semirings .In this case we say that R is a ternary semiring with zero

Definition (1.3) [6] An additive subsemigroup I of ternary semiring R is called a left (resp.,right and lateral) ideal of R if

$s_1 s_2 i (resp., i s_1 s_2, s_1 i s_2) \in I \quad \forall s_1 s_2 \in R \, and \quad i \in I$

If I is both left and right ideal of R, then I is called a two sided ideal of R.
If I is a left ,a right and a lateral ideal of R, then I is called an ideal of R.

Definition (1.4) [4] A function μ from a non empty set X to the interval [0,1] is called a fuzzy subset of X .
Definition (1.5) [4] The complement of a fuzzy subset μ of a set X is denoted by μ^c and defined as

$\mu^c(x) = 1 - \mu(x), \forall x \in X$

Definition (1.6)[3] Let ν and μ be any two fuzzy subsets of X then
$\nu \cap \mu$ *and* $\nu \cup \mu$ are fuzzy subset of X and defined by

$(\nu \cap \mu)(x) = \min\{\nu(x), \mu(x)\}$

$(\nu \cup \mu)(x) = \max\{\nu(x), \mu(x)\}, \forall x \in X$

Definition (1.7) [3] A fuzzy subsemigroup μ of a ternary semiring R is called a fuzzy ideal of R if the function $\mu : R \to [0,1]$ satisfying the following conditions:

$(i)\quad \mu(x+y) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in R$

$(ii)\quad \mu(xyz) \geq \mu(z)$

$(iii)\quad \mu(xyz) \geq \mu(x)$

$(iv)\quad \mu(xyz) \geq \mu(y), \quad \forall x, y, z \in R$

A fuzzy set μ with conditions (i)and (ii) is called an fuzzy left ideal of R. If a fuzzy set μ satisfies (i) and (iii) then it is called a fuzzy right ideal of R. Also if μ satisfy (i) and (iv) then it is called a fuzzy lateral ideal of R.
If μ is fuzzy left ideal, fuzzy right ideal and fuzzy lateral ideal then it is called fuzzy ideal of a ternary semiring R.

Definition (1.8)[7] A fuzzy ideal μ of a ternary semiring R is said to be a fuzzy k-ideal of R if

$\mu(x) \geq \min\{\mu(x+y), \mu(y)\}, \forall x, y \in R$

Definition(1.9)[7] Let S and R be two ternary semirings.a mapping $f : S \to R$ is said to be a homomorphism if f(x+y)=f(x)+ f(y) and f(xyz)=f(x)f(y)f(z) $, \forall x, y, z \in S$
If S and R are ternary semirings with zero 0 ,then f(0)=0.

Definition (1.10)[7] Let $f : S \to R$ be a homomorphism of ternary semirings and μ be a fuzzy subset of S ,we define a fuzzy subset $f(\mu)$ of R by

$$f(\mu)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} & if \quad f^{-1}(y) \neq \phi \\ 0 & other \quad wise \end{cases}$$

We call $f(\mu)$ the image of μ under *f*.
Definition (1.11)[7] Let R_1 and R_2 be two ternary semirings and f be a function of R_1 into R_2. If μ is a fuzzy subset of R_2, then the preimage of μ under f is a fuzzy subset of R_1 defined by

$f^{-1}(\mu)(x) = \mu(f(x)) \qquad \forall x \in R_1$.

Definition (1.12)[9] Let $\phi : R_1 \to R_2$ be any function. An anti fuzzy ideal μ of R_1 is called ϕ-invariant if $\phi(x) = \phi(y)$ implies μ *(x)*= μ*(y)* where *x,y*$\in R_1$.

2-The Main Results

Definition (2.1) A fuzzy subset μ of a ternary semiring R is said to be an anti fuzzy left (right, lateral) ideal of R if

1- $\mu(x + y) \leq max\ \{\mu(x), \mu(y)\}$,

2- $\mu(xyz) \leq \mu(z)$, $[\ \mu(xyz) \leq \mu(x), \mu(xyz) \leq \mu(y)]$, *for all* $x, y, z \in R$

μ is an anti fuzzy ideal of R if it is anti fuzzy left ideal , anti fuzzy right ideal and anti fuzzy lateral ideal of R.

Definition (2.2) An anti fuzzy ideal μ of a ternary semiring R is said to be an anti fuzzy k-ideal of R if

$\mu(x) \leq max\ \{\mu(x+y), \mu(y)\}$, *for all* $x, y \in R$

Example (2.3) Let R be the set of non-positive integer with zero. R is ternary semiring with the usual addition and ternary multiplication,

Let μ a fuzzy subset of R defined by

$$\mu(x) = \begin{cases} 0 & if\ x\ \ is \text{ an } even\ \ or\ 0 \\ 1 & if\ \ x\ \ is \text{ an } odd \end{cases}$$

Then μ is an anti fuzzy k-ideal of R.

Proposition (2.4) Let R be a ternary semiring and μ be a fuzzy subset of R. Then μ is an anti fuzzy k-ideal of R if and only if μ^c is a fuzzy k-ideal of R.

Proof:

Suppose μ be an anti fuzzy k-ideal of R.

Let x, y ,z $\in$ R,

$\mu^c\ (x + y) = 1- \mu\ (x + y)$, since μ is an anti fuzzy k-ideal of R

$\geq 1 - max\{\mu(x), \mu(y)\}$

$= min\{1 - \mu(x), 1 - \mu(y)\}$

$= min\{\ \mu^c\ (x), \mu^c\ (y)\}$,

$\mu^c\ (xyz) = 1- \mu(xyz)$ since μ is an anti fuzzy left k-ideal of R

$\geq 1 - \mu(z)$

$= \mu^c\ (z)$. Hence μ^c is fuzzy left ideal of R

$\mu^c\ (xyz) = 1- \mu(xyz)$

$\geq 1 - \mu(x)$

$= \mu^c$ (x). Hence μ^c is fuzzy right ideal of R

$\mu^c\ (xyz) = 1- \mu(xyz)$

$\geq 1 - \mu(y)$

$= \mu^c$ (y). Hence μ^c is fuzzy lateral ideal of R

Then μ^c is a fuzzy ideal of R

Let $x,y \in R$ Then

$\mu^c\ (x) = 1- \mu(x)$

$\geq 1 - max\ \{\mu(x+y), \mu(y)\}$

$= min\ \{1 - \mu(x+y), 1 - \mu(y)\}$

$= min\{\ \mu^c\ (x+y), \mu^c\ (y)\}$.

Hence μ^c is a fuzzy k-ideal of R.

Conversely, let μ^c be a fuzzy k-ideal of R

For $x, y, z \in R$, we have

$\mu(x + y) = 1- \mu^c\ (x + y)$

$\leq 1 - min\{\ \mu^c\ (x), \mu^c\ (y)\}$

$= max\{\mu(x), \mu(y)\}$

$\mu(\ xyz\ ) = 1- \mu^c\ (xyz)$

$\leq 1 - \mu^c\ (z)$

$= \mu(z)$. Hence μ is an anti fuzzy left ideal of R

Similarly we can prove that μ is an anti fuzzy right and lateral ideal of R then μ is an anti fuzzy ideal of R

Let $x,y \in R$ Then

$\mu(x) = 1- \mu^c\ (x)$

$\leq 1 - \min\{\mu^c(x+y), \mu^c(y)\}$
$= \max\{\mu(x+y), \mu(y)\}$
Hence μ is an anti fuzzy k-ideal of R.

Proposition (2.5) Let μ and ν are anti fuzzy k- ideal of ternary semiring R. Then $\mu \cup \nu$ is also an anti fuzzy k- ideal of ternary semiring R.

Proof

Let μ and ν be two anti fuzzy k- ideals of a ternary semiring R and x, y ,z $\in$ R. Then we have

$$\begin{aligned}(\mu \cup \nu)(x+y) &= \max\{\mu(x+y), \nu(x+y)\}\\ &\leq \max\{\max\{\mu(x), \mu(y)\}, \max\{\nu(x), \nu(y)\}\}\\ &= \max\{\max\{\mu(x), \nu(x)\}, \max\{\mu(y), \nu(y)\}\}\\ &= \max\{(\mu \cup \nu)(x), (\mu \cup \nu)(y)\}\end{aligned}$$

$$\begin{aligned}(\mu \cup \nu)(xyz) &= \max\{\mu(xyz), \nu(xyz)\}\\ &\leq \max\{\mu(z), \nu(z)\}\\ &= (\mu \cup \nu)(z),\end{aligned}$$

Hence $\mu \cup \nu$ is an anti fuzzy left ideal of R similarly we can prove that $\mu \cup \nu$ is an anti fuzzy right and lateral ideal of R then $\mu \cup \nu$ is an anti fuzzy ideal of R

Let x,y $\in$ R , since μ and ν are anti fuzzy k- ideal Then
$\mu(x) \leq \max\{\mu(x+y), \mu(y)\}$ and $\nu(x) \leq \max\{\nu(x+y), \nu(y)\}$,

$$\begin{aligned}(\mu \cup \nu)(x) &= \max\{\mu(x), \nu(x)\}\\ &\leq \max\{\max\{\mu(x+y), \mu(y), \max\{\nu(x+y), \nu(y)\}\}\\ &= \max\{\max\{\mu(x+y), \nu(x+y)\}, \max\{\mu(y), \nu(y)\}\}\\ &= \max\{(\mu \cup \nu)(x+y), (\mu \cup \nu)(y)\}\end{aligned}$$

then $\mu \cup \nu$ is an anti fuzzy k-ideal of R.

Theorem (2.6) Let f : $R_1 \rightarrow R_2$ be an onto homomorphism of ternary semirings R_1 and R_2 . If μ is an anti fuzzy k-ideal of R_2, then $f^{-1}(\mu)$ is an anti fuzzy k-ideal of R_1.

Proof Let μ be an anti fuzzy k-ideal of R_2 and let $x, y, z \in R_1$,
Then we have
$f^{-1}(\mu)(x+y) = \mu(f(x+y))$,since f is a homomorphism then
$= \mu(f(x) + f(y))$
$\leq \max\{\mu(f(x)), \mu(f(y))\}$
$= \max\{f^{-1}(\mu)(x), f^{-1}(\mu)(y)\}$

$f^{-1}(\mu)(xyz) = \mu(f(xyz))$
$\leq \mu(f(z))$
$= f^{-1}(\mu)(z),$

Hence $f^{-1}(\mu)$ is an anti fuzzy left ideal of R_1

Similarly we $f^{-1}(\mu)$ is an anti fuzzy right and lateral ideal of R_1 then $f^{-1}(\mu)$ is an anti fuzzy ideal of R_1

Let $x,y \in R_1$ Then
$f^{-1}(\mu)(x) = \mu(f(x))$
$\leq \max\{\mu(f(x)+f(y)), \mu(f(y))\}$,
since f is a homomorphism then
$= \max\{\mu(f(x+y)), \mu(f(y))\}$

$\max\{f^{-1}(\mu)(x+y), f^{-1}(\mu)(y)\}$

Hence $f^{-1}(\mu)$ is an anti fuzzy k-ideal of R_1.

Lemma (2.7)[7] Let R_1 and R_2 be two ternary semirings and $\phi: R_1 \to R_2$ be a homomorphism. Let μ be a ϕ-invariant anti fuzzy ideal of R_1 if $x= x \in R_2$ such that $x = \phi(a), a \in R_1$ then $\phi(\mu)(x)= \mu(a)$.

Proof If $r \in \phi^{-1}(x)$,then $\phi(r)=x=\phi(a)$

Since μ is a ϕ- invariant $\mu(r)= \mu(a)$, then by definition (1.10), we have

$\phi(\mu)(x)= \sup_{r\in\phi^{-1}(x)} \mu(r) =\mu(a)$

Hence $\phi(\mu)(x)= \mu(a)$

Theorem (2.8) Let $\phi: R_1 \to R_2$ be an onto homomorphism of ternary semirings R_1 and R_2. If μ is a ϕ-invariant anti fuzzy k-ideal of R_1, then $\phi(\mu)$ is an anti fuzzy k-ideal of R_2.

Proof: Let $\phi: R_1 \to R_2$ be an onto homomorphism
and μ is ϕ- invariant anti fuzzy k-ideal of R_1,

Let $x, y, z \in R_2$ .Since ϕ is surjective then there exist $a, b, c \in R_1$ such that $\phi(a) = x$, $\phi(b) = y$ and $\phi(c) = z$
since ϕ is a homomorphism then
$x + y = \phi(a)+\phi(b)=\phi(a + b)$ and $xyz = \phi(a)\phi(b)\phi(c)=\phi(a bc)$. Then we have

$\phi(\mu)(x + y) = \mu(a + b)$

$\leq max\{\mu(a), \mu(b)\}$ Since μ is ϕ-invariant by lemma (2.9)

$= max(\phi(\mu)(x), \phi(\mu)(y)),$

$\phi(\mu)(xyz) = \mu(abc)$

$\leq \mu(c)$ Since μ is ϕ-invariant by lemma (2.9)

$= \phi(\mu)(z).$

Hence $\phi(\mu)$ is an anti fuzzy left ideal of R_2 Similarly
$\phi(\mu)$is an anti fuzzy right and lateral ideal of R_2
then $\phi(\mu)$ is an anti fuzzy ideal of R_2
Let $x,y \in R_2$ since ϕ is onto then there exists $a, b \in R_1$ such that
$\phi(a) = x$ and $\phi(b) = y$

$\phi(\mu)(x) = \mu(a) \leq max\{\mu(a+b), \mu(b)\}$

$= max\{\phi(\mu)(x+y), \phi(\mu)(y)).$

Hence $\phi(\mu)$ is an anti fuzzy k-ideal of R_2.

Definition (2.9) An anti fuzzy k-ideal μ of a ternary semiring R is said to be normal if $\mu(0) = 1$.

Theorem (2.10) Let μ be an anti fuzzy k- ideal of a ternary semiring Rand μ^* be a fuzzy subset of R defined by $\mu^*(x) = \mu(x)+1-\mu(0)$ for all $x \in R$.
Then μ^* is a normal anti fuzzy k-ideal of R.

Proof. Let $x, y, z \in R$.Then
$\mu^*(x + y) = \mu(x + y) + 1 - \mu(0)$,since μ be an anti fuzzy k- ideal

$\leq max\{\mu(x), \mu(y)\} + 1 - \mu(0)$

$= max\{\mu(x) + 1 - \mu(0), \mu(y) + 1 - \mu(0)\}$

$= max\{\mu^*(x), \mu^*(y)\},$

$\mu^*(xyz) = \mu(xyz) + 1 - \mu(0)$,since μ be an anti fuzzy left k- ideal then

$\leq \mu(z) + 1 - \mu(0)$

$= \mu^* (z)$.

Hence μ^* is an anti fuzzy left ideal of R similarly we can prove that μ^* is an anti fuzzy right and lateral ideal of R then μ^* is an anti fuzzy ideal of R

$$\mu^* (x) = \mu(x) + 1 - \mu(0)$$
$$\leq \max\{\mu(x+y), \mu(y)\} + 1 - \mu(0)$$
$$= \max\{\mu(x+y) + 1 - \mu(0), \mu(y) + 1 - \mu(0)\}$$
$$= \max\{\mu^* (x+y), \mu^* (y)\}.$$

Hence μ^* is an anti fuzzy k-ideal of R.

Since $\mu^* (x) = \mu(x) + 1 - \mu(0)$

$\mu^* (0) = \mu(0) + 1 - \mu(0)=1$ Hence μ^* is a normal anti fuzzy k-ideal of R

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ضد المثالي –k الضبابي على شبه الحلقة ذات الضرب الثلاثي

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الخلاصة

في هذا البحث قمنا بتعريف ضد المثالي -k الضبابي
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