Compactly f- closed and compactly f-k-closed sets

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Abstract

In this paper , we introduce the concept of compactly f-closed set. In this work , we have also shown that where X is a T_2-spase,then a subset A of X is compactly f-closed if and only if A is an f-closed set .

Moreover , we have studied the concept of compactly f-k-closed set and shown that if the space is T2, then compactly if – closed is equivalent to compactly f-k-closed.

Introduction

In [1], Njastad, O. introduced the concept "$\alpha - open\ set$" in topology (a subset A of a space X is called an $\alpha - open\ set$ if $A \subseteq \overline{A^{0}}^{0}$, and he proved that the family of all "$\alpha - open\ sets$" in a topological space (X,T) is a topology on X .

In [2].Navalagi, G.. gives the concept " feebly open set " in topology (a subset A of a space X
is called feebly open set if there exists an open subset U of X such that $U \subseteq A \subseteq \overline{U}^{s}$, where $\overline{U}^{s}$ stands for the intersection of all semi-closed subset of X which contains U.

In [3] , Jankovic , D.S. proved that the concept " feebly open " and " $\alpha - open$ " conincide .

This paper consists of two section .Section one contains Basic concepts in general topology. Section two recalls the definitions of the compactly f- closed set and the compactly f-k-closed set and the relation between them.

1.Basic concepts

Definition 1.1,[2].
A subset B of a space X is called feebly open (f-open) set if there exists an open subset U of X such that $U \subseteq B \subseteq \overline{U}^{s}$.
The complement of a feebly open set is defined to be a feebly closed (f-closed)set .

Definition 1.2,[4]
A space X is called f-compact if every f-open cover of X has a finite sub cover.

Definition 1.3,[5,6,7]
Let X and Y be spaces and $f : X \to Y$ be a function .Then f is called feebly irresolute (f-irresolute) function if $f^{-1}(A)$ is an f- open in X , for every f-open set A in Y .

Definition 1.4,[8]
Let X and Y be spaces, the function $f : X \to Y$ is called a st-f-compcat function if the inverse image of each f-compact set Y is f-compact set in X .

Remark(1.5),[8]
Every f-irresolute function is an f-continuous function .

Proposition(1.6),[9]
Let Y be a subspace of a space X .Then if A is a pre-open subset of X and B is an f-open set in X, then $A \cap B$ is f-open in A.

Proposition (1.7),[8]
Let X be a space and $A \subseteq X$, $x \in X$.

Then $x \in \overline{A}^{f}$ if and only if there exists a net $(\chi_d)_{d \in D}$ in A and $\chi_d \xrightarrow{f} x$.

Proposition (1.8),[8]

(i) Every f-closed subset of an f-compact space is f- compact.

(ii) Every f-compact subset of an f-T_2 space is f-closed **.**

Proposition(1.9),[8]

In any space X, the intersection of any f-closed with any f-compact set is f-compact.

Proposition(1.10),[4]

Let X and Y be a space and $f : X \to Y$ be a function , then if f is f-irresolute function, then an image f(X) of any f-compact space X is an f-compact space.

Proposition(1.11),[8]

Let Y be an f-open subspace of space X and $A \subseteq Y$ .Then A is an f-compact set in Y if and only if A is f-compact set in X .

2- The main results

Definition (2.1) :

Let X be a space .A subset W of X is called a compactly f – closed set if for every f – compact set K in X ,$W \cap K$ is f – compact .

Example (2.2):

(i) Every finite subset of a space X is compactly f – closed set .

(ii) Every subset of indiscrete space is compactly f – closed set .

Proposition (2.3):

Every f – closed subset of a space X is compactly f – closed set .

Proof :

Let A be an f – closed subset of a space X and let K be an f- compact set in X . Then by proposition (1.9) , $A \cap K$ is an f – compact . Thus A is compactly f – closed set .

Theorem(2.4) :

Let X be a T_2 – space . A subset A of X is compactly f – closed if and only if A is f – closed set .

Proof :

$\Rightarrow$) Let A be a compactly f – closed set in X . Let, $x \in \overline{A}^f$, then by proposition (1.7) , there exists a net $(\chi_d)_{\alpha \in D}$ in A such that $\chi_d \xrightarrow{f} x$. Then $F = \{\chi_\alpha , x\}$ is an f – compact set .

Since A is an compactly f – closed , then $A \cap F$ is an f – compact set . So by proposition (1.8,ii) , $A \cap F$ is f – closed set . Since $\chi_d \xrightarrow{f} x$ and $\chi_d \in A \cap F$, then by proposition (1.7) , $x \in A \cap F \to x \in A$. Hence $\overline{A}^f \subseteq A$, Therefore A is f – closed set .

$\Leftarrow$) By proposition (2.3) .

Proposition (2.5):

Let $f : X \to Y$ be an f – irresolute , st – f – compact , one to one , function . Then A is compactly f – closed set in X if and only if f (A) is compactly f – closed set in Y .

Proof :

$\Rightarrow$) Let A be compactly f – closed in X and let K be an f – compact in Y . Since f is a st –f– compact function,then $f^{-1}(K)$ is f – compact set in X. So $A \cap f^{-1}(K)$ is an f – compact set . Then by proposition(1.10), $f(A \cap f^{-1}(K))$ is f – compact set .
But $f(A \cap f^{-1}(K)) = f(A) \cap K$,then

$f(A)\cap K$ is f – compact set .Hence f (A) is compactly f – closed set .

Conversely

$\Leftarrow$) Let f(A) be a compactly f – closed set in Y , (To prove A is compactly f – closed set in X), let K be an f – compact set in X . Since f is f – irresolute function , then by proposition (1.10) , f (K) is f–compact in Y,So $f(A)\cap f(K)$ is f–compact set . Since f is st–f– compact function, then

$f^{-1}(f(A)\cap f(K)) =$

$f^{-1}(f(A)\cap f^{-1}(f(K))$ is f – compact set in X .

Since f is one to one function, then $A = f^{-1}(f(A))$ and

$K = f^{-1}(f(K))$, thus

$A\cap K = f^{-1}(f(A))\cap f^{-1}(f(K))$. Hence $A\cap K$ is f – compact set in X . Therefore A is compactly f – closed set in X .

Proposition(2.6) :

Let B be an f – open subspace of a space X . Then B is compactly f – closed if and only if the inclusion function $i_B : B \to X$ is st – f – compact .

Proof :

$\Rightarrow$) Let K be an f – compact set in X , then $B\cap K$ is an f – compact set in X , thus by proposition (1.11) , $B\cap K$ is an f – compact set in B .

But $i_B^{-1}(K) = B\cap K$, then $i_B^{-1}(K)$ is an f – compact set in B . Hence $i_B : B \to X$ is a st – f – compact function .

Conversely :

$\Leftarrow$) Let K be an f – compact set in X . Since $i_B : B \to X$ is a st – f – compact function,then $i_B^{-1}(K)$ is an f – compact set in B.Thus by proposition (1.11) , $i_B^{-1}(K)$ is an f – compact set in X .

But $i_B^{-1}(K) = B\cap K$, then $B\cap K$ is an f – compact set in X , for every f – compact set K in X . Therefore B is a compactly f – closed set in X.

Definition (2.7) :

Let X be a space . Then a subset A of X is called compactly f – k – closed if for every f – compact set K in X, $A\cap K$ is f – closed set

Example (2.8) :

Every subset of a discrete space is compactly f – k - closed set .

Proposition(2.9) :

Every compactly f–k–closed subset of a space X is compactly f– closed.

Proof :

Let A be a compactly f – k – closed subset of X and let K be an f – compact set in X , then $A\cap K$ is f – closed set . Since $A\cap K \subseteq K$ and K is f – compact set , then by proposition (1.8, i) $A\cap K$ is f – compact set .

Therefore A is compactly f – closed set .

Theorem (2.10):

Let X be T_2 – space and A is a subset of X . Then the following statement are equivalent :

(i) A is compactly f – closed .

(ii) A is compactly f-k-closed .

(iii) A is f- closed .

Proof :

(i $\leftrightarrow$ ii)

(i $\rightarrow$ ii) Let A be a compactly f – closed subset of X and let K be an f– compact set in X . Then $A \cap K$ is f – compact set . Since X is T2 – space, then by proposition (1.8 ,ii) , $A \cap K$ is f – closed set .

Hence A is a compactly f – k – closed set .

(ii $\rightarrow$ i) By proposition (2.9) .

(iii $\leftrightarrow$ i) By Theorem (2.4)

Remark (2.11):

If X is not T2 – space then need not that every compactly f – closed set is compactly f – k – closed set as the following example .

Example (2.12):

Let $X = \{a, b, c\}, A = \{a, b\}$, $B = \{a, c\}$ be a sets and let (X , T) is indiscrete topological space . Then X is not T_2 – space and A is compactly f – closed set, but A is not compactly f – k – closed set , Since B is f – compact set in X , then $A \cap B = \{a\}$ is not f – closed set .

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