On b-Syndetic Sets

Ahmed L.M. Al-Hindawe

Department of Mathematics

College of Mathematics and Computer Sciences

Kufa University

Abstract

This paper introduce the notion of b-syndetic sets and give new results by using the

work of W. Gottschalk (1955), H. Lee (1985), D. Andrijevic´(1996) and S. Al-Kutaibi (1997).

Introduction

Some preliminaries and basic definitions are follows:

Let X be a topological space, a subset A of X is said to be b-open set if and only if $A \subseteq cl(int\ A) \cup int(cl\ A)$[2].The subset B of X is said to be b-closed if its complement is b-open[2] .Let $A \subseteq X$ and β be a family of b-open subsets of X , then β is called a b-open cover of A if $A \subseteq \bigcup_{B\in\beta} B$, A is said to be b-compact if and only if any b-open cover has a finite sub cover . Clear that, every open set is b-open and then every b-compact set is compact also, the union of two b-compact subsets of X is b-compact .A topological group is a set G which carries a group structure and a topology and satisfies the two axioms :(i) The map $(x, y) \rightarrow xy$ of $G \times G$ into G is continuous.(That is, the operation of G is continuous).(ii) The map $x \rightarrow x^{-1}$ (The inversion map) of G into G is continuous[4].

Proposition .1:

If K is b-compact set in a topological group G, then K^{-1} is b-compact.

Proof:

Let $f : G \longrightarrow G$ be the inversion map ,that is, $f(x) = x^{-1}$ for all x in G, let β be a b-open cover of K^{-1} then $f(\beta)$ is a b-open cover of $f(K^{-1}) = (K^{-1})^{-1} = K$, but K is b-compact which implies, $f(\beta)$ has a finite subcover β^* then $f(\beta^*)$ covers $f(K) = K^{-1}$. Hence, K^{-1} is b-compact set.

Definition.2:[3]

Let A be a subset of a topological group G, then A is called left (right) syndetic if there exists a compact subset K of G such that $AK = G$ $(KA = G)$.

Similarly, we obtain the following definition:

Definition.3:

Let A be a subset of a topological group G, then A is called left (right) b-syndetic if there exists a b-compact subset K of G such that $AK = G$ $(KA = G)$.

Remark.4:

Directly from the above definition we can obtain that, every b-syndetic set is syndetic .

Note.5:

In the following results we will prove the case of left-syndetic and the case of right syndetic will be similar.

Proposition.6:

Let G be a topological group and let $A \subseteq G$, then A is left (right) b-syndetic set in G if and only if there exists a b-compact subset $K$of G such that every left(right) translation of K intersects A .

Proof:

Sufficiency, suppose A is a left b-syndetic set then there exists a b-compact subset K of G such that $AK = G$, let $g \in G$ then there exists $a \in A, k \in K$ such that $g = ak$ which implies $a = gk^{-1}$ and then $a \in gK^{-1}$ but K^{-1} is b-compact .Hence, $gK^{-1} \cap A \neq \phi$ (i.e.K^{-1} is the b-compact set we need).

Efficiency, let $g \in G$, there exists a b-compact subset K of G such that $gK \cap A \neq \phi$, for each g in G, there exists $a \in A, k \in K$ such that, $gk = a$ so, $g = ak^{-1}$ which implies $G = AK^{-1}$ and since K^{-1} is b-compact then A is a left b-syndetic set.

Proposition .7:

Let A be a subset of a topological group G then A is left (right) b-syndetic in G if and only if A^{-1} is right (left) b-syndetic .

Proof:

Let A be a left b-syndetic then there exists b-compact subset K of G such that $AK = G$. Since $G = G^{-1} = (AK)^{-1} = K^{-1}A^{-1}$ and since K^{-1} is b-compact then A^{-1} is right b-syndetic.

Proposition .8:

Let G be a topological group, let A, B are two subsets of G such that $A \subseteq B$. If A is a left (right) b-syndetic set then so is B.

Proof:

Let A be a left b-syndetic set, then there exists a b-compact subset K of G such that $AK = G$, since $A \subseteq B$ then $BK = G$ which implies that B is left b-syndetic.

Proposition.9:

Let G be a topological group then :

If A is a left (right) b-syndetic set in G

1- then, $cl\ A$ and $bcl\ A$ are left (right) b-syndetic sets in G.(where $bcl\ A$ denotes the b-closure of A ,that is, the smallest b-closed set containing A)

The union of any family of left (right) b-syndetic sets is left (right) b-syndetic.

Proof:

Directly from proposition.8.

Proposition .10:

Let G be a topological group, let A, B are two left (right) b-syndetic subsets of G then $A \cap B$ is a left (right) b-syndetic .

Proof:

Since both A and B are left b-syndetic then there exist b-compact sets K and H such that $AK = G$ and $BH = G$ and then $(A \cap B)(K \cup H) = A(K \cup H) \cap B(K \cup H) = G \cap G = G$ and since $(K \cup H)$ is b-compact then $(A \cap B)$ is left b-syndetic.

Proposition .11:

Let A be a subset of a topological group G.If A is a subgroup of G or if G is an abelian group, then A is left b-syndetic in G if and only if A is a right b-syndetic in G.

Proof:

Let A be a left b-syndetic subgroup of G, then $G = AK$, that means $G^{-1} = K^{-1}A^{-1}$ and hence, $G = K^{-1}A$,but K^{-1} is b-compact which implies A is right b-syndetic in G.

Proposition .12:

Let A be a b-syndetic subgroup of a topological group G, then the quotient space G/A is compact .

Proof:

Let A be a b-syndetic subgroup of a topological group G,then there exists a b-compact set K such that $KA = G$. Let $p: G \to G/A$ be the canonical projection.

Clear that $p(K) \subset G/A$ .Let $gA \in G/A$ then, $gA \subset G$ which implies $gA \subset KA$ that is, $gA \in p(K)$ and then, $G/A \subset p(K)$. Hence, $G/A = p(K)$.

Since p is continuous and K is b-compact and hence is compact. Then $p(K) = G/A$ is compact set.

References:

1- Alkutaibi,S.H., On semi-syndetic and feebl subsets, Journal Of Sciences, College Of Education, Tikrit University, Vo 1998.

2- Andrijevic,D.,On b-open sets,Mat.Vesnik,48,1996,59-64.

3- Gottschalk,W.H., Hedlund,G.A., Topologic Dynamics, American Mathematical Society, 1955.

4- Higgins,P.J., An Introduction to topologica Cambridge University Press,1979

5- Hu,S.T., Elements of General Topology,Hc Day,Inc,1964.

حول المجموعات الرابطة من النمط -b

أحمد لفتة موسى الهنداوي

قسم الرياضيات

كلية الرياضيات وعلوم الحاسوب

جامعة الكوفة

<u>الملخص :</u>

نقدم في هذا البحث المجموعة الرابطة من النمط -b مع بعض النتائج التي حصلنا عليها معتمدين في ذلك على المفاهيم التي قدمها كوتسجوك في هذا الموضوع (1955) واندريجفيك (1996) والكتبي (1997) .