ON almost Tℓ- continuous multifunctions

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Abstract

Using the concept of T- open set we introduce and study the concept of Almost Tℓ-continuous multifunctions which is stronger than the concept of Almost ℓ- continuous multifunctions. Several properties and characterizations of this new concept are proved .

1. Introduction:

The notion of Almost ℓ-continuous functions(briefly, a. ℓ-c.f.)between topological spaces was introduced by KonstadilakiSavvopoulou and Reilly[6].This class of functions lies between almost continuous functions [11]and c-continuous functions[5]. Then A. kanibir and I.L. Reilly have extended this concept to multifunction [2].In this paper ,using the concept of T-open sets we get the concept of T- Lindelöf and by using the concept of T- Lindelöf we introduce the concept of Almost Tℓ- continuous multifunctions(briefly, a.Tℓ-c.m.f.) which is stronger than the concept of Almost ℓ-continuous multifunctions (a. ℓ-c.m.f.) .Through out this paper , the closure (resp. interior) of a subset B in a topological space (Y, τ) is denoted by $\mathrm{cl}B$ $(\mathrm{resp.\,int}B)$ .while B is called regular open if $\mathrm{B} = \mathrm{int}(\mathrm{cl}B)$[1].

2. Preliminaries:

In this section, we introduce and recall the basic definition and facts needed in this work .

2.1 Definition: Let (X, τ, T)be an operator topological space [9], then X is called T-Lindelöf if every T-open cover of X has a countable subcover .(for the definition of T-open set, see[7]).

2.2 Remark :

Thefamily of all T- open subsets of X does not form in general a topology on X [7] .

2.3 Definition:[8]

By amultifunction $F: (X, \sigma) \longrightarrow (Y, \tau)$, we mean a point –to-set correspondence from$(X, \sigma)$to(Y, τ),and we always assume that $F(x) \neq \emptyset, \forall\, x \in X$.

2.4 Definition:[8]

Let$F: (X, \sigma) \longrightarrow (Y, \tau)$ be a topological multifunction,$A \subseteq X$, $B \subseteq Y$,then

i) $F^{+}(B) = \{x \in X : F(x) \subseteq B\}$is called the upper inverse of B.
ii) $F^{-}(B) = \{x \in X : F(x) \cap B \neq \emptyset\}$is called the lower inverse of the set B.
iii) $F(A) = \bigcup_{x \in A} F(x)$is called the image of the set A.

2.5 Definition:[8]

The multifunction $F: (X, \sigma) \longrightarrow (Y, \tau)$ is called upper semicontinuous brieflyu.s.c.(resp.lower semicontinuous briefly l.s.c.) if $F^{+}(B)$ (resp.$F^{-}(B)$is open in(X, σ) for every V open set of (Y, τ).

2.6 Definition:

i) For a topological space(Y,τ), the cocompact topology ofτ on $Y$is denoted by $c(\tau)$,and defined by$c(\tau) = \{\emptyset\} \cup \{U \in \tau, U^C iscompact\}$.[5]

ii) For an operator topological space(Y,τ,T), the co T-compact topology ofτ on Y is denoted by $c(\tau,T)$m and defined by$c(\tau,T) = \{\emptyset\} \cup \{U \in \tau, U^C isT - compact\}$.

iii) The almost compact topology ofτ on Y is denoted by $e(\tau)$,and it has a base $e'(\tau) = \{U \in RO(Y,\tau) : U^C iscompact\}$.[5]

iv) The almost T-compact topology ofτ on Y is denoted by $e(\tau,T)$,and it has a base $e'(\tau,T) = \{U \in T - RO(Y,\tau,T)\colon U^C isT - compact\}$.

3. Almost Tℓ- continuous multifunctions(simply a.Tℓ-c.m.f.):

We now introduce a new class of multifunctions between topological spaces with the following definition .

3.1 Definition :

A multifunction F: $(X,\sigma) \longrightarrow (Y,\tau,T)$ is defined to be

i) Upper Almost $T\ell$-continuous or$u.a.T\ell$-c. ata point$x \in X$, if for eachT-regular open subset V(briefly TR.O) of Y with $F(x) \subseteq V$ and having T-Lindelöfcomplement ,there exist an open neighbourhood$U$of x such that $F(U) \subseteq V$.

ii) Lower Almost $T\ell$-continuous or$l.a.T\ell$-c. at a point$x \in X$, if for each T-regular open subset V of Y with $F(x) \cap V \neq \emptyset$ and havingT-Lindelöfcomplement , there exist an open neighbourhood$U$ of x such that $F(z) \cap V \neq \emptyset$ for every point $z \in U$.

iii) Almost $T\ell$-continuous, at a point$x \in X$,if it is both $u.a.T\ell$-c. and $l.a.T\ell$-c. at $x \in X$.

iv) Almost $T\ell$-continuous ($resp.\ u.a.T\ell$-c. , $l.a.T\ell$c.) if it is Almost $T\ell$-continuous ($resp.\ u.a.T\ell$-c. , $l.a.T\ell$c.) ateach point of$X$.

3.2 Theorem:

The following conditions are equivalent for a multifunctionsF: $(X,\sigma) \longrightarrow (Y,\tau,T)$.

a) $F$is upper almost $T\ell$-continuous.

b) $F^+(V)$is open for any T-regular open set V having T-Lindelöf complement in Y.

c) $F^+(V)$is open for any $V \in q'(\tau,T)$.

d) $F^-(V)$is closed for any T-regular closed Lindelöf set $VofY$.

e) For each$x \in X$, and each net(x_α) which converges to x in X, and for each T-regular open subset V with T-Lindelöf complement V^C, such that$x \in F^+(V)$, the net (x_α) is eventually in$F^+(V)$.

Proof:

(a) $\Longrightarrow$ (b)

Let V be any TR.O subset in Y having T-Lindelöf complement .Let $x \in F^+(V)$.Then there exist an open set U containing x, such that $F(U) \subseteq V$ hence $x \in U \subseteq F^+(V)$ .This show that $F^+(V)$ is open.

(b) $\Longrightarrow$ (a) . Let $x \in X$ and V be any TR.O subset of Y having T- Lindelöf complementwith $F(x) \subseteq V$, then $x \in F^+(V)$ and $F^+(V)$ is open. Put $U = F^+(V)$. Hence U is an open nbh.ofx and $F(U) \subseteq V$.

(b) $\Longrightarrow$ (c) .Let $V \in q'(\tau,T)$. henceV is TR.O subset in Y, V^C is T- Lindelöf by (b) $F^+(V)$ is open.

(c) $\Longrightarrow$ (b) . let V is TR.O subset of Y, hence T-Lindelöf in Y.

$\therefore V \in q'(\tau, T)$. $\therefore F^{+}(V)$ is open.

(b) $\Longrightarrow$ (d) . Let V be any TR.C. T- Lindelöf subset of Y.consider V^{C} is TR.O subset of Y having T- Lindelöf complement, by (b) we have $F^{-}(V^{C})$ is open. Hence by the fact $F^{+}(V^{C}) = (F^{-}(V))^{C}$, we have

(d) $\Longrightarrow$ (b) . Let V be any TR.O subset of Y having T- Lindelöf complement .consider V^{C} is TR.C T- Lindelöf , by(d) $F^{-}(V^{C})$ is closed ,hence $(F^{-}(V^{C}))^{C}$ is open , hence $F^{+}(V)$ is open.

(a) $\Longrightarrow$ (e). Let $J = (x_{\alpha})$ be a net which converge to $x \in X$ and let V be any TR.O subset of Y having T- Lindelöf complement V^{C} such that $x \in F^{+}(V)$,then there exist an open set $U \subseteq X$ containing x such that $U \subseteq F^{+}(V)$

Since (x_{α}) converge to x it follows that there exist $\alpha_0 \in \Omega$ such that $\alpha_{\alpha} \in U$ for all $\alpha \geq \alpha_0$. Therefore $x_{\alpha} \in F^{+}(V)$ for all $\alpha \geq \alpha_0$. Hence the net (x_{α}) is eventually in $F^{+}(V)$.

(d) $\Longrightarrow$ (a). suppose that (a) is not true .then there exist $x \in X$ and a TR.O subset V of Y having T- Lindelöf complement with $F(x) \subseteq V$ such that $F(U) \nsubseteq V$ for each open set $U \subseteq X$ containing x .Therefore the nbh net(x_U), $x_U \to x$, but (x_U) is not eventually in $F^{+}(V)$ .This is a contradiction.■

Similarly , we can obtain the following conditions for the lower almost continuous multifunctions .

3.3 theorem :

The following conditions are equivalent for a multifunctions $F: (X, \sigma) \longrightarrow (Y, \tau, T)$.

a) F is lower almost $T\ell$-continuous.
b) $F^{-}(V)$ is open for any T-regular open set V having T-Lindelöf complement in Y.
c) $F^{-}(V)$ is open for any $V \in q'(\tau, T)$.
d) $F^{+}(V)$ is closed for any T-regular closed T-Lindelöf set V of Y.
e) For each $x \in X$, and each net(x_{α}) which converges to x in X, and for each T-regular open subset V with T-Lindelöf complement V^{C}, such that $x \in F^{-}(V)$, the net (x_{α}) is eventually in $F^{-}(V)$.

3.4 Remark :

Let $F: (X, \sigma) \longrightarrow (Y, \tau, T)$ be a multifunction then we have:

i- If F is u.s.c. then F is u.a.$T\ell$-continuous.

ii- If F is l.s.c. then F is l.a.$T\ell$-continuous.

3.5 Example:

Consider $(\mathbb{R}, \tau_u)$ and $(\mathbb{R}, \tau_{CC}, T)$ be two topological spaces where τ_u is the usual topology on $\mathbb{R}$, τ_{CC} is the co countable on $\mathbb{R}$ and T is the identity operator on $P(\mathbb{R})$.

Define the multifunction $F: (X, \sigma) \longrightarrow (Y, \tau, T)$ as follow:

$$F(\mathbf{x}) = \begin{cases} \{x\} , & \text{if x is irrational} \\ Q , & \text{if x is rational} \end{cases}$$

Then the multifunction F is u.a. $T\ell$-continuous ,since $q'(\tau_{CC}, T) = \{\mathbb{R}, \emptyset\}$. In fact F is a. $T\ell$-continuous.However F is not u.s.c. or l.s.c., since $V = Q^{c}$ is open in $(\mathbb{R}, \tau_{CC})$, but $F^{+}(V)$ and $F^{-}(V)$ are not open in $(\mathbb{R}, \tau_u)$.

3.6 Theorem :

Let $F: (X, \sigma) \longrightarrow (Y, \tau, T)$ be a multifunction then,it is l.a. $T\ell$-continuous iff $F_q: (X, \sigma) \longrightarrow (Y, q(\tau, T), T)$ is l.s.c.(where $F_q = F$).

Proof$\Longrightarrow$:

AssumeFisl.a. $T\ell$-continuous. Let $V \in q(\tau, T)$ we can write $V \in \bigcup_{\alpha \in \Omega} V_\alpha$ where V_α is a T.R.O set having T-Lindelöf complement in Yfor $\alpha \in \Omega$. Where $F_q^-(V) = F_q^-(\bigcup_{\alpha \in \Omega} V_\alpha) = \bigcup_{\alpha \in \Omega} F_q^-(V_\alpha) = \bigcup_{\alpha \in \Omega} F^-(V_\alpha)$ but $F^-(V_\alpha)$ is an open set for $\alpha \in \Omega$ by theorem (3.3), so $F_q^-(V)$ is an open set . Hence $F_q: (X, \sigma) \longrightarrow (Y, q(\tau, T), T)$ is l.s.c.

$\Longleftarrow$abvious. l.s.c. $\longrightarrow$ l.a. $T\ell$-continuous■

The theorem (3.6) does not hold for upper almost continuousmultifunctions as the following example shows .

3.7 Example :

Consider $X = \{1,2,3,4\}$with the topology $\sigma = \{\emptyset, X, \{1\}, \{1,3,4\}\}$, and $Y = \{1,2,3,4\}$with the topology $\tau = \{\emptyset, Y, \{1\}, \{2\}, \{1,2\}\}$. let F be defined as $F(1) = \{4\}$, $F(2) = \{1,2\}$, $F(3) = \{3\}$, $F(4) = \{4\}$.

Let T= identity operator then $q'(\tau, T) = q'(\tau)=\{\emptyset, Y, \{1\}, \{2\}\}$.

The family $q'(\tau)$is abase consisting of R.O. sets having T-Lindelöf complement in Yfor $q(\tau) = \tau$. Then for any $V \in q'(\tau)$ we have $F^+(V) \in \sigma$, therefore $F: (X, \sigma) \longrightarrow (Y, \tau, T)$is u.a. $T\ell$-continuous . The topology $q(\tau)$ contains the set $\{1,2\}$ but $F_q^+(\{1,2\}) = \{2\} \notin \sigma$. Hence $F_q: (X, \sigma) \longrightarrow (Y, q(\tau, T), T)$ is not u.s.c.

3.8proposition:[3]

If(Y, τ) is Lindelöf ,and semiregular, then $q(\tau) = \tau$.

Now we introduce the following propositions:

3.9 proposition:

If(Y, τ, T) is T-Lindelöf ,and semiregular, then $q(\tau, T) = \tau$.

Proof:

Since Y is T- semiregular, then $(\tau_s, T) = \tau$. where(τ_s, T) is the topology generated by the family of all T- R.O. set in Y. Now $q(\tau, T)$ is the topology on Y generated by the family of all T-R.O. with T-Lindelöf complement .consider Vwhich is T-R.O. then V is R.O. and T-open . HenceV^C is T-closed in Y which is T-Lindelöf , this mean thatV^C is also T-Lindelöf. Hence $(\tau_s, T) = q(\tau, T)$, but $(\tau_s, T) = \tau$. hence $q(\tau, T) = \tau$. ■

The proof of the following theorem is clear.

3.10 theorem:

i) Let$F: (X, \sigma) \longrightarrow (Y, \tau, T)$ be a multifunction where$Y$ is T-Lindelöf and T-semiregular(briefly T-S.R.) then F is l.a.$T\ell$-continuous if and only if$F_q: (X, \sigma) \longrightarrow (Y, q(\tau, T), T)$ is l.s.c.
ii) Let$F: (X, \sigma) \longrightarrow (Y, \tau)$ be a multifunction where$Y$ is Lindelöf and semiregular then F is l.a. ℓ-continuous if and only if$F_q: (X, \sigma) \longrightarrow (Y, q(\tau, T))$ is l.s.c.

3.11 Remark:

A result analogous to theorem (3.10) does not hold for u.a. $T\ell$-continuous , as example 3.7 shows:

3.12 Definition:

Let(Y, , T) be an operator topological space we say that Y is T- compact if and only if every T-open cover of Y has a finite subcover. We have the following diagram :

$$\begin{tikzcd} \text{Compact} \arrow[r] \arrow[d] & \text{T- compact} \arrow[d] \\ \text{Lindelöf} \arrow[r] & \text{T-Lindelöf} \end{tikzcd}$$

Now we introducanew class of multifunction.

3.13 Definition :

A multifunction $F: (X, \sigma) \longrightarrow (Y, \tau, T)$ is defined to be

i) Upper Almost Tc-continuous (briefly,$u.a.Tc$-c.) at a point$x \in X$, if for each T-regular open subset V(briefly, TR.O) of Y with $F(x) \subseteq V$ and having T-compactcomplement , there exist an open neighbourhood$U$ of x such that $F(U) \subseteq V$.

ii) Lower Almost Tc-continuous (briefly, $l.a.Tc$-c.) at a point$x \in X$, if for each T-regular open subset V of Y with $F(x) \cap V \neq \emptyset$ and having T-compact complement , there exist an open neighbourhood$U$ of x such that $F(z) \cap V \neq \emptyset$ for every point $z \in U$

iii) Almost Tc-continuous, at a point$x \in X$,if it is both $u.a.Tc$-c. and $l.a.Tc$-c. at $x \in X$.

iv) Almost Tc-continuous ($resp.$ $u.a.Tc$-c. , $l.a.Tc$-c.) if it is Almost Tc-continuous ($resp.$ $u.a.Tc$-c. , $l.a.Tc$-c.) at each point of X.

3.14 Remarks:

i) If $F: (X, \sigma) \longrightarrow (Y, \tau)$ is almost ℓ-continuous,then$F$is$c$-continuous[2].

ii) If $F: (X, \sigma) \longrightarrow (Y, \tau)$ is almost$T\ell$-continuous, then$F$is Tc-continuous .

iii) The reverse implication does not hold ingeneral, as the following example shows :

Example:

Let X be the set of real numbers and let $\sigma$and τ be the usual and discrete topology on X , respectively. Define i : $(X, \sigma) \longrightarrow (Y, \tau)$ to be the identity functionfrom(X, σ) to operator topological space(X, σ, T). Then i is almost Tc –continuous but not almost Tℓ - continuous .

The proofs of the following two theorems are analogous to the proof of theorem (3.2) and theorem (3.3), respectively.

3.15 Theorem:

The following conditions are equivalent for a multifunctions$F: (X, \sigma) \longrightarrow (Y, \tau, T)$.

a) $F$is upper almost Tc-continuous(briefly, u.a.Tc.c).

b) $F^+(V)$is open for any T-regular open set V having T-compact complement in Y.

c) $F^+(V)$is open for any $V \in e'(\tau, T)$.

d) $F^-(V)$is closed for any T-regular closed T-compactsubset $V of Y$.

e) For each$x \in X$, and each net(x_α) which converges to x in X, and for each T-regular open subset V with T-compact complement V^C, such that$x \in F^+(V)$, the net (x_α) is eventually in$F^+(V)$.

3.16Teorem:

The following conditions are equivalent for a multifunctions$F: (X, \sigma) \longrightarrow (Y, \tau, T)$.

a) $F$is lower almost Tc-continuous(briefly, l.a.Tc.c) .

b) $F^-(V)$is open for any T-regular open set V having T-compact complement in Y.

c) $F^-(V)$is open for any $V \in e'(\tau, T)$.

d) $F^+(V)$is closed for any T-regular closed T-compact subset $V of Y$.

e) For each$x \in X$, and each net(x_α) which converges to x in X, and for each T-regular open subset V with T-compact complement V^C, such that$x \in F^-(V)$, the net (x_α) is eventually in$F^-(V)$.

3.17Theorem:

Let $F: (X,\sigma) \longrightarrow (Y,\tau,T)$ be a multifunction .then $F: (X,\sigma) \longrightarrow (Y,\tau,T)$ is lower almost Tc –continuous (briefly, l.a. Tc.c.) if and only if $F_e: (X,\sigma) \longrightarrow (Y, e(\tau,T), T)$is lower semicontinuous(briefly, l.s.c.).

Proof:

$\Longrightarrow$ Let $V \in e(\tau,T)$ we can write $V = \bigcup_{\alpha\in\Omega} V_\alpha$, where$V_\alpha \in e'(\tau,T)$for $\alpha \in \Omega$ .We have $F^-(\bigcup_\alpha V_\alpha) = \bigcup_\alpha F^-(V_\alpha)$,from theorem 3.16$F^-(V_\alpha)$ is an open set for $\alpha \in \Omega$, so$F^-(V)$ is an open set , $F_e^-(V) = F^-(V)$ is an open set .Hence $F_e: (X,\sigma) \longrightarrow (Y, e(\tau,T), T)$ is l.s.c.

$\Longleftarrow$obvious.■

3.18 Remark:

Aresult analogous to theorem (3.17) for upper almost Tc –continuous (briefly, u.a. Tc.c.does not hold for as example(3.7) shows.

3.19 Proposition:

If (Y,τ,T) is T-compact and T-semiregular then$e(\tau,T) = \tau$.

3.20 Theorem:

Let $F: (X,\sigma) \longrightarrow (Y,\tau,T)$ be a multifunction where (Y,τ,T) is T-compact and T-semiregular .then $F: (X,\sigma) \longrightarrow (Y,\tau,T)$ is lower almost TC –continuous (briefly, l.a. TC.c.) if and only if $F_e: (X,\sigma) \longrightarrow (Y, e(\tau,T), T)$ is lower semicontinuous(briefly, l.s.c.).

Proof:obvious from proposition 3.19 and theorem 3.17. ■

A result analogous to theorem (3.20) for upper almost TC –continuous(briefly, l.a. Tc.c.) does not hold for as example(3.7) shows.

3.21 Proposition:

Let $F: (X,\sigma) \longrightarrow (Y,\tau,T)$ be a multifunction . If $F_q: (X,\sigma) \longrightarrow (Y, q(\tau,T), T)$ is upper semicontinuous(briefly, u.s.c.)(lowersemicontinuous(briefly, l.s.c.)).then if $F_e: (X,\sigma) \longrightarrow (Y, e(\tau,T), T)$is upper semicontinuous (briefly, u.s.c.)(lowersemicontinuous(briefly, l.s.c.)).

Proof:

This follows from thefact $e(\tau,T) \subseteq q(\tau,T)$.■

Now before we state the next theorem, we recall that:

i) A topological space whose Lindelöf subset are closed is called an LC-space [10].
ii) An operator topological space whose T- Lindelöf subsets are T-closed is called a TLC- space.

3.22 Theorem:

An operator topological space is TLC- space if and only if the following is true.for any topological space$(X,\sigma)$and any multifunction $F: (X,\sigma) \longrightarrow (Y,\tau,T)$where is upper $T\ell$ –continuous (lower $T\ell$ –continuous) if and only if $F^-(L)(F^+(L))$ is T-closed for any T-Lindelöf $L \subseteq Y$.

Proof:

$\Longrightarrow$ Let F be upper $T\ell$ – continuous and let L be any TL- subset of is a TLC- space, L is T-closed since F is upper $T\ell$–continuous $F^-(L)$ is closed. Now letV be a T- open set having TL-complement in by hypothesis $(F^-(V))^C$ is closed . But $(F^+(V))^C = F^-(V^C)$ so $F^+(V)$ is open. Hence F is upper $T\ell$–continuous. The proof for lower $T\ell$–continuous is similar .

$\Longleftarrow$suppose$(Y,\tau,T)$is not a TLC-space . It is sufficient to find a single-valued function $f: (X,\sigma) \longrightarrow (Y,\tau,T)$ such that f is $T\ell$–continuous (T is the identity operator) but$f^-(L)$ is not closed for some Lindelöf set $L \subseteq Y$.

Consider $(\mathbb{R},\tau_u)$,whereτ_u is the usual topology on $\mathbb{R}$, and consider $(\mathbb{R},\tau_c,T)$ to be the cofinite topology on $\mathbb{R}$ and T is the Identity operator

Let $f:(\mathbb{R},\tau_u) \longrightarrow (\mathbb{R},\tau_c)$ be the identity function, and $\mathbb{Q}$ the set of all rational number , then f is TL- continuous $\equiv$ L- continuous, and $\mathbb{Q}$Lindelöf subset of Y ,but$f^{-1}(\mathbb{Q})$ is not closed, $f^{-1}(\mathbb{Q}) = \mathbb{Q} \subset (\mathbb{R},\tau_u)$

$\therefore$ $\mathbb{Q}$ is not closed in $\mathbb{R}$, this is a contradiction, hence is $(Y,\tau,T)$is TLC- space.■

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