

Incidence and Non-incidence Soft Topology with Application

Iman Abbas Ali

*Department of Mathematics,
Faculty of education for women, University
of Kufa ,Najaf, Iraq
emana.alyousify@student.uokufa.edu.iq
[Orcid.org/0009-0002-1953-2296](https://orcid.org/0009-0002-1953-2296)*

Asmhan Flieh Hassan

*Department of Mathematics,
Faculty of education for women, University
of Kufa ,Najaf, Iraq
asmhanf.alzuhairy@uokufa.edu.iq
[Orcid.org/0000-0002-0855-1873](https://orcid.org/0000-0002-0855-1873)*

DOI: <http://dx.doi.org/10.31642/JoKMC/2018/130111>

Received Dec. 23, 2025. Accepted for publication Feb.11, 2026

Abstract :- *In this work, incidence and non-incidence relations are used to create a new type of soft topology on soft graphs. Every edge in the graph is used as a parameter, and ordered pairs are used to build soft sets. The first part of an ordered pair is the edge, and the second part is the point that is connected to the edge. The sub-base, base, and soft topologies are explained, and then properties, theorems, and a full example are given to show how they work. This method gives a combinatorial structure encoding and makes it possible to do new kinds of study in soft topological spaces.*

Keyword. *Soft graph, Soft topology, Incidence and non-incidence relations, Soft Topology on Soft Graphs*

I. INTRODUCTION

The study of graphs, which are combinatorial structures made up of vertices and edges that represent pairwise relationships between discrete objects, is the primary focus of graph theory. As a cornerstone of discrete mathematics, graphs are instrumental in modeling various relational datasets. Leonhard Euler, a Swiss mathematician, is credited with laying the groundwork for graph theory in the 18th century with his analysis of the well-known Königsberg Bridge Problem.

In order to tackle issues involving uncertainty, D. Molodtsov presented the theory of soft sets in 1999 [1] as a new mathematical framework. With its efficient use of parameterization—a trait that fuzzy set theory mostly lacks—soft set theory offers greater flexibility than more conventional methods like fuzzy set theory and probability theory.

This parameterization makes it possible for soft sets to manage uncertainty in a broader way. P.K. Maji, A.R. Roy, and R. Biswas are among the researchers who have expanded Molodtsov's original framework and effectively applied it to decision-making problems [2], [3].

The notion of soft graphs was introduced by Rajesh K. Thumbakara and Bobin George in 2014, offering a

parameterized interpretation of common graph structures [4]. They also introduced additional concepts such as soft trees, soft subgraphs, and other operations on soft graphs. Soft graphs were redefined in 2015 by Muhammad Akram and Saira Nawaz, who further elaborated on graph-theoretic concepts such as soft bridges, soft cut vertices, and soft cycles [5]. R.S. Jain, B.S. Reddy, and J.D. Thenge made further contributions to the development of soft graph theory with their, particularly with regard to linked soft graphs, providing a deeper comprehension of the characteristics and applications of soft graph topologies [6], [7], [8]. Additionally, their study looked at fundamental graph-theoretic characteristics, such as the examination of graph invariants like radius, diameter, and center in the setting of soft graphs. Numerous topologies generated from graphs are included in a large body of study, as references [9], [10], and [11] demonstrate.

Our work presents a novel form of soft topology on soft graphs that is based on the dual relations of non-incidence and incidence. This new topology demonstrates a precise and structured way to represent combinatorial relationships between vertices and edges, paving the way for novel applications in the study of soft graphs and decision-making systems under uncertainty.

II. PRELIMINARIES

This section introduces soft graph theory and soft topology basic definitions and preliminaries. All of these soft concepts are widely used and are found in references.

Definition 2.1[12]

A simple-graph is defined as a graph that does not have any loops or parallel edges and is represented by the equation $Y=(\mathcal{U},\mathcal{E})$.

Definition 2.2[4]

Let $Y=(\mathcal{U},\mathcal{E})$ represent a simple-graph, and let Λ be any non-empty set. Examine a general binary relevance \mathcal{R} between Λ and the vertices of \mathcal{U} . $\mathcal{R} \subseteq \Lambda \times \mathcal{U}$. Let $\mathcal{F}: \Lambda \rightarrow \mathcal{P}_p(\mathcal{U})$ be a set-valued function defined by $\mathcal{F}(x) = \{y \in \mathcal{U} \mid x\mathcal{R}y\}$. The set \mathcal{F}_Λ is a soft set over \mathcal{U} .

Definition 2.3:[4], [7]

Let Λ represent any set that is not empty and $Y=(\mathcal{U}, \mathcal{E})$ represent a simple-graph. Assume that the relation from Λ to \mathcal{U} is random and that $\mathcal{R} \subseteq \Lambda \times \mathcal{U}$ is a subset of that relation. You can define a mapping $\mathcal{F}^f: \Lambda \rightarrow \mathcal{P}_p(\mathcal{U})$ by $\mathcal{F}^f(x) = \{\lambda \in \mathcal{U} \mid x\mathcal{R}\lambda\}$ and a mapping $\Pi^f: \Lambda \rightarrow \mathcal{P}_p(\mathcal{E})$ by $\Pi^f(x) = \{\partial \ell \in \mathcal{E}/\{\partial, \ell\} \subseteq \mathcal{F}^f(x)\}$.

A 4-tuple $Y=(Y, \mathcal{F}^f, \Pi^f, \Lambda)$ is named a soft graph of Y if it has these features:

- i. $Y=(\mathcal{U}, \mathcal{E})$ is a simple graph.
- ii. Λ is a set of parameters and $\Lambda \neq \emptyset$.
- iii. \mathcal{F}_Λ^f is a soft set over \mathcal{U} .
- iv. Π_Λ^f is a soft set over \mathcal{E} .
- v. $(\mathcal{F}^f(w), \Pi^f(w))$ is a subgraph of Y for all $w \in \Lambda$.

The symbol $\mathcal{S}(\Psi)$ represents the set of all soft graph of Y .

Definition 2.4:[1]

Let \mathcal{X}_x represent a universal set and Σ a collection of parameters. Define $\mathcal{P}_p(\mathcal{X}_x)$ as the power set of \mathcal{X}_x , with Λ being a non-empty subset of \mathcal{E} . A soft set, denoted as \mathcal{F}_Λ^f , is a pair where \mathcal{F}^f is a function $\mathcal{F}^f: \Lambda \rightarrow \mathcal{P}_p(\mathcal{X}_x)$. In essence, a soft set over \mathcal{X}_x is a parameterized collection of subsets of \mathcal{X}_x . For each $\delta \in \Lambda$, $\mathcal{F}^f(\delta)$ represents the set of elements approximately corresponding to the parameter δ . Importantly, a soft set is distinct from a conventional set.

Definition 2.5:[12]

Let $Y=(\mathcal{U},\mathcal{E})$ represent a simple-graph, where \mathcal{U} is the set of vertices and \mathcal{E} is the set of edges. An edge $e^\vartheta = \{u^\vartheta, v^\vartheta\} \in \mathcal{E}$ is said to be incident with the vertices u^ϑ and v^ϑ . Conversely, a vertex $v^\vartheta \in \mathcal{U}$ is incident with an edge $e^\vartheta \in \mathcal{E}$ if v^ϑ is one of the endpoints of e^ϑ . The relation between vertices and edges defined in this way is called the incidence relation of the graph.

Definition 2.6:[12]

Let $Y=(\mathcal{U},\mathcal{E})$ represent a simple-graph, where \mathcal{U} is the set of vertices and \mathcal{E} is the set of edges are said to be in a state of non-incidence if v^ϑ is not an endpoint of e^ϑ .

III. Incidence and Non-incidence Soft Topology

Definition 3.1:

Let $\Psi=(Y, \mathcal{F}^f, \Pi^f, \Sigma)$ represent a soft graph, where Σ is a set of parameters. The soft topology is constructed using incidence and non-incidence relations and named as Incidence and Non-incidence Soft Topology it is written simply $\mathcal{IN} - \mathcal{ST}_\vartheta$ denoted by $\mathcal{IT}_\mathcal{N}(\Psi)$. The soft sub-base consists of these sets $\mathcal{IN} - \mathcal{S}_s$ denoted by $(\mathcal{IS}_\mathcal{N})$, whose elements are the soft sets $\mathcal{R}_\mathcal{E}^J$ and $\mathcal{R}_\mathcal{E}^N$, where $\mathcal{R}: \mathcal{E} \rightarrow \mathcal{P}_p(\mathcal{U})$.

Remark 3.2:

It is natural to take the parameter set as the edge set, since incidence and non-incidence relations accurately reflect the relationships between vertices and edges.

- ✚ There are different types of soft graphs based on how many vertices, edges, connections, and general structure they have.

Example 3.3:

Let Ψ be a path soft graph $s\mathbb{P}_3$ with vertex set $\mathcal{U}(\Psi) = \{\overset{5}{g}\overset{5}{\zeta}_1, \overset{5}{g}\overset{5}{\zeta}_2, \overset{5}{g}\overset{5}{\zeta}_3, \overset{5}{g}\overset{5}{\zeta}_4\}$ and $\mathcal{E}(\Psi) = \{\Gamma_1^{\delta^\circ}, \Gamma_2^{\delta^\circ}, \Gamma_3^{\delta^\circ}\}$ shown in the fig;(1). The parameters set are edges set, where: -



Fig 1: path soft graph $s\mathbb{P}_3$

$\mathcal{R}^J: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^J is incidence relation of edges, and $\mathcal{R}^N: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^N is non-incidence relation of edges.

$$\begin{aligned} \mathcal{R}^J(\Gamma_1^{\delta^\circ}) &= \{\xi_1, \xi_2\} \Rightarrow (\Gamma_1^{\delta^\circ}, \{\xi_1, \xi_2\}); \\ \mathcal{R}^J(\Gamma_2^{\delta^\circ}) &= \{\xi_2, \xi_3\} \Rightarrow (\Gamma_2^{\delta^\circ}, \{\xi_2, \xi_3\}); \\ \mathcal{R}^J(\Gamma_3^{\delta^\circ}) &= \{\xi_3, \xi_4\} \Rightarrow (\Gamma_3^{\delta^\circ}, \{\xi_3, \xi_4\}); \\ \mathcal{R}^N(\Gamma_1^{\delta^\circ}) &= \{\xi_3, \xi_4\} \Rightarrow (\Gamma_1^{\delta^\circ}, \{\xi_3, \xi_4\}); \\ \mathcal{R}^N(\Gamma_2^{\delta^\circ}) &= \{\xi_1, \xi_4\} \Rightarrow (\Gamma_2^{\delta^\circ}, \{\xi_1, \xi_4\}); \\ \mathcal{R}^N(\Gamma_3^{\delta^\circ}) &= \{\xi_1, \xi_2\} \Rightarrow (\Gamma_3^{\delta^\circ}, \{\xi_1, \xi_2\}); \\ \mathcal{R}_{\mathfrak{E}}^J &= \{(\Gamma_1^{\delta^\circ}, \{\xi_1, \xi_2\}), (\Gamma_2^{\delta^\circ}, \{\xi_2, \xi_3\}), (\Gamma_3^{\delta^\circ}, \{\xi_3, \xi_4\}), (\Gamma_1^{\delta^\circ}, \{\xi_3, \xi_4\})\}; \\ \mathcal{R}_{\mathfrak{E}}^N &= \{(\Gamma_1^{\delta^\circ}, \{\xi_3, \xi_4\}), (\Gamma_2^{\delta^\circ}, \{\xi_1, \xi_4\}), (\Gamma_3^{\delta^\circ}, \{\xi_1, \xi_2\})\}. \end{aligned}$$

The sub base for $\mathcal{J}\mathcal{T}_N(\Psi)$ is: -

$$\mathcal{J}\mathcal{S}_N = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

After taking finitely intersection, base is produced:

$$\mathcal{J}\mathcal{B}_N = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N, \emptyset \}.$$

When all unions are taken into account, $\mathcal{J}\mathcal{T}_N(\Psi)$ can be expressed as:

$$\mathcal{J}\mathcal{T}_N(\Psi) = \{ \emptyset, \emptyset, \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

Example 3. 4.

Let Ψ be a cycle soft graph $s\mathcal{C}_3$ with vertex set $\mathcal{U}(\Psi) = \{\xi_1, \xi_2, \xi_3\}$ and $\mathfrak{E}(\Psi) = \{\Gamma_1^{\delta^\circ}, \Gamma_2^{\delta^\circ}, \Gamma_3^{\delta^\circ}\}$ shown in the fig;(2). The parameters set are edges set, where: -

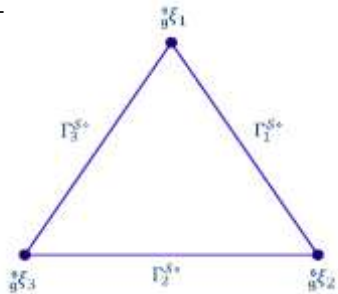


Fig 2: cycle soft graph $s\mathcal{C}_3$

$\mathcal{R}^J: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^J is incidence relation of edges, and $\mathcal{R}^N: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^N is non-incidence relation of edges.

$$\begin{aligned} \mathcal{R}_{\mathfrak{E}}^J &= \{(\Gamma_1^{\delta^\circ}, \{\xi_1, \xi_2\}), (\Gamma_2^{\delta^\circ}, \{\xi_2, \xi_3\}), (\Gamma_3^{\delta^\circ}, \{\xi_3, \xi_1\})\}; \\ \mathcal{R}_{\mathfrak{E}}^N &= \{(\Gamma_1^{\delta^\circ}, \{\xi_3\}), (\Gamma_2^{\delta^\circ}, \{\xi_1\}), (\Gamma_3^{\delta^\circ}, \{\xi_2\})\}. \end{aligned}$$

The sub base for $\mathcal{J}\mathcal{T}_N(\Psi)$ is: -

$$\mathcal{J}\mathcal{S}_N = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

After taking finitely intersection, base is produced:

$$\mathcal{J}\mathcal{B}_N = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N, \emptyset \}.$$

When all unions are taken into account, $\mathcal{J}\mathcal{T}_N(\Psi)$ can be expressed as:

$$\mathcal{J}\mathcal{T}_N(\Psi) = \{ \emptyset, \emptyset, \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

Example 3.5:

Let Ψ be a path soft graph $s\mathcal{P}_5$ with vertex set $\mathcal{U}(\Psi) = \{\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6\}$ and $\mathfrak{E}(\Psi) = \{\Gamma_1^{\delta^\circ}, \Gamma_2^{\delta^\circ}, \Gamma_3^{\delta^\circ}, \Gamma_4^{\delta^\circ}, \Gamma_5^{\delta^\circ}\}$ shown in the fig;(3). The parameters set are edges set, where: -

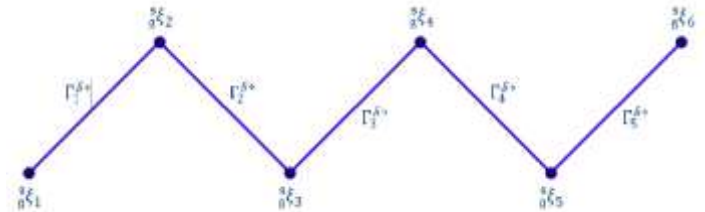


Fig 3: path soft graph $s\mathcal{P}_5$

$\mathcal{R}^J: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^J is incidence relation of edges, and $\mathcal{R}^N: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^N is non-incidence relation of edges.

$$\begin{aligned} \mathcal{R}_{\mathfrak{E}}^J &= \{(\Gamma_1^{\delta^\circ}, \{\xi_1, \xi_2\}), (\Gamma_2^{\delta^\circ}, \{\xi_2, \xi_3\}), (\Gamma_3^{\delta^\circ}, \{\xi_3, \xi_4\}), (\Gamma_4^{\delta^\circ}, \{\xi_4, \xi_5\}), (\Gamma_5^{\delta^\circ}, \{\xi_5, \xi_6\})\}; \\ \mathcal{R}_{\mathfrak{E}}^N &= \{(\Gamma_1^{\delta^\circ}, \{\xi_3, \xi_4, \xi_5, \xi_6\}), (\Gamma_2^{\delta^\circ}, \{\xi_1, \xi_4, \xi_5, \xi_6\}), (\Gamma_3^{\delta^\circ}, \{\xi_1, \xi_2, \xi_5, \xi_6\}), (\Gamma_4^{\delta^\circ}, \{\xi_1, \xi_2, \xi_3, \xi_6\}), (\Gamma_5^{\delta^\circ}, \{\xi_1, \xi_2, \xi_3, \xi_4\})\}. \end{aligned}$$

The sub base for $\mathcal{J}\mathcal{T}_N(\Psi)$ is: -

$$\mathcal{J}\mathcal{S}_N = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

After taking finitely intersection, base is produced:

$$\mathcal{J}\mathcal{B}_N = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N, \emptyset \}.$$

When all unions are taken into account, $\mathcal{J}\mathcal{T}_N(\Psi)$ can be expressed as:

$$\mathcal{J}\mathcal{T}_N(\Psi) = \{ \emptyset, \emptyset, \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

Example 3.6:

Let Ψ be a complete soft graph with vertex set $\mathcal{U}(\Psi) = \{\xi_1, \xi_2, \xi_3, \xi_4, \xi_5\}$ and $\mathfrak{E}(\Psi) = \{\Gamma_1^{\delta^\circ}, \Gamma_2^{\delta^\circ}, \Gamma_3^{\delta^\circ}, \Gamma_4^{\delta^\circ}, \Gamma_5^{\delta^\circ}, \Gamma_6^{\delta^\circ}, \Gamma_7^{\delta^\circ}, \Gamma_8^{\delta^\circ}, \Gamma_9^{\delta^\circ}, \Gamma_{10}^{\delta^\circ}\}$ shown in the fig;(4). The parameters set are edges set, where: -

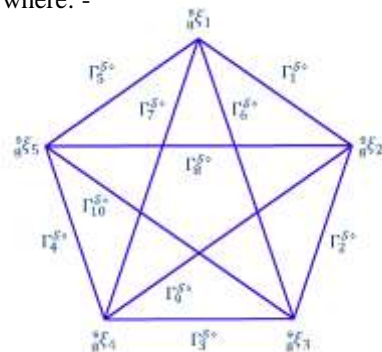


Fig 4: complete soft graph

$\mathcal{R}^J: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^J is incidence relation of edges, and $\mathcal{R}^N: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^N is non-incidence relation of edges.

$$\mathcal{R}_{\mathfrak{E}}^J = \{(\Gamma_1^{\delta^\circ}, \{\xi_1, \xi_2\}), (\Gamma_2^{\delta^\circ}, \{\xi_2, \xi_3\}), (\Gamma_3^{\delta^\circ}, \{\xi_3, \xi_4\}), (\Gamma_4^{\delta^\circ}, \{\xi_4, \xi_5\}), (\Gamma_5^{\delta^\circ}, \{\xi_1, \xi_5\}), (\Gamma_6^{\delta^\circ}, \{\xi_1, \xi_3\}), (\Gamma_7^{\delta^\circ}, \{\xi_1, \xi_4\}), (\Gamma_8^{\delta^\circ}, \{\xi_2, \xi_5\}), (\Gamma_9^{\delta^\circ}, \{\xi_2, \xi_4\}), (\Gamma_{10}^{\delta^\circ}, \{\xi_3, \xi_5\})\}.$$

$$\mathcal{R}_{\mathfrak{E}}^N = \{(\Gamma_1^{\delta^\circ}, \{\xi_3, \xi_4, \xi_5\}), (\Gamma_2^{\delta^\circ}, \{\xi_1, \xi_4, \xi_5\}), (\Gamma_3^{\delta^\circ}, \{\xi_1, \xi_2, \xi_5\}), (\Gamma_4^{\delta^\circ}, \{\xi_1, \xi_2, \xi_3\}), (\Gamma_5^{\delta^\circ}, \{\xi_2, \xi_3, \xi_4\}), (\Gamma_6^{\delta^\circ}, \{\xi_2, \xi_4, \xi_5\}), (\Gamma_7^{\delta^\circ}, \{\xi_2, \xi_3, \xi_5\}), (\Gamma_8^{\delta^\circ}, \{\xi_1, \xi_3, \xi_4\}), (\Gamma_9^{\delta^\circ}, \{\xi_1, \xi_3, \xi_5\}), (\Gamma_{10}^{\delta^\circ}, \{\xi_1, \xi_2, \xi_4\})\}.$$

The sub base for $\mathcal{J}\mathcal{T}_N(\Psi)$ is: -

$$\mathcal{J}\mathcal{S}_N = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

After taking finitely intersection, base is produced:

$$\mathcal{J}\mathcal{B}_N = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N, \bar{\varphi} \}.$$

When all unions are taken into account, $\mathcal{J}\mathcal{T}_N(\Psi)$ can be expressed as:

$$\mathcal{J}\mathcal{T}_N(\Psi) = \{ \bar{\mathcal{U}}, \bar{\varphi}, \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

Example 3.7:

Let Ψ be a bipartite soft graph with vertex set $\mathcal{U}(\Psi) = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ and $\mathfrak{E}(\Psi) = \{\Gamma_1^{\delta^\circ}, \Gamma_2^{\delta^\circ}, \Gamma_3^{\delta^\circ}\}$ shown in the fig;(5). The parameters set are edges set, where: -

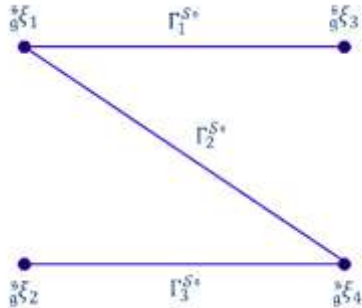


Fig 5: bipartite soft graph

$\mathcal{R}^J: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^J is incidence relation of edges, and $\mathcal{R}^N: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^N is non-incidence relation of edges.

$$\mathcal{R}_{\mathfrak{E}}^J = \{(\Gamma_1^{\delta^\circ}, \{\xi_1, \xi_3\}), (\Gamma_2^{\delta^\circ}, \{\xi_1, \xi_4\}), (\Gamma_3^{\delta^\circ}, \{\xi_2, \xi_4\})\}.$$

$$\mathcal{R}_{\mathfrak{E}}^N = \{(\Gamma_1^{\delta^\circ}, \{\xi_2, \xi_4\}), (\Gamma_2^{\delta^\circ}, \{\xi_2, \xi_3\}), (\Gamma_3^{\delta^\circ}, \{\xi_1, \xi_3\})\}.$$

The sub base for $\mathcal{J}\mathcal{T}_N(\Psi)$ is: -

$$\mathcal{J}\mathcal{S}_N = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

After taking finitely intersection, base is produced:

$$\mathcal{J}\mathcal{B}_N = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N, \bar{\varphi} \}.$$

When all unions are taken into account, $\mathcal{J}\mathcal{T}_N(\Psi)$ can be expressed as:

$$\mathcal{J}\mathcal{T}_N(\Psi) = \{ \bar{\mathcal{U}}, \bar{\varphi}, \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

Example 3.8:

Let Ψ be a star soft graph with vertex set $\mathcal{U}(\Psi) = \{\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6, \xi_7\}$ and $\mathfrak{E}(\Psi) = \{\Gamma_1^{\delta^\circ}, \Gamma_2^{\delta^\circ}, \Gamma_3^{\delta^\circ}, \Gamma_4^{\delta^\circ}, \Gamma_5^{\delta^\circ}, \Gamma_6^{\delta^\circ}\}$ shown in the fig;(6). The parameters set are edges set, where: -

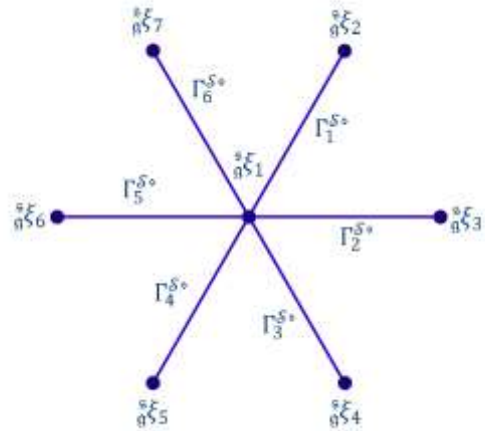


Fig 6: star soft graph

$\mathcal{R}^J: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^J is incidence relation of edges, and $\mathcal{R}^N: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^N is non-incidence relation of edges.

$$\mathcal{R}_{\mathfrak{E}}^J = \{(\Gamma_1^{\delta^\circ}, \{\xi_1, \xi_2\}), (\Gamma_2^{\delta^\circ}, \{\xi_1, \xi_3\}), (\Gamma_3^{\delta^\circ}, \{\xi_1, \xi_4\}), (\Gamma_4^{\delta^\circ}, \{\xi_1, \xi_5\}), (\Gamma_5^{\delta^\circ}, \{\xi_1, \xi_6\}), (\Gamma_6^{\delta^\circ}, \{\xi_1, \xi_7\})\}.$$

$$\mathcal{R}_{\mathfrak{E}}^N = \{(\Gamma_1^{\delta^\circ}, \{\xi_3, \xi_4, \xi_5, \xi_6, \xi_7\}), (\Gamma_2^{\delta^\circ}, \{\xi_2, \xi_4, \xi_5, \xi_6, \xi_7\}), (\Gamma_3^{\delta^\circ}, \{\xi_2, \xi_3, \xi_5, \xi_6, \xi_7\}), (\Gamma_4^{\delta^\circ}, \{\xi_2, \xi_3, \xi_4, \xi_6, \xi_7\}), (\Gamma_5^{\delta^\circ}, \{\xi_2, \xi_3, \xi_4, \xi_5, \xi_7\}), (\Gamma_6^{\delta^\circ}, \{\xi_2, \xi_3, \xi_4, \xi_5, \xi_6\})\}.$$

The sub base for $\mathcal{J}\mathcal{T}_N(\Psi)$ is: -

$$\mathcal{J}\mathcal{S}_N = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

After taking finitely intersection, base is produced:

$$\mathcal{J}\mathcal{B}_N = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N, \bar{\varphi} \}.$$

When all unions are taken into account, $\mathcal{J}\mathcal{T}_N(\Psi)$ can be expressed as:

$$\mathcal{J}\mathcal{T}_N(\Psi) = \{ \bar{\mathcal{U}}, \bar{\varphi}, \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

Example 3.9:

Let Ψ be a tree soft graph with vertex set $\mathcal{U}(\Psi) = \{\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6, \xi_7\}$ and

$\mathfrak{E}(\Psi) = \{\Gamma_1^{\delta^\circ}, \Gamma_2^{\delta^\circ}, \Gamma_3^{\delta^\circ}, \Gamma_4^{\delta^\circ}, \Gamma_5^{\delta^\circ}, \Gamma_6^{\delta^\circ}\}$ shown in the fig:(7). The parameters set are edges set, where: -

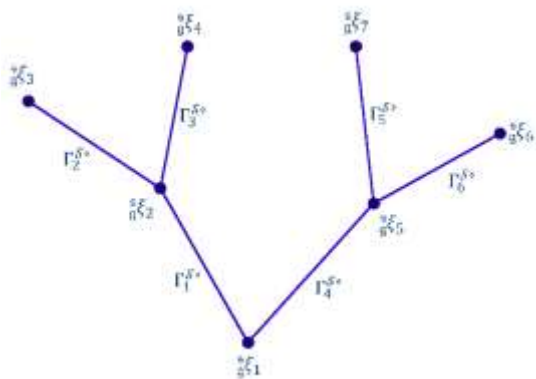


Fig 7: tree soft graph

$\mathcal{R}^J: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^J is incidence relation of edges, and $\mathcal{R}^N: \mathfrak{E} \rightarrow \mathcal{P}_p(\mathcal{U})$, where is \mathcal{R}^N is non-incidence relation of edges.

$$\mathcal{R}_{\mathfrak{E}}^J = \{(\Gamma_1^{\delta^\circ}, \{g^{\xi_1}, g^{\xi_2}\}), (\Gamma_2^{\delta^\circ}, \{g^{\xi_2}, g^{\xi_3}\}), (\Gamma_3^{\delta^\circ}, \{g^{\xi_2}, g^{\xi_4}\}), (\Gamma_4^{\delta^\circ}, \{g^{\xi_1}, g^{\xi_5}\}), (\Gamma_5^{\delta^\circ}, \{g^{\xi_5}, g^{\xi_7}\}), (\Gamma_6^{\delta^\circ}, \{g^{\xi_5}, g^{\xi_6}\})\}.$$

$$\mathcal{R}_{\mathfrak{E}}^N = \{(\Gamma_1^{\delta^\circ}, \{g^{\xi_3}, g^{\xi_4}, g^{\xi_5}, g^{\xi_6}, g^{\xi_7}\}), (\Gamma_2^{\delta^\circ}, \{g^{\xi_1}, g^{\xi_4}, g^{\xi_5}, g^{\xi_6}, g^{\xi_7}\}), (\Gamma_3^{\delta^\circ}, \{g^{\xi_1}, g^{\xi_3}, g^{\xi_5}, g^{\xi_6}, g^{\xi_7}\}), (\Gamma_4^{\delta^\circ}, \{g^{\xi_2}, g^{\xi_3}, g^{\xi_4}, g^{\xi_6}, g^{\xi_7}\}), (\Gamma_5^{\delta^\circ}, \{g^{\xi_1}, g^{\xi_2}, g^{\xi_3}, g^{\xi_4}, g^{\xi_6}\}), (\Gamma_6^{\delta^\circ}, \{g^{\xi_1}, g^{\xi_2}, g^{\xi_3}, g^{\xi_4}, g^{\xi_7}\})\}.$$

The sub base for $\mathcal{J}\mathcal{T}_{\mathcal{N}}(\Psi)$ is: -

$$\mathcal{J}\mathcal{S}_{\mathcal{N}} = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

After taking finitely intersection, base is produced:

$$\mathcal{J}\mathcal{B}_{\mathcal{N}} = \{ \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N, \emptyset \}.$$

When all unions are taken into account, $\mathcal{J}\mathcal{T}_{\mathcal{N}}(\Psi)$ can be expressed as:

$$\mathcal{J}\mathcal{T}_{\mathcal{N}}(\Psi) = \{ \emptyset, \emptyset, \mathcal{R}_{\mathfrak{E}}^J, \mathcal{R}_{\mathfrak{E}}^N \}.$$

IV. Application Warfarin–CYP2C9–Fluconazole Interaction

Introduction 4.1: The clinical interaction between Warfarin, CYP2C9, and Fluconazole is a well-documented drug–enzyme–inhibitor case. Traditionally modeled as a simple interaction graph, this system can be enriched by using the incidence/non-incidence soft topology. In this framework, incidence relations identify drug–enzyme pairs actively involved in a reaction, while non-incidence relations highlight the excluded entities.

Combining both yields a more comprehensive pathway analysis that clarifies active clusters and isolated nodes in the biochemical network [[13], [14][15], [16].

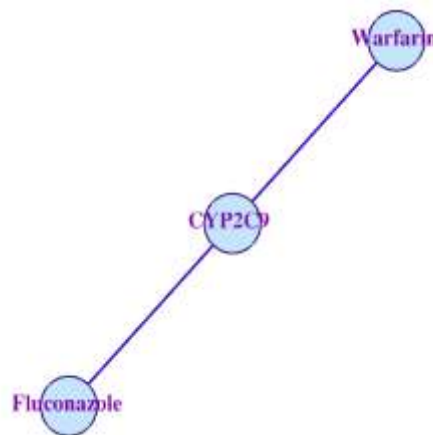


Fig 8: Original Drug–Enzyme–Inhibitor Interaction Graph.

Soft Topology Construction 4.2:

Let $\mathcal{U} = \{ \text{Warfarin } (g^{\xi_1}), \text{CYP2C9 } (g^{\xi_2}), \text{Fluconazole } (g^{\xi_3}) \}.$

The parameter (edge) set is $\mathfrak{E} = \{ \Gamma_1^{\delta^\circ}, \Gamma_2^{\delta^\circ}, \Gamma_3^{\delta^\circ} \}.$

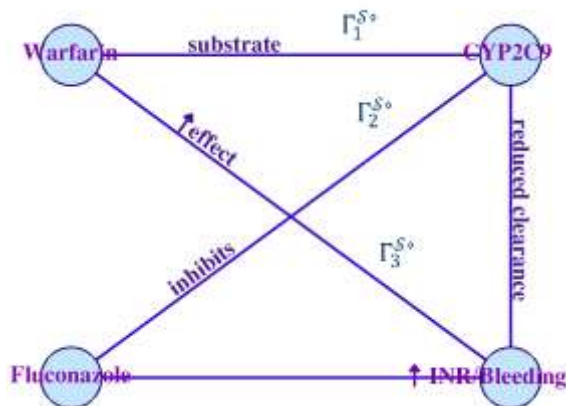


Fig 9: Directed conceptual pathway ($\Gamma_1^{\delta^\circ}$ substrate, $\Gamma_2^{\delta^\circ}$ inhibition, $\Gamma_3^{\delta^\circ}$ clinical outcome).

The incidence and non-incidence relations are:

$$\mathcal{R}^J(\Gamma_1^{\delta^\circ}) = \{g^{\xi_1}, g^{\xi_2}\}, \quad \mathcal{R}^N(\Gamma_1^{\delta^\circ}) = \{g^{\xi_3}\};$$

$$\mathcal{R}^J(\Gamma_2^{\delta^\circ}) = \{ {}_9^{\xi_2}, {}_9^{\xi_3} \}, \mathcal{R}^N(\Gamma_2^{\delta^\circ}) = \{ {}_9^{\xi_1} \};$$

$$\mathcal{R}^J(\Gamma_3^{\delta^\circ}) = \{ {}_9^{\xi_1}, {}_9^{\xi_3} \}, \mathcal{R}^N(\Gamma_3^{\delta^\circ}) = \{ {}_9^{\xi_2} \};$$

$$\mathcal{R}_{\mathbb{E}}^J = \{ (\Gamma_1^{\delta^\circ}, \{ {}_9^{\xi_1}, {}_9^{\xi_2} \}), (\Gamma_2^{\delta^\circ}, \{ {}_9^{\xi_2}, {}_9^{\xi_3} \}), (\Gamma_3^{\delta^\circ}, \{ {}_9^{\xi_1}, {}_9^{\xi_3} \}) \}.$$

$$\mathcal{R}_{\mathbb{E}}^N = \{ (\Gamma_1^{\delta^\circ}, \{ {}_9^{\xi_3} \}), (\Gamma_2^{\delta^\circ}, \{ {}_9^{\xi_1} \}), (\Gamma_3^{\delta^\circ}, \{ {}_9^{\xi_2} \}) \}.$$

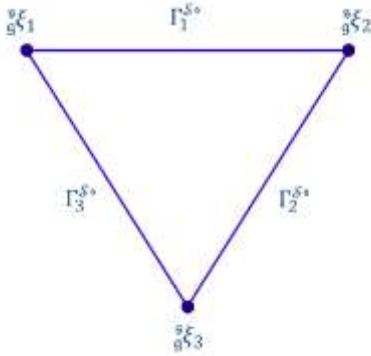


Fig 10: Soft incidence/non-incidence representation used for soft topology construction.

The sub base for $\mathcal{IT}_{\mathcal{N}}(\Psi)$ is: -
 $\mathcal{IS}_{\mathcal{N}} = \{ \mathcal{R}_{\mathbb{E}}^J, \mathcal{R}_{\mathbb{E}}^N \}.$
 After taking finitely intersection, base is produced:
 $\mathcal{IB}_{\mathcal{N}} = \{ \mathcal{R}_{\mathbb{E}}^J, \mathcal{R}_{\mathbb{E}}^N, \emptyset \}.$
 When all unions are taken into account, $\mathcal{IT}_{\mathcal{N}}(\Psi)$ can be expressed as:
 $\mathcal{IT}_{\mathcal{N}}(\Psi) = \{ \emptyset, \emptyset, \mathcal{R}_{\mathbb{E}}^J, \mathcal{R}_{\mathbb{E}}^N \}.$

Comparison Between Graphs 4.3:-

Aspect	Original Graph	Soft Graph
Representation	Edges show only direct links	Edges + explicit excluded nodes
Focus	Interaction presence	Both presence and absence
What is Missing	No info on excluded elements	Shows non-incidence clearly
Clinical Relevance	Limited interpretation	Helps identify control points & risks

Benefits of Using Soft Topology 4.4:-

The use of incidence/non-incidence soft topology in this drug–enzyme–inhibitor interaction provides several current and future advantages:-

- Explicit representation of both incidence and non-incidence, reducing ambiguity.
- Clear identification of biochemical clusters through soft open sets.
- Structured reasoning for clinical safety: non-incidence singletons indicate precise isolation.
- Future extensibility: dosage, timing, and genotype can be included as parameters.

Statement of Novelty 4.5:-

To the best of our knowledge, no prior work has modeled the Warfarin–CYP2C9–Fluconazole interaction using an incidence/non-incidence soft topology. While theoretical work exists on soft topologies and incidence/non-incidence relations, and biological networks have been studied with graphs and machine learning applying a soft-topological framework to this specific drug–enzyme–inhibitor triad appears to be novel.

V. CONCLUSION

This research introduced incidence and non-incidence soft topologies within soft set theory. By constructing relations on soft graphs, we showed how sub-bases, bases, and topological spaces can be derived. Incidence soft topology reflects adjacency and interaction, while non-incidence soft topology captures independence and separation. Applications to drug–protein networks highlighted their practical value, where incidence tracks possible bindings and non-incidence reveals non-interactions. In sum, these dual perspectives enrich soft set theory and offer promising tools for future studies in mathematics and applied sciences.

REFERENCES

- [1] D. Molodtsov, "Soft Set Theory First Results," 1999.
- [2] A. R. Roy and R. Biswas, "An Application of Soft Sets in A Decision Making Problem," 2002. [Online]. Available: www.elsevier.com/locate/camwa
- [3] "R-AsmhanHassan".
- [4] Lenovo, "6. GMN-480[2]-V21N2." [Online]. Available: <http://www.geman.in>
- [5] M. Akram and S. Nawaz, "Operations on Soft Graphs," *Fuzzy Information and Engineering*, vol. 7, no. 4, pp. 423–449, Dec. 2015, doi: 10.1016/j.fiae.2015.11.003.
- [6] K. Palani, T. Jones, and V. Maheswari, "Soft Graphs of Certain Graphs," in *Journal of Physics: Conference Series*, IOP Publishing Ltd, Aug. 2021. doi: 10.1088/1742-6596/1947/1/012045.
- [7] R. K. Thumbakara, J. Jose, and B. George, "Hamiltonian Soft Graphs."
- [8] J. D. Thenge, B. S. Reddy, and R. S. Jain, "Connected Soft Graph," *New Mathematics and Natural Computation*, vol. 16, no. 2, pp. 305–318, Jul. 2020, doi: 10.1142/S1793005720500180.
- [9] S. Hussain and B. Ahmad, "Soft separation axioms in soft topological spaces," *Hacettepe Journal of Mathematics and Statistics*, vol. 44, no. 3, pp. 559–568, 2015, doi: 10.15672/HJMS.2015449426.
- [10] P. Sharma, N. Bhardwaj, and G. Dhiman, "WITHDRAWN: Alexandroff soft topological spaces," *Mater Today Proc*, Feb. 2021, doi: 10.1016/j.matpr.2021.01.351.
- [11] Z. I. Mousa, A. F. Hassan, and A. M. Jafar, "II-Bitopology, II-Induced Topology and II-Separation Axioms on Locally Finite Graphs," *Mathematical Modelling of Engineering Problems*, vol. 12, no. 3, pp. 1064–1070, 2025, doi: 10.18280/MMEP.120333.
- [12] K. Ruohonen, "GRAPH THEORY."
- [13] "Warfarin-DDI-Reference_Bimonthly-Resource-FINAL-061716".
- [14] P. L. Mar *et al.*, "Drug Interactions Affecting Oral Anticoagulant Use," Jun. 01, 2022, *Lippincott Williams and Wilkins*. doi: 10.1161/CIRCEP.121.007956.
- [15] M. Safety Queensland and P. Division, "Appendix 2 - Warfarin drug interactions."
- [16] A. M. Holbrook, J. A. Pereira, R. Labiris, and H. McDonald, "Systematic Overview of Warfarin and Its Drug and Food Interactions." [Online]. Available: www.archinternmed.com