On Almost contra T*-continuous functions

Dr. Hadi .Jaber.mustafa and Layth .Muhsin.Habeeb

Department of Mathematics,

Faculty of Mathematics and computer Sciences, University of Kufa

Abstract:

Using the concept of T*-open set, we introduce and study Almost contra –T* -continuous functions
Several properties and characterizations of these function are given.

1 – Introduction:

In 1996, Dontchev[2] introduced contra-continuous functions .Jafariand Noiri[4] introduced and studied anew form of function called contra –precontinuous functions. In this paper, we introduce and study Almost contra T*-continuous functions as a generalization of almost contra-precontinuous functions.

Throughout this paper, all spaces X and Y (or (X,τ) and (Y,δ)) are topologicalspaces.

2- Basic definitions

In this section, weintroduce and recall the basic definition needed in this work.

2.1Definition[10]: A subset A of a spaces X is said to be regular open if A = int (cl (A)) where cl (A) and int (A) denoted the closure and interior of A. a subsetA of X is called regular closed if A= cl (int(A)).

2.2Definition[6]: A subset A of a space is said to be pre open if A $\subset$int(cl(A)),the complement of a pre open set is said to be pre closed the family of all regular open (respectively regular closed ,pre open ,pre closed) set of X is denoted by RO(X) (respectively RC(X),PO(X),PC(X)).

2.3Definition [1]: A subset A of a space X is said to be semi open if A $\subset$ cl (int (A)). The complement of a semi open set is called semi closed.

2.4Definition[10]: Let (X, τ) be a topological space and let $T:p(X)\to p(X)$ be a function (where p(X) is the power set of X) we say that T is an operator associated with the topology τ on X if $W\subseteq T(W)$ $(W\in\tau)$ and the triple (X, τ, T) is called an operator topological space .

2.5Definition [10]: Let (X, τ, T) be an operator topological space, let $A \subseteq X$

i) A is called T-open if given $x\in A$, then there exists $V \in \tau$ there exists $x \in V \subseteq T(V) \subseteq A$.

ii) A is called T*-open if $A \subseteq T(A)$ (A is not necessarily open)

.**2.6 Examples:**

i) Let (X,τ)be space and$T:p(X)\to p(X)$be define as follows :T(A) =Cl(Int(A)),$A\subseteq X$ now if w is open , then w= Int(w)

$\subseteq$Cl (Int(w)) = T(w) hence T is an operator associated with the topology τ on X ,
consider A =[0,1) in (R, t_u). Notice that A$\subseteq$Cl(Int(A)) hence A is T*-open), so if T(A) = Cl(Int(A)) the T*-open sets are exactly the semi-open sets.

ii) Let (X,τ)be space and T:p(X)→ p(X)
be define as follows T(A)=Int(Cl(A)) now if w is open
then w$\subseteq$Cl(A) hence w$\subseteq$IntCl(w)
hence T is an operator associated with the topology τ on
X , consider Q in (R,t_u) .
Notice that Q $\subseteq$IntCl (Q) (Q is not open)
hence Q is T*-open so if T(A) =Int(Cl(A)) the T*-open sets
are exactly the pre-open sets .

3- Almost contra T*-continuous function

In this section, we introduce the concept of
Almost contra –T*-continuous functions
3.1Definition[9]: A functionf :X→Y is called contra per-continuous if f^{-1}(V)is pre closed in X for each open set V of Y.

Now, we generalize this definition as follows:

3.2Definition:A function f :(X, τ ,T)→(Y,δ) from an operator topological space (X, τ ,T) into a topology space (Y,δ) is called contra-T*-continuous if f^{-1}(V) is T*-closed for each open set V of Y .
3.3Definition[2]: Afunction f:X→Y is called contra –continuous if f^{-1}(V) is closed in X for each open set V of Y.

3.4Definition[8]:A function f: X→Y is called perfectly continuous if f^{-1} (V) is clopen in X for every open set V of Y.

3.5Definition:A function f :(X, τ ,T) →(Y,δ) from an operator topological space (X, τ ,T) into a topology space (Y,δ) is called T*-perfectly function if f^{-1}(V) is T*-clopen (T-closed and T-open) for every open set V∈Y.

3.6Definition[3]: A function f: X→Y is called almost perfectly-function iff^{-1}(V) is clopen for every V∈ RO(Y).

3.7Definition:A function f :(X, τ ,T)→(Y,δ) from an operator topological space (X, τ ,T) into a topology space (Y,δ) is called almost T*-perfectly function if f^{-1}(V) is T*-clopen for every V∈RO(Y).

3.8Definition[7]:A function f: X→Y is said to be almost pre continuous if f^{-1}(V) is pre open in X for every regular open set V of Y.
Now we generalize this definition as follows:
3.9Definition: A function f: (X,τ, T) → (Y,δ) is said to be almost T*-continuous if f^{-1}(V) is T*-open in X for every regular open set V in Y.

3.10Definition[5]:A function f: X→Y is said to be pre-continuous if f^{-1}(V) is pre open in X for every open set V of Y.

3.11Definition:A function f: (X,τ,T) → (Y,δ) is said to be T*-continuous if f^{-1}(V) is T*-open in X for every open set V in Y.

3.12Definition[7]:A function f: X → Y is said to be almost contra –pre-continuous iff^{-1}(V) ∈ PC(X) for each V∈ RO(Y).

Now we generalize this definition as follow:

3.13Definition: A function f: (X,τ,T) → (Y,δ) is said to be almost contra –T*-continuous if f^{-1}(V) ∈ T*-closed (X) for each V∈ RO(Y).

3.14Theorem[10]: Let (X ,τ) and (Y,δ) be topological spaces. The following statements are equivalent for a function f: X → Y:

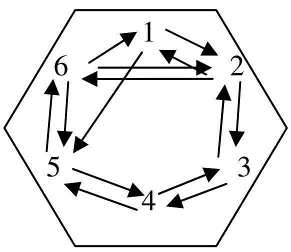
(1) f is almost contra pre-continuous;
(2) f^{-1}(F) ? PO(X) for every F ? RC(Y);
(3) For each x ? X and each regular closed set F in Y
Containing f(x), there exists apreopen set U in X containing x
Such that f (U) ?F;
(4) For each x ? X and each regular open set V in Y non-containing
f(x), there existsa pre-closed set K in X non-containing x such

That $f^{-1}(V) \subseteq K$;
(5) $f^{-1}(int(cl\ (G))) \in PC(X)$ for every open subset G of Y;
(6) $f^{-1}(cl(int\ (F))) \in PO(X)$ for every closed subset F of Y.

3.15Propositions: Let (X,τ,T) be an operator topological space then the union of any family of T*-open is also T*-open .
Proof:
Let $\mathcal{F}=\{ W_\alpha \mid \alpha \in \Lambda \}$ be any family of T*-open sets,
Let $W = U_\alpha W_\alpha$
Now $W_\alpha \subseteq T(W_\alpha)$
Hence $W = U_\alpha W_\alpha \subseteq U_\alpha T(W_\alpha)$
$= T(U_\alpha W_\alpha)$
$\subseteq T(W)$
$W \subseteq T(W)$
Hence W is T*-open .

Now we generalize the above theorem as follows:

3.16Theorem: Let $f: (X, \tau, T) \rightarrow (Y,\delta, L)$ from an operator topology space (X, τ, T) into topological space (Y,δ) then the following statements are equivalent:
(1) f is almost contra-T*continuous;
(2) $f^{-1}(F) \in$ T*-open in X for every $F \in RC(Y)$;
(3) for each $x \in X$ and each regular closed set F in Y containing f (x) , there exists a T*-open set U in X containing x such that$f(U) \subseteq F$;
(4) for each $x \in X$ and each regular open set V in Y non-containing f (x) , there exists a T*-closed set K in X non-containing x such that $f^{-1}(V) \subseteq K$;
(5) $f^{-1}(int\ (cl\ (G))) \in T^*\text{-}C(X)$ for every open subset G of Y;
(6)$F^{-1}(cl\ (int\ (F))) \in T^*\text{-}O(X)$ for every closed subset F of Y.



Proof:(1)⇒ (2) Let $F \in RC(Y)$. Then $F^c \in RO(Y)$. By (1), $f^{-1}(F^c) = (f^{-1}(F^c)$ is T*-C(X). Which mean that $f^{-1}(F)$ is T*-O(X).
(2)⇒ (1) the proof is similar.

(2)⇒(3)let F be any regular closed set in Y containing f (x). By (2), $f^{-1}(F)$ is T*-Open in X and $x \in f^{-1}(F)$. Take $U=f^{-1}(F)$. Then $f(U) \subset F$.
(3)⇒(2) Let F be regular closed in Y and $x \in f^{-1}(F)$ from (3),then there exists a T*-open set U_x in X containing x such that $U_x \subset f^{-1}(F)$. We have
$f^{-1}(F) = \bigcup_{x \in f^{-1}(F).} U_x$thus $f^{-1}(F)$ is T*-Open**.**
(Notice that the union of any family of T*-open is T*-open**propositions (3.15)**)
(3)⇒(4) Let V be any regular open set in Y non –containing f(x) .then V^c is a regular closed set containing f(x) .By (3) then there exists a T*-open set U in X containing x such that $f(U) \subseteq V^c$.hence $U \subseteq f^{-1}(V^c)=f^{-1}(V)^c$,then $U \subseteq f^{-1}(V)^c$ then $f^{-1}(V) \subseteq U^c$. Take $k= U^c$, we obtain that k is a T*-open in X non–containing x such that $f^{-1}(V) \subseteq k$.

(4)⇒(3) Let $V=F^c$ then V is a regular open set non –containing f(x) then there exists a T*-closed set k in X non –containing x ,$f^{-1}(V) \subseteq k$,let $U= k^c$ then (where U is T*-open containing x) ,
$k^c \subseteq (f^{-1}(V))^c= f^{-1}(V^c) \Rightarrow U \subseteq = f^{-1}(V^c)$,
$f(U) \subseteq V^c=F$.
(1)⇒ (5)let G be open subset of Y. Since int (cl (G)) is regular open, then by (1), it follows that f^{-1} (int (cl (G))) is T*-Closed in X.
(2)⇒ (6) Let F closed in Y we will show that f-1(cl intF)is T*-open in X , c lint F is regular closed in Y then by (2),f^{-1}(c lint F)is T*-open.
(6)⇒ (2) Let F be regular closed in Y, then F =Clint F then f^{-1}(ClintF)is T*-open. i.e.$f^{-1}(F)$ is T*-open.

3.17Remarks:
1) For function $f: X \rightarrow Y$ we have the following diagram**[9]**

Perfectly continuous ⇒ contra-continuous ⇒ contra-precontinuous
⇓ ⇓
Almost perfectly continuous ⇒ almost contra-precontinuous

2) For function f :(X, τ, T) → (Y,δ) we have the following diagram

$$\begin{tikzcd} \text{T*- perfectly continuous} \arrow[r, Rightarrow] \arrow[d, Rightarrow] & \text{contra T*-continuous} \arrow[d, Rightarrow] \\ \text{Almost T*-perfectly continuous} \arrow[r, Rightarrow] & \text{almost contra T*-continuous} \end{tikzcd}$$

<u>3.18 Examples:</u>

1) Let X = {a, b, c},
τ = {X ,∅,{a},{b},{a, c},{a, b}} and Y={a ,b ,c} ,
δ= {X ,∅,{b}‚{c},{b, c}}. Then the identity functionf: (X, τ)→ (Y, δ) is almost contra pre continuous. But it is not almost perfectly continuous.

2) Let X = {a, b, c},
τ = {X ,∅,{a},{b},{a, c},{a, b}} defined T:p(x)→p(x) as follows T(A) =int cl A and let Y={a, b, c},δ= {X ,∅,{b}‚{c},{b, c}}. Then the identity functionf: (X, τ, T)→ (Y, δ) is almost contra T*- continuous. But it is not regular open T*-closed..

3) Let X = {a, b, c}, τ={X,∅,{a},{a, b},{a, c}} andY=X,δ={X,∅,{a},{a, b}} and
f : (X ,τ) → (Y,δ) be the identity function. Then f is almost contra pre continuous function which is not contra-pre continuous.

4) Let X = {a, b, c},
τ = {X ,∅,{a},{b},{a, c},{a, b}} defined T:p(x)→p(x) as follows T(A) =int cl A and let Y={a, b, c} ,δ= {X ,∅,{b}‚{c},{b, c}}. Then the identity function f: (X, τ, T)→ (Y, δ) is almost contraT*-continuous. But it is not contraT*-continuous.

<u>3.19Theorem[9]</u>: If f: X → Y is almost contra-pre continuous function and A is a semi open subset of X, then the restrictionf | $_A$: A → Y is almost contra-pre continuous.

<u>3.20 Theorem</u> : Let (X, ,τ,T) be an operator topological spaces .where T(A)=intcl(A) and let (Y,δ)be topological space,if f: (X, ,τ,T) → (Y,δ) is almost contra T*-continuous function and A is a semi –open subset of X ,then the restriction g=f/$_A$:A→ Y is almost contra T*-continuous .

<u>Proof:</u> Let F be regular closed in Y since f is almost contraT*-continuous then f^{-1} (F) is T*-open in X (Notice that T*-open sets in X are exactly the pre-open sets in X) since A is semi open then g^{-1}(F)=A ∩f^{-1}(F) by [13] g^{-1}(F) will be pre –open in X (T*-open) there for g is almost contra –T*-continuous function .

<u>3.21Remark:</u> If A is not semi –open then f/A may not be almost contra pre-continuous function.

<u>3.22 Definition</u>:A cover £= {Uα│α ∈*I}*of sub set of X is called a T*-cover if U_α isT*-open for each α∈*I*. If T (A) =int cl A then the T*-cover of X will be a pre – open cover of X (p-cover).

<u>3.23Defintion:</u>Let (X,τ,T) be an operator topological spaces where w∈ τ , w⊆ T(w) and let U ⊆X then τ_1={w∩U│w∈T} T:p(X)→p(X), T_1: p(U) → p(U) let A⊆U⊆X,T_1(A)=T(A)∩U⊆U then (U, τ_1 T_1) operator topological sub spaces of (X, ,τ,T) such that (U, τ_1 T_1)⊆ (X, ,C,T).

<u>3.24Lemma:</u>

1) Let (X, τ) be a topological spaces, let U⊆X .If V is pre open in U and U is pre open in X then V is pre open in X.

2) If (X, τ, T) is an operator topological spaces and (U, τ_1,T_1) is an operator topological sub spaces of (X, τ, T) if Vis T^*_1-open in U and U is T*-open in X then V is T*-open in X.

<u>Prove 2:</u>

V⊆ T_1(V)

T_1(V) = T (V)∩ U

V⊆T (V) ∩U⊆T (V)

Then

V⊆T (V)

Vis T*-open in X

<u>3.25Theorem:</u> Let f : (X,τ,T) → (Y,δ) be function and let (U_α,τ_α,T_α) be an operator topological sub spaces of (X,τ,T) for each α let £ = { $U_\alpha \mid \alpha \in I$ } be a T*-cover of X [U_α is T*-open] suppose $g_\alpha = f \mid U_\alpha = (U_\alpha, \tau_\alpha, T_\alpha) \to (Y,\delta)$ is almost contra T*-continuous for each $\alpha \in I$ .then f : (X,τ,T) → (Y,δ) is almost contra T*-continuous function .

Proof: Let Vbe a regular closed set in Y,
Since gα= f $\mid U_\alpha$ is almost contra T*-continuous for each $\alpha \in I$,
$(g_\alpha)^{-1}(V)$ is T^*_α–open in U_α
But $U_\alpha$is T*-open in X bylemma (**3.24.2**)
$(g_\alpha)^{-1}(V)$ T*-open in X
V_α T^*_α-open in U_α
$V_\alpha \subseteq T_\alpha(V_\alpha) = T(V_\alpha) \cap U_\alpha \subseteq T(V_\alpha)$
$V_\alpha \subseteq T(V_\alpha)$, V_α T*-open in X
But $f^{-1}(V) = U\ (g_\alpha)^{-1}(V)$, hence $f^{-1}(V)$ is T*-open

3.26Theorem[9]: Let f : (X,τ) → (Y,δ) be function and let g : X → X x Y be the graph function of fdefinition by g(x) =(x , f(x)) for every x ∈ X if g is almost is almost contra pre-continuous then f is almost contra pre-continuous .

3.27Theorem: Let f : (X,τ,T) → (Y,δ) be function and let g: (X,τ,T)→(X x Y ,L)[where L is the product topology on X x Y] be graph function of f definition by g(x)=(x,f(x)) if g is almost contra T*-continuous then f is almost contra T*-continuous .
Proof: Let V be regular closed in Y
Notice that Xx Y is regular closed in X x Y
Since g is almost contra T*-continuous
Then g^{-1}(Xx V) is T*-open in X
But $f^{-1}(V) = g^{-1}$(Xx V)
Hence $f^{-1}(V)$ is T*-open in X
Which mean that f is almost contra T*-continuous.

3.28Theorem[9]: If a function f:(X, τ)→(Y,δ) is almost contra-precontinuous and almost continuous, then f is almost perfectly continuous

3.29Theorem: If a function f:(X, τ, T)→(Y,δ) is almost contra T*-continuous and almost continuous, then f is almost T*-perfectly continuous.
Proof: Let V is regular open in Y.
Since f is almost contra-T*continuous and almost continuous,
f^{-1} (V) is T*-closed and T*-open.
Hence, f^{-1} (V) is clopen.
We obtain that f is almost T*-perfectly continuous.

3.30Theoerm: Let f:(X, τ, T)→ (Y,δ) be a function and x ∈ X. If there exists U is T*-open in X such that x□U and the restriction of f to U is an almost contra T*-continuous at x,then f is almost contra-T*continuous at x.

Proof: Suppose that *F* isregular closed containing f(x). Since $f\mid_U$ is almost contra T*-continuous at x,
There exists *V* is T*-open in U containing x
Such that f (V) = $(f\mid_U)(V) \subset F$.
Since U is T*-open in X containing x
ThatV is T*-open in X containing x
This shows clearly that f is almost contra T*-continuous.

3.31Theorem: Let f : (X,τ,T)→ (Y,δ,L) and g:(Y,δ,L) → (Z, φ) be function ,then the following properties is hold :
(1) If f is almost contra T*-continuous and g is almost L-perfectly continuous,then gof: (X, τ, T) → (Z, φ) is almost contra T*-continuous and almost T*-continuous.
(2) If f is almost contra T*-continuous and g is perfectly continuous, then gof: (X, τ, T) → (Z, φ) is T*-continuous and contra T*-continuous.
(3) If f is contra T*-continuous and g is almost perfectly continuous, then gof:(X, τ, T) → (Z, φ) is almost contra T*-continuous and almost T*-continuous.
Proof:

(1) Let V be any regular open in Z
Since g is almost L-perfectly continuous
$g^{-1}(V)$ is L-clopen(L-open and L-closed)

Since f is almost contra T*-continuous

$(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is T*-open and T*-closed

There for gof is almost contra T*-continuous and almost T*-continuous.

(2) Let V be open in (Z, φ)

Since g is perfectly continuous,

Then $g^{-1}(V)$ is clopen in (Z, φ)

This mean that $g^{-1}(V)$ is regular open in (Z, φ) and regular closed in (Z, φ)

Then $f^{-1}(g^{-1}(V))$ is T*-closed and T*-open

Now $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$

Hence gof is almost T*-continuous and contra T*-continuous.

(3) Let V be regular open in (Z, φ)

Since g is almost perfectly continuous,

Then $g^{-1}(V)$ is clopen

Hence $g^{-1}(V)$ is regular open and regular closed

Now f is contra T*-continuous

Then $f^{-1}(g^{-1}(V))$ is T*-closed

Hence $(gof)^{-1}(V)$ is T*-continuous

Hence gof is almost contra T*-continuous

Notice that $g^{-1}(V)$ is regular closed

Then $f^{-1}(g^{-1}(V))$ is T*-open

That is $(gof)^{-1}(V)$ is T*open

Then gof is almost contra T*-continuous and almost T*-continuous.

References

[1]. S. G. Crossley and S. K. Hildebrand, Semi-closure, Texas J. Sci. 22 (1971), 99–112.

[2]. J. Dontchev, Contra-continuous functions and strongly S-closed spaces, Internat J. Math. Math. Sci. 19 (1996), 303–310.

[3]. J. Dontchev, M. Ganster and I. Reilly, More on almost s-continuity, Indian J. Math. 41 (1999),139–146.

[4]. S. Jafari and T. Noiri, On contra-precontinuous functions, Bull. Malaysian Math. Sc. Soc.25 (2002), 115–128.

[5]. A.S. Mashhour, I.A. Hasanein and S.N. El-Deeb, A note on semi-continuity and precontinuity,Indian J. Pure Appl. Math. 13 (1982), 1119–1123.

[6]. A.S. Mashhour, M.E.A. El-Monsef and I.A. Hasanein, On pretopological spaces, Bull. Math.Soc. Sci. R S R. 28 (1984), 39–45.

[7]. A.A. Nasef and T. Noiri, Some weak forms of almost continuity, Acta. Math. Hungar.74 (1997), 211–219.

[8]. T. Noiri, super-continuity and some strong forms of continuity, Indian J. Pure Appl. Math.15 (1984), 241–250.

[9].E. Ekici, "Almost contra-precontinuous functions", Bull. Malaysian Math. Sc. Soc. 27(1) (2004), 53-65.

[10]. HadiJ.Mustafa ,and A.A.Hassan , T-open sets ,M.SC thesis ,mu'ta university Jordan ,2004