Fuzzy - α– $T_{1/2}$ space

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Abstract

In this paper , we introduce the definition of Fuzzy- α– $T_{1/2}$ space and its some proposition about this subject .

Introduction

In A. S. Bin Shahna [1] the concept fuzzy α -open sets. The concepts fuzzy α -continuous and fuzzy α- irresolute function were introduced by Hakeem A. othman and A. S. Bin Shahna in [9 ,1] . We introduce new definitions of Fuzzy - α - generalized – continuous , Fuzzy - α - generalized irresolute and Fuzzy - α *- generalized irresolute functions which we need in Propositions of Fuzzy - α– $T_{1/2}$ space.

1- Basic Concepts

Definition 1.1 [1]

A fuzzy set A in a fuzzy topological space "fts" is called :

(i) fuzzy α -open or fuzzy feebly open if $A \le \text{int}(cl(\text{int}(A)))$

(ii) fuzzy α -closed or fuzzy feebly closed if $cl(\text{int}(cl(A))) \le A$

Definition 1.2 [2,3]

Let A be any fuzzy set in a fts X . The closure of A is the intersection of all fuzzy closed Sets containing A , denoted by cl(A) .That is $cl(A) = \wedge\{B : B$ is fuzzy closed set , $B \ge A\}$

Definition 1.3 [4,1 ,5]

Let B be a fuzzy set in a fts X .Then fuzzy α – closure of B is denoted and defined by $\alpha\, cl(B) = \wedge\{\nu : B \le \nu, \nu$ is fuzzy α –closed set }

Definition 1.4 [6, 7,8]

A fuzzy set E in a fts X is called :

(i) fuzzy generalized closed set , if $cl(E) \le H$ whenever $E \le H$ and H is fuzzy open ,We briefly denoted it as Fg – closed set

(ii) fuzzy generalized α - closed set , if $\alpha\, cl(E) \le H$ whenever $E \le H$ and H is fuzzy open We briefly denoted it as Fg α – closed set .

(iii) fuzzy α - generalized closed set , if $\alpha\, cl(E) \le H$ whenever $E \le H$ and H is fuzzy α - open ,We briefly denoted it as Fαg – closed set

Proposition 1.5 [6,4,5]

In a fuzzy topological space (X,T) , the following hold :

(i) Every fuzzy closed set is Fα– closed .

(ii) Every fuzzy closed set is Fg – closed .

(iii) Every fuzzy α - closed set is Fαg – closed.

Definition 1.6 [6]

A fuzzy topological space (X,T) is said to be a fuzzy – $T_{1/2}$ space if every Fg – closed set is fuzzy closed .

Definition 1.7 [9,2 ,1]

A mapping $f : X \rightarrow Y$ is said to be :

(i) fuzzy continuous if $f^{-1}(V)$ is fuzzy open (fuzzy closed) set in X for each fuzzy open (fuzzy closed) set V in Y.

(ii) fuzzy α - continuous if $f^{-1}(V)$ is fuzzy α - open (fuzzy α- closed) set in X for each fuzzy open (fuzzy closed) set V in Y.

(iii) fuzzy α – irresolute if $f^{-1}(V)$ is fuzzy α - open (fuzzy α- closed) set in X for each fuzzy α -open (fuzzy α- closed) set V in Y.

Definition 1.8[2 ,9]

A mapping $f : X \rightarrow Y$ is said to be :

(i) fuzzy – open (fuzzy closed) if $f(U)$ is fuzzy open (fuzzy closed) set in Y for each fuzzy open (fuzzy closed) set U in X .

(ii) fuzzy α – open (fuzzy α - closed) if $f(U)$ is fuzzy α -open (fuzzy α - closed) set in Y for each fuzzy open (fuzzy closed) set U in X .

(iii) fuzzy α *- open (fuzzy α *- closed) if $f(U)$ is fuzzy α - open (fuzzy α- closed) set in Y for each fuzzy α- open (fuzzy α- closed) set U in X .

2- The main results

Definition (2.1) :

If every fuzzy - α - generalized closed (briefly F – α - g– closed) set in X fuzzy - α – closed (briefly F – α - closed) set in X , then the space is fuzzy - α – $T_{1/2}$ space which denoted by F- α – $T_{1/2}$ space .

Theorem (2.2)

A fuzzy topological space (X , T) is F- α – $T_{1/2}$ space iff F α O (X , T) = F α G O (X , T) .

Proof

$\rightarrow$ Let (X , T) be F- α – $T_{1/2}$ space , let A be F α G O set in X , then 1- A is F α g – closed set in X . since (X , T) is F- α – $T_{1/2}$ space , then 1 – A is a fuzzy – α – closed set (F – α – closed set) , thus $A \leq F\alpha$ O (X ,T) .

since F α O(X , T)≤ F α G O(X , T) , then F α G O(X , T) = F α O(X , T) .

$\leftarrow$ Let F α O(X , T) = F α GO(X , T) , let A is F- α- g – closed set , then 1- A is F- α- g open set . By hypothesis , 1- A ≤ F α O(X , T) . Hence (X , T) is F- α – $T_{1/2}$ space .

Definition (2.3)

A $f : X \rightarrow Y$ be a function from afts X to afts Y is called:

1- Fuzzy - α - generalized – continuous (F – α – g – continuous) function if $f^{-1}(V)$ is F – α – g - closed set in X , for every fuzzy closed set V in Y .

2 - Fuzzy - α - generalized irresolute (F – α – g – irresolute) function if $f^{-1}(V)$ is F – α – g - closed set in X , for every F – α – g - closed set V in Y .

3 - Fuzzy - α *- generalized irresolute (F – α *- g – irresolute) function if $f^{-1}(V)$ is F – α –closed set in X , for every F – α – g - closed set V in Y .

Proposition (2.4)

Every F – α – g – irresolute function is F – α – g – continuous function.

Proof

Let $f : X \rightarrow Y$ be F – α – g – irresolute function and let U be fuzzy closed set in Y, by Proposition 1.5 (i,iii) , then U is F – α – g – closed set in Y. Since f is F – α – g – irresolute function , then $f^{-1}(U)$ is F – α – g – closed set in X . Hence f is F – α – g – continuous function .

Proposition (2.5)

Let $f : X \to Y$ be a function from afts X to afts Y ,then :

(i) If f is F – α – g – continuous function and X is F- α – $T_{1/2}$ space .Then f is fuzzy α - continuous function .

(ii) If f is F – α – g – irresolute function and X is F- α – $T_{1/2}$ space .Then f is fuzzy α - continuous function .

(iii) If f is F – α – g – irresolute function and X is F- α – $T_{1/2}$ space .Then f is fuzzy – α *- g – irresolute function .

Proof

(i) Let V be fuzzy closed set of Y , since f is F – α – g – continuous function , then $f^{-1}(V)$ is F – α – g – closed set of X . since X is F- α – $T_{1/2}$ space , then $f^{-1}(V)$ is fuzzy – α – closed set . Therefore $f : X \to Y$ is fuzzy α - continuous function .

(ii) By Proposition (2.4)

(iii) Let U be fuzzy- α - g –closed set of Y , since f is F – α – g – irresolute function , then $f^{-1}(U)$ is F – α – g – closed set of X . since X is F- α – $T_{1/2}$ space , then $f^{-1}(V)$ is fuzzy – α – closed set . Hence $f : X \to Y$ is fuzzy – α *- g -irresolute function .

Proposition (2.6)

Let X ,Y and Z be ftss, and $f : X \to Y$, $g : Y \to Z$ and $gof : X \to Z$ be function , then :

(i) If f is fuzzy α –irresolute function and $g : Y \to Z$ is F – α – g – irresolute function such that Y is F- α – $T_{1/2}$ space .Then gof is fuzzy – α *-g irresolute function .

(ii) If f is fuzzy α –irresolute function and $g : Y \to Z$ is F – α – g – continuous function , such that Y is F- α – $T_{1/2}$ space .Then gof is fuzzy α- continuous function .

Proof

(i) Let U be F - α – g – closed set in Z ,since g is F – α – g – irresolute function , then $g^{-1}(U)$ is F - α – g – closed set in Y . since Y is F- α – $T_{1/2}$ space ,then $g^{-1}(U)$ is F - α – closed set in Y. But f is fuzzy α – irresolute function , then $f^{-1}(g^{-1}(U))$ is is F - α – closed set in X.

But $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$. Therefore gof fuzzy is fuzzy – α *-g - irresolute function .

(ii)) Let U be fuzzy closed set in Z ,since g is F – α – g – continuous function , then $g^{-1}(U)$ is F - α – g – closed set in Y . since Y is F- α – $T_{1/2}$ space ,then $g^{-1}(U)$ is F - α – closed set in Y. But f is fuzzy α – irresolute function , then $f^{-1}(g^{-1}(U))$ is is F - α – closed set in X.

But $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$. Therefore gof is fuzzy α- continuous function .

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Proposition (2.7)

Let $f : (X,T) \to (Y,T^{/})$ be onto , Fuzzy - α - generalized irresolute and fuzzy α *- closed function . If X is F- α – $T_{1/2}$ space , then (Y , $T^{/}$) is F- α – $T_{1/2}$ space .

Proof

Let A be F – α – g - closed set in Y , since f is Fuzzy - α - generalized irresolute function , then $f^{-1}(A)$ is F – α – g - closed set in X . since X is F- α – $T_{1/2}$ space , then $f^{-1}(A)$ is F – α –closed set in X .

But f is fuzzy α * - closed function , then f ($f^{-1}(A)$) is F – α –closed set in Y . since f is onto , then f ($f^{-1}(A)$) = A . Thus A is F – α –closed set in Y. Therefore (Y, $T^{/}$) is F- α – $T_{1/2}$ space .

Proposition (2.8)

Let X,Y and Z be ftss, and let $f : X \to Y$, $g : Y \to Z$ and $gof : X \to Z$ be function , then :

(i) If $gof : X \to Z$ be F – α – g – continuous function and $f : X \to Y$ is fuzzy α *- closed function such that X is F- α – $T_{1/2}$ space ,then $g : Y \to Z$ is fuzzy α - continuous function .

(ii) If $gof : X \to Z$ be F – α – g – irresolute function and $f : X \to Y$ is fuzzy α *- closed function such that X is F- α – $T_{1/2}$ space , then $g : Y \to Z$ is fuzzy - α *-g- irresolute function .

Proof

(i) Let U be fuzzy closed set in Z, since gof is F – α – g – continuous function , then $(gof)^{-1}(U)$ is F – α – g – closed set in X . But X is F- α – $T_{1/2}$ space , then $(gof)^{-1}(U)$ F – α –closed set in X. Since f is fuzzy α *- closed function ,then $f(gof)^{-1}(U)$ is F – α –closed set in Y. But

$$f(gof)^{-1}(U) = f(f^{-1}(g^{-1}(U)) = g^{-1}(U) ,$$

Then $g^{-1}(U)$ is F – α –closed set in Y. Hence $g : Y \to Z$ is fuzzy α - continuous function .

(ii) Let U be F – α – g - closed set in Z, since gof is F – α – g – irresolute function , then $(gof)^{-1}(U)$ is F – α – g – closed set in X . But X is F- α – $T_{1/2}$ space , then $(gof)^{-1}(U)$ F – α –closed set in X. Since f is fuzzy α *- closed function ,then $f(gof)^{-1}(U)$ is F – α –closed set in Y. But

$$f(gof)^{-1}(U) = f(f^{-1}(g^{-1}(U)) = g^{-1}(U) ,$$

Then $g^{-1}(U)$ is F – α –closed set in Y. Hence $g : Y \to Z$ is fuzzy α *- g- irresolute function .

Definition (2.9)

If a space is called fuzzy – α * -$T_{1/3}$ space if every fuzzy – α – g – closed set is fuzzy closed set .

Proposition (2.10)

Every fuzzy – α * -$T_{1/3}$ space is F- α – $T_{1/2}$ space .

Proof

Let A be F – α – g - closed set in X , since (X ,T) is fuzzy – α * -$T_{1/3}$, then A is fuzzy closed set in X, ,thus A fuzzy – α - closed set in X . Hence (X ,T) F- α – $T_{1/2}$ space .

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