Journal of Kufa for Mathematics and Computer **Vol.1, No.6, December, 2012, pp.86—87**

ON The Similarity of Matrices

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Abstract

In this paper we shall discuss when an invertible matrix and its inverse are similar.We shall give theorems for such case.

1 Introduction

The similarity is one of the important concept of matrices in most fields.This importance yield from the properties of the similar matrices where they have the same eigenvalues , determinant , trace ,and rank.

2 Definition[1]

A square matrix A is said to be idempotent if $A^2 = A$.

3 Example

The following matrix is idempotent:

$$\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$$

4 Definition[2]

If A and B are nxn matrices , we say that B is similar to A if there is a nonsingular matrix P such that $B=P^{-1} A P$.

5 Example

The following matrices are similar:

$$A=\begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}, B=\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

6 Remark[2]

The similarity is symmetric relation on M_n(□).

Main Results

7 Theorem

If A is nonsingular idempotent matrix , then A is similar to A^{-1}.

Proof:

Since A is idempotent , then $A^2 = A$. Also A^{-1} exists.

Now

$A^{-1} =(A A^{-1})A^{-1}$

$=A(A^{-1} A^{-1})$

$=A(AA)^{-1}$

$=A A^{-1}$ for A is an idempotent matrix

$=AA A^{-1}$ for A is an idempotent matrix

$=(A^{-1})^{-1} A A^{-1}$

Hence A is similar to A^{-1}.

8 Remark

The converse of the this theorem is not true:

The matrix

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

is similar to its inverse but it is not an idempotent matrix.

9 Theorem

If A is similar to a nonsingular idempotent matrix B , then A is similar to A^{-1}.

Proof:

Since A is similar to B, then there is a nonsingular matrix P such that

$A=P^{-1} B P$

$\Rightarrow A^{-1} = P^{-1} B^{-1} P$

Thus A^{-1} is similar to B^{-1}

Since the similarity is symmetric relation and by Theorem 7 ,we have

B^{-1} is similar to B

$\Rightarrow A^{-1}$ is similar to B

And since B is similar to A

Then A^{-1} is similar to A

Hence A is similar to A^{-1}.

References

[1]Shanti Narayan&P.K.Mittal,A Textbook of Matrices,S.Chand &Company Ltd.,(2009).

[2] Bernard Kolman & David R. Hill, Elementary Linear Algebra With Applications, Ninth Ed.,Pearson , (2008).