On Mixing Properties in Topological Dynamics

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Abstract

Mixing property was classified in the first book of topological dynamics written by Gottschalk & Hedlund ,[4], (1955) as one of the regionally transitive properties . It is concerned with the existence of a compact subset K of the phase group G of a topological transformation group (X,G,π) such that for every U,V non empty open subsets of X ,there exists a compact subset K of G such that, for every

$g \in K^C, \Rightarrow Ug \cap V \neq \phi$

In this work we introduced new different types of mixing ,and studied there relations with other transitive properties. Moreover we studied some relations between them and recursion dynamical properties such as periodicity , almost periodicity and recurrence. we gave conditions to transfer some types of mixing under suitable homomorphism.

Introduction

There are three types of studies in dynamics, local, global ,and abstraction. The field of our work is the third one .It is a study of topological transformation groups It is the study of the action of a topological on a topological space.

In this work we introduced new different types of mixing such as mixing at appoint, point mixing, point wise mixing, uniformly mixing ,regionally mixing ,extensively mixing and regionally extensively mixing ,and studied these relations with some other dynamical properties, such as transitive at a point ,point transitive, point wise transitive, regionally transitive, uniformly transitive , extensively transitive and regionally extensively transitive . Moreover we studied some relations between them and minimal ,periodicity, and recursion dynamical properties such as almost periodicity and recurrence. we gave conditions to transfer some types of mixing under suitable homomorphism .

Section 1

Preliminaries

In this section we recall the basic definitions and facts needed in this work .

We assume that all our spaces are T_2 .

(1-1)Definition:- [6]

The group $(G,*)$ is said to be a topological group , if (G,τ_G) is a topological space , such that the function $f: G\times G \to G$ defined by $f(g_1,g_2)=g_1*g_2^{-1}$ is continuous

(1-2)Examples:-

1-$(R,+)$ is topological group with the usual topology and usual addition .

2- Every group with discrete topology is a topological group .

3- Every subgroup of a topological group is a topological group .

i.e.:- $(Z,+)$ is a topological group .

(1-3)Definition:- [4]

A subset A of $(G,*)$ is said to be {left} {right} syndetic in $(G,*)$ provided that $\{G = AK\}$ $\{G = KA\}$ for some compact subset K of G .

(1-4)Definition:- [4]

The subset N of a topological group $(G,*)$ is said to be Replete if for every compact subset K of G there exists $g_1,g_2 \in G$ such that $g_1Kg_2 \subset N$.

(1-5)Definition:- [4]

The subset A of a topological group $(G,*)$ is said to be Extensive if the intersection of Replete semi group N in G with A is not empty set $(A\cap N \neq \phi)$.

(1-6)Definition:- [4]

A topological transformation group , or more briefly , a transformation group , is defined by the ordered triple (X,G,π) consisting of a non empty topological space X , a topological group G and a mapping $\pi : X\times G \to X$ such that :-

(1)$(x,e)\pi = x$, $(x\in X)$ where e is the identity element of G .

(2) $((x,g_1)\pi, g_2)\pi = (x, g_1g_2)\pi$ $(x\in X; g_1,g_2 \in G)$.

(3) π is continuous .

If (X,G,π) is a transformation group , then X is called the phase space , and G is called the phase group , and π is called the action .

(1-7)Remark:-

The following statements are used to mean same thing as follows :-

(a) (X,G,π) is a topological transformation group .

(b) (X,G) is a transformation group .

(c) (X,G) is a transformation group under π .

(d) (X,G) is a transformation group with respect to π .

(e) G is a transformation group on X under π .

(f) G is a transformation group on X with respect to π .

(g) G is a transformation group of X under π .

(h) G is a transformation group of X with respect to π .

(i) G act as a transformation group on X under π .

(j) G act as a transformation group on X with respect to π .

(k) G act as a transformation group of X under π .

(l) G act as a transformation group of X with respect to π .

(1-8)Definitions:- [8]

1- (X,G) is called Dynamical system , If X is a metric space , $G=R$.

2- (X,G) is called Continuous Flow , If $G=R$.

3- (X,G) is called Discrete Flow or (cascade) , If $G=Z$.

(1-9)Definition:- [8]

Every homeomorphism $f:X\to X$ induced a discrete flow (X,Z,π) denoted by (X,f) where :

$$(x,n)\pi = f^{n}(x) = \underbrace{fofofo...of}_{\text{n-times}}(x) .$$

(1-10)Definition:- [8]

If (X,G,π) is a topological transformation group , then :-

1- By g-transition π^{g} , $\forall g\in G$ we mean the function $\pi|(X\times\{g\})$ from X onto X .

2- π^{e} is the identity map on X .

3- If $g,r\in G$, then $\pi^{g}o\pi^{r}=\pi^{g*r}$.

4- If $g\in G$, then π^{g} is a one to one , onto , and $(\pi^{g})^{-1}=\pi^{g^{-1}}$.

5- The transition projection of (X,G) is $\lambda:G\to T$ such that $\lambda(g)=\pi^{g}$, $\forall g\in G$ is homomorphism .

6- The set $T=\{\pi^{g}:g\in G\}$ is called transition group .

7- By x-motion under G π_x , $\forall x \in X$ we mean the function $\pi|(\{x\}\times G)$ from G into X .

8- The motion injection of (X,G) is the map $\theta: X \to M$ such that $\theta(x)=\pi_x$, $\forall x \in X$.

9- The image of G under π_x is the orbit of x under G and denoted by $\pi_x(G)$ or xG .

i.e.:- , the orbit of x is the set

$$xG=\{xg : g\in G\}=\{(x,g)\pi : g\in G\}=\{\pi_x(g): g\in G\}$$

.

10- The closure of xG $(\overline{xG})$ is called the orbit closure of x under G .

11- The set $M=\{\pi_x : x\in X\}$ is called motion space .

(1-11)<u>Example:-</u>

In the topological transformation group (R,Z,π) The orbit of the point $1/5$ in $[0,1]$, where f is a function from $[0,1]$ into itself defined by $(x)f=\sqrt{x}, \forall x\in[0,1]$ is :-

$1/5Z = \{\ldots, 1/125, 1/25, 1/5, 1/\sqrt{5}, 1/\sqrt[4]{5}, \ldots\}$ under Z .

The orbit closure $1/5$ is :-

$$\overline{1/5Z} = \{\ldots, 1/125, 1/5, 1/5, 1/\sqrt{5}, 1/\sqrt[4]{5}, \ldots\} \cup \{0,1\}$$

(1-12)<u>Definition:-</u> [4]

If (X,G) is a transformation group and K is subset of X . K is called invariant subset of X under G if and only if $KG\subset K$, where :- $KG=\{(k,g)\pi : k\in K, g\in G\}$

(1-13)<u>Definition:-</u> [4]

If (X,G) is a transformation group . A subset A of X is called minimal under G , if $A\neq\phi$, closed and invariant under G , and A does not contain a proper subset which is non empty closed and invariant under G .

(1-14)<u>Definition:-</u> [8]

If (X,G) is a transformation group and U is open subset of X . The set $UG=\{(u,g)\pi : u\in U, g\in G\}$ is called a tube .

(1-15)<u>Definition:-</u> [4]

If $x\in X$, then x is said to be recurrent under G if for each U a neighborhood of x there exists an extensive subset A of G such that $xA\subset U$.

(1-16)<u>Definition:-</u> [4]

If (X,G) is a transformation group and $x\in X$. The period of x under G is defined to be the greatest subset P of G such that $xP=x$.

(1-17)Example:-

Suppose that $X = [0,1] \subset R$ with usual topology and $f : [0,1] \to [0,1]$ defined by $f(x) = 1 - x$, then f is homeomorphism . So

$$f(\frac{1}{2}) = \frac{1}{2}$$

$$f^2(\frac{1}{2}) = f(f(\frac{1}{2})) = \frac{1}{2}$$

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$$f^n(\frac{1}{2}) = \underbrace{f(f(f\ldots(f(\frac{1}{2}))))}_{\text{n-times}} = \frac{1}{2}$$

Then the period of point $\frac{1}{2}$ is Z because it's fixed . The period of $x \neq \frac{1}{2}$ is $2Z$.

(1-18)Definition:- [4]

Let $x \in X$. The transformation group G is said to be periodic at x and the point x is said to be periodic under G provided that the period of G at x is a syndetic subset of G . The transformation group G is said to point wise periodic provided that G is periodic at every point of X .

(1-19)Example:-

If (X, Z, f) is a transformation group, where X is a unit circle defined by :

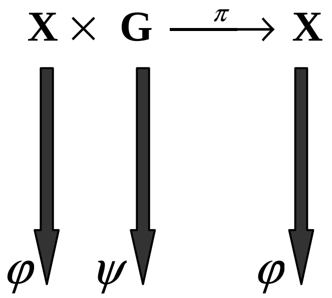
$X = \{(1,\theta) : 0 \le \theta \le 360°\}$ and f is an action of Z on X defined by : $(x)f^n = (r,\theta)f^n = (1, \theta + n\psi)$. If $x = (1,\theta)$ and $\psi = \frac{\pi}{6} = 30°$. Then there exists 12 points in the orbit of x under f . i.e.:- the orbit of x is $(1,\theta), (1,\theta+\psi), (1,\theta+2\psi), \ldots, (1,\theta+11\psi)$. Then this point is periodic under f and by period $A = 12Z$ which is the set $\{\ldots, -24, -12, 0, 12, 24, \ldots\}$ subset of Z and there exists compact subset K of G such that $K = \{0,1,2,\ldots,12\}$ and $12Z + K = Z$.
So $12Z$ is syndetic set and it's greatest syndetic set such that $(x)12Z = x$.

(1-20)Definition:- [8]

If (X, G) is a transformation group and $x \in X$, then x is said to be almost periodic under G , if for each U a neighborhood of x there exists a syndetic subset A of G such that $xA \subset U$.

(1-21)Definition:- [1]

The map $(\varphi, \psi) : (X, G, \pi) \to (Y, H, \sigma)$ is called a homomorphism between the two topological transformation groups (X, G, π) and (Y, H, σ) if $\varphi : X \to Y$ is continuous and $\psi : G \to H$ is continuous homomorphism , if $((x,g)\pi)\varphi) = ((x,g)(\varphi,\psi))\sigma$.
Or the following diagram commutative .



(1-22)Definition:- [1]

The homomorphism (φ,ψ) between two transformation groups (X, G_1, π_1) and (Y, G_2, π_2) is said to be :-

1- Indomorphism if $(\varphi,\psi):(X,G_1,\pi_1)\rightarrow(X,G_1,\pi_1)$.

2- Epimorphism if φ,ψ are onto .

3- Monomorphism if φ,ψ are one to one .

4- Isomorphism if φ,ψ are one to one and onto . The Isomorphism is called Automorphism if $(\varphi,\psi):(X,G_1,\pi_1)\rightarrow(X,G_1,\pi_1)$.

(1-23)Example:-

If $([0,1], Z, \pi_1)$ and $([0,1], Z, \pi_2)$ are transformation groups where :-

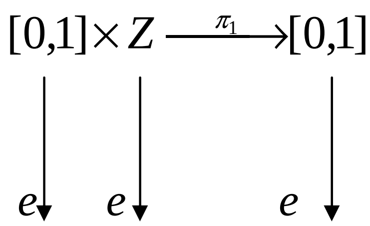
$(x,n)\pi_1 = xf^n$ for every $x\in[0,1]$ and $n\in Z$, where $f:[0,1]\rightarrow[0,1]$, $(x)f = x^3$

$(x,n)\pi_2 = xg^n$ for every $x\in[0,1]$ and $n\in Z$, where $g:[0,1]\rightarrow[0,1]$, $(x)g = x^2$. We note that (e,e) is not homomorphism because

$$((\frac{1}{2},2)\pi_1)e = (\frac{1}{2})f \circ f = (\frac{1}{2})^9$$

$$((\frac{1}{2})e,(2)e)\pi_2 = (\frac{1}{2},2)\pi_2 = (\frac{1}{2})g \circ g = (\frac{1}{2})^4$$

That is the following diagram is not commutative:



Section 2

Certain mixing properties

In this section we study and introduce several types of mixing properties.

(2-1)Definitions:-[4]

1-Let $x \in X$ .The transformation group (X,G) is said to be transitive at x and the point x is said to be transitive under (X,G) provided that if U is a non empty open subset of X ,then there exists $t \in G$ such that $xt \in U$. The transformation group (X,G) is said to be {point wise}{point} transitive provided that (X,G) is transitive at {every}{some} point of X .

2- The transformation group (X,G) is said to be (regionally) transitive provided that if U and V are non empty open subsets of X , then there exists $t \in G$ such that $Ut \cap V \neq \phi$.

3- Let $x \in X$. The transformation group (X,G) is said to be extensively transitive at x and the point x is said to be extensively transitive under (X,G) provided that if U is a non empty open subset of X , then there exists an extensive subset A of G such that $xA \subset U$. The transformation group (X,G) is said to be {point wise} {point} extensively transitive provided that (X,G) is extensively transitive at {every}{some} Point of X .

4- The transformation group (X,G) is said to be universally transitive provided that if $x, y \in X$,then there exists $t \in G$ such that $xt = y$. it's clear that , if (X,G) is universally transitive , then it's point wise transitive .

5- The transformation group (X,G) is said to be extensively (regionally) transitive provided that if U and V are non empty open subsets of X , then there exists an extensive subset A of G such that $t \in A$ implies $Ut \cap V \neq \phi$.

(2-2)Propositions:- [7]

1- If (X,G) is a point transitive , then it is (regionally) transitive .

2- If (X,G) is a point wise transitive , then it is (regionally) transitive.

(2-3)Remarks:-

If (X,G) is an extensively transitive at x , then:-

1- For every open subset V of X , there exists an extensive subset A of G such that $U_x A \cap V \neq \phi$ (where U_x is a neighborhood of x) $\Leftrightarrow$ $\exists t \in A \subset G \ni U_x t \cap V \neq \phi$.

2- For every neighborhood U_x of x in X , there exists an extensive subset A of G such that $xA \subset U_x$. And consequently we get the following remark :-

(2-4)Remark:-

If x is an extensively transitive point of a transformation group (X,G), then x is recurrent point under G.

(2-5)Proposition:- [3]

If (X,G) is a transformation group, X is locally compact T_2 and $x \in X$, then x is almost periodic under G if and only if $\overline{xG}$ is compact minimal subset of X.

Proof:-

$(\Rightarrow)$ Let X be almost periodic under G, and U be a compact neighborhood of X, then there exists a compact subset K of G such that $AK = G$, where $A = \{g \mid xg \in U\}$ (A is syndetic), then $xG \subset xgK = UK$ which is compact $\Rightarrow \overline{xG} \subset UK \Rightarrow \overline{xG}$ is compact. Now if $y \in xG$, then as above $y \in \overline{xG} \subset UK$, thus $yG \cap U \neq \phi$ and U arbitrary neighborhood of $x \Rightarrow x$ is a limit point of $yG \Rightarrow x \in \overline{yG} \Rightarrow \overline{xG} = \overline{yG} \Rightarrow \overline{xG}$ minimal.

$(\Leftarrow)$ Suppose that $\overline{xG}$ is compact minimal subset of X, and let U be an open neighborhood of X. Since $\overline{xG} - UG$ is closed, invariant proper subset of $\overline{xG}$ and $\overline{xG}$ minimal, then $\overline{xG} - UG = \phi$. Then $\overline{xG} \subset UG \Rightarrow \exists F$ finite subset of G such that $\overline{xG} \subset UF$.

Let $A = \{g \in G \mid xg \in U\}$, then $\forall g \in G, \exists u \in U, h \in G, \ni xg = uh$ thus $xgh^{-1} \in U \Rightarrow gh^{-1} \in A$ and $g \in AF \Rightarrow G = AF \Rightarrow A$ is syndetic $\Rightarrow x$ almost periodic.

(2-6)Definition:-

Let $x \in X$. The transformation group (X,G) is said to be mixing at x and the point x is said to be mixing under (X,G) provided that if U is a non empty open subset of X, then there exists a compact subset K of G such that $t \in G - K$ implies $xt \in U$. The transformation group (X,G) is said to be {point wise} {point} mixing provided that (X,G) is mixing at {every} {some} point of X.

(2-7)Proposition:-

The transformation group (X,G) is mixing at the point x of X if and only if $\overline{x(G-K)} = X$ where K is a compact subset of G.

Proof:-

If x is a mixing point of a transformation group (X,G), then for every open set V in X, $x(G-K) \cap V \neq \phi \Leftrightarrow x(G-K)$ is dense in $X \Leftrightarrow \overline{x(G-K)} = X$.

(2-8)Corollary:-

The transformation group (X,G) is mixing at the point x of X if and only if $\overline{U_x(G-K)}=X$ where K is a compact subset of G and U_x is a neighborhood of x .

That is , $\forall U_x, \forall V$, $U_x(G-K)\cap V\neq\phi$ $\Leftrightarrow$ $\exists t\in G-K \ni U_x t\cap V\neq\phi$.

Moreover , every point of xG is mixing .

(2-9) Proposition:-

If the transformation group (X,G) is point wise mixing , then it is point wise transitive .

Proof:-

Let (X,G) be a point wise mixing transformation group and let $x\in X$ and U is a non empty open subset of X . Since (X,G) is point wise mixing , then x is mixing point . Then there exists a compact subset K of G such that $t\in G-K$ implies $xt\in U$. But $t\in G$ implies $xt\in U$. So (X,G) is point wise transitive .

(2-10)Proposition:-

If the transformation group (X,G) is mixing at $x\in X$ and X is compact minimal , then x is almost periodic under G .

Proof:-

Let $x\in X$ is mixing point and X is compact minimal space . Since every mixing point is transitive point (by (2-8)proposition) and X is compact minimal . Then x is almost periodic (by (2-5)proposition) .

(2-11)Definition:- [4]

The transformation group (X,G) is said to be (regionally) mixing provided that if U and V are non empty open subsets of X , then there exists a compact subset K of G such that $t\in G-K$ implies $Ut\cap V\neq\phi$.

(2-12)Proposition:-

If (X,G) is a point wise mixing , then it's (regionally) mixing .

Proof:-

Let (X,G) be a point wise mixing and let U and V are non empty open subsets of X . Since (X,G) is point wise mixing .Then it's mixing at every point in U and V . That is , there exists a compact subset K of G such that $t\in G-K$ implies $xt\in V$, where $x\in U$. But $xt\in Ut$, for every $t\in G-K$. So $Ut\cap V\neq\phi$. Then (X,G) is (regionally) mixing .

(2-13)Definition:-

The transformation group (X,G) is said to be universally mixing provided that if $x,y\in X$, then there exists a compact subset K of G such that $t\in G-K$ implies $xt=y$.

(2-14)<u>**Remark:-**</u>

If (X,G) is universally mixing , then $\forall x,y \in X, \exists t_x$, $t_y \in G-K \ni (U_x)t_x \cap U_y \neq \phi, U_x \cap (U_y)t_y \neq \phi$ (where U_x is a neighborhood of x and U_y is a neighborhood of y and K is a compact subset of G) .

(2-15)<u>**Proposition:-**</u>

If (X,G) is universally mixing . Then it's point wise mixing .

Proof:-

Let (X,G) be a universally mixing and let $x \in X$ and let U be a non empty open subset of X . Since (X,G) is universally mixing . Then for every $y \in U$, there exists a compact subset K of G such that $t \in G-K$ implies $xt = y$. That is , $xt \in U$, for every $t \in G-K$. Then (X,G) is point wise mixing .

(2-16)<u>**Proposition:-**</u>

If (X,G) is universally mixing . Then it's (regionally) mixing .

Proof:-

Let (X,G) be a universally mixing and let U and V are non empty open subsets of X . Since (X,G) is universally mixing . So if $x \in U$ and $y \in V$, then there exists a compact subset K of G such that $t \in G-K$ implies $xt = y$. So $xt \in V$.

But $xt \in Ut$, for every $t \in G-K$. So $Ut \cap V \neq \phi$, for every $t \in G-K$ .Then (X,G) is (regionally) mixing .

(2-17)<u>**Definition:-**</u>

Let $x \in X$. The transformation group (X,G) is said to be extensively mixing at x and the point x is said to be extensively mixing under (X,G) provided that if U is a non empty open subset of X , then there exists a compact subset K of G such that there exists an extensive subset A of $G-K$ implies $xA \subset U$. The transformation group (X,G) is said to be {point wise} {point} extensively mixing provided that (X,G) is extensively mixing at {every} {some} point of X .

(2-18)<u>**Remarks:-**</u>

If (X,G) is an extensively mixing at x , then:-

1- For every open subset V of X , there exists an extensive subset A of $G-K$ where K is a compact subset of G such that $U_x A \cap V \neq \phi$ (where U_x is a neighborhood of x) $\Leftrightarrow$ $\exists t \in A \subset G-K \ni U_x t \cap V \neq \phi$.

2- For every neighborhood U_x of x in X , there exists an extensive subset A of $G-K$ where K is a compact subset of G such that $xA \subset U_x$.

(2-19)Definition:-

The transformation group (X,G) is said to be extensively (regionally) mixing provided that if U and V are non empty open subsets of X , then there exists a compact subset K of G such that there exists an extensive subset A of $G-K$ such that $t\in A$ implies $Ut\cap V\neq\phi$.

(2-20)Proposition:-

If the transformation group (X,G) is a point wise extensively mixing , then it is extensively (regionally) mixing .

Proof:-

Let (X,G) be a point wise extensively mixing transformation group and let U and V are non empty open subsets of X . Take $x\in U$, then there exists a compact subset K of G such that there exists an extensive subset A of $G-K$ implies $xA\subset V$. But $xA\subset UA$. So $UA\cap V\neq\phi$. Then (X,G) is extensively (regionally) mixing.

(2-21)Proposition:-

If (φ,ψ) is onto and φ is one to one homomorphism from (X,G,π) into (Y,H,σ) and X is universally mixing under G , then Y is universally mixing under H .

Proof:-

Suppose that $y_1,y_2\in Y$ and since φ is onto , then there exists $x_1,x_2\in X$ such that $x_1\varphi=y_1$, $x_2\varphi=y_2$. Since X is universally mixing , then there exists a compact subset K of G such that $t\in G-K$ implies $x_1t=x_2$. So $y_2=x_2\varphi=x_1t\varphi=x_1\varphi t\psi=y_1(t)\psi$. But $(t)\psi\in(G-K)\psi=G\psi-K\psi=H-K\psi$ and $K\psi$ is compact in H (because ψ is continuous) implies $y_2=y_1(t)\varphi$. Then Y is universally mixing under H .

(2-22)Proposition:-

If (φ,ψ) is onto homomorphism from (X,G,π) into (Y,H,σ) and x is point mixing under G , then $x\varphi$ is point mixing under H .

Proof:-

Since x is mixing point , then $\overline{x(G-K)}=X$ and this means that $Y=X\varphi=\overline{x(G-K)}\varphi\subset\overline{x(G-K)\varphi}=$

$\overline{x\varphi(G-K)\psi}=\overline{x\varphi(G\psi-K\psi)}=\overline{x\varphi(H-K\psi)}$, where $K\psi$ is compact subset of H , and since $\overline{x\varphi(H-K\psi)}\subset Y$, then $\overline{x\varphi(H-K\psi)}=Y$. Then $x\varphi$ is mixing point .

(2-23)Corollary:-

If (φ,ψ) is onto homomorphism from (X,G,π) into (Y,H,σ) and X is point wise mixing , then Y is point wise mixing .

Proof:-

Easy .

(2-24)Remark:-

If $\{(X_i,G_i,\pi)\}_{i=1}^{n}$ is a sequence of topological transformation groups , then $(\prod_1^n X_i,\prod_1^n G_i,\pi^{/})$ is a topological transformation group , where $\prod_1^n X_i$ and $\prod_1^n G_i$ are finite product of phase spaces X_i and phase groups G_i and $\pi^{/}:\prod_1^n X_i\times\prod_1^n G_i\to\prod_1^n X_i$ such that (($x_1, x_2, x_3,\dots, x_n$), $(g_1,g_2,g_3,\dots,g_n))\pi^{/}=((x_1,g_1)\pi,(x_2,g_2)\pi,(x_3,g_3)\pi,\dots,(x_n,g_n)\pi)$ where $(x_1,x_2,x_3,\dots,x_n)\in\prod_1^n X_i$ and $(g_1,g_2,g_3,\dots,g_n)\in\prod_1^n G_i$. Its clear that , $\pi^{/}=\underbrace{(\pi,\pi,\pi,\dots,\pi)}_{\text{n-times}}$.

(2-25)Definition:-

Let $x\in\prod_1^n X_i$. The transformation group $(\prod_1^n X_i,\prod_1^n G_i,\pi^{/})$ is said to be mixing at x and the point x is said to be mixing under $(\prod_1^n X_i,\prod_1^n G_i,\pi^{/})$ provided that if U is a nonempty open subset of $\prod_1^n X_i$, then there exists a compact subset K of $\prod_1^n G_i$ such that $t\in\prod_1^n G_i-K$ implies $xt\in U$. Without losing of generality we can assume $U=U_1\times U_2\times U_3\times\dots\times U_n$. The transformation group $(\prod_1^n X_i,\prod_1^n G_i,\pi^{/})$ is said to be {point wise} {point} mixing provided that $(\prod_1^n X_i,\prod_1^n G_i,\pi^{/})$ is mixing at {every} {some} point of $\prod_1^n X_i$.

(2-26)Proposition:-

If $\{(X_i,G_i,\pi)\}_{i=1}^{n}$ is a sequence of point wise mixing transformation groups , then $(\prod_1^n X_i,\prod_1^n G_i,\pi^{/})$ is point wise mixing transformation group .

Proof:-

Let $\{(X_i,G_i,\pi)\}_{i=1}^{n}$ be a sequence of point wise mixing transformation group and let

$U = \{U_1 \times U_2 \times U_3 \times ... \times U_n, U_i \in T_i, i = 1,2,3,...n\} \subset \prod_1^n X_i$,

where $U_i \subset X_i$. Let $x = (x_1, x_2, x_3,..., x_n) \in \prod_1^n X_i$, where $x_i \in X_i$. Since $x_i \in X_i$ then there exists a compact subsets K_i of G_i such that $t_i \in G_i - K_i$ this implies that $x_i t_i \subset U_i$. Take $K = \prod_1^n K_i$, then $t = (t_1, t_2, t_3,..., t_n) \in \prod_1^n G_i - \prod_1^n K_i$. So

$xt = (x_1, x_2, x_3,..., x_n)(t_1, t_2, t_3,..., t_n) = (x_1t_1, x_2t_2, x_3t_3,..., x_nt_n)$

$\subset (U_1, U_2, U_3,..., U_n) = U$. Then $(\prod_1^n X_i, \prod_1^n G_i, \pi')$ is point wise mixing transformation group .

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