

Study of The Breakup Channel Effect on The Semiclassical and Quantum Mechanical Calculations for The Light and Medium System

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ABSTRACT

A semiclassical and full quantum mechanical approaches have been used to study the effect of channel coupling on the calculations of the total fusion reaction cross section σ_{fus} , the fusion barrier distribution D_{fus} and the reaction probability P_{fus} for the systems $^{11}\text{B} + ^{237}\text{Np}$, $^{15}\text{N} + ^{54}\text{Fe}$ and $^{58}\text{Ni} + ^{54}\text{Fe}$. The semiclassical approach used in the present work based on the method of the Alder and Winther for Coulomb excitation. The full quantum mechanical approach was based on solving the time dependent Schrödinger equation including the coupling effect. A comparison of our semiclassical calculations and full quantum mechanical calculations with the corresponding experimental data shows good agreement, above and below the Coulomb barrier.

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دراسة تأثير قنوات التفكك على الحسابات شبه الكلاسيكية والكمية للأنظمة الخفيفة والمتوسطة

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الكلمات المفتاحية:

- تفاعل الاندماج
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- المعالجة الشبه كلاسيكية
- معالجة كمية
- الانظمة الخفيفة
- الانظمة المتوسطة

الخلاصة

لقد تم استخدام التقريبين الشبه كلاسيكي والكمي لدراسة تأثير اقتران قنوات التفاعل على حسابات مساحة المقطع العرضي وتوزيع حاجز الجهد واحتمالية التفاعل لتفاعلات الاندماج النووي للأنظمة. التقريب شبه الكلاسيكي مبني على استخدام طريقة ايلدر ووينذر للتهيج الكولومي. التقريب الكمي كان مبني على حل معادلة شرودنجر المعتمدة على الزمن. مقارنة نتائج الحسابات الشبه كلاسيكية والكمية اثبتت توافقها مع النتائج العملية اعلى واسفل حاجز الجهد.

1. Introduction

One of the most important fields that attracted many researchers all over the world is the study of the collisions of both stable and radioactive weakly bound nuclei. In the nuclear

fusion two nuclei are interact trough the collision to produce different ones [1], these reactions can be studied theoretically and comparing the theoretical results with the available experimental ones in order to be convinced that we investigate on the right way.

For the tightly bound systems, the collision is very influence by the breakup channel [2]. It is important to choose system with different masses in order to investigate the dependence of the reaction modes on the bombarding energy and nucleons number in more detail [3]. The fusion cross sections at sub-barrier energies was improved by both inelastic excitations of bound states and direct transfer channels. There is a strong coupling between the breakup and the elastic channel because of the weak binding, in addition to this coupling, the usual couplings with inelastic and transfer channels will have a strong effect on the reaction cross section [4].

The Coulomb barrier can be considered as the minimum kinetic energy that the projectile should have to generate a nuclear reaction [5]. The energy and many other quantities are conserved through the nuclear reactions which is associated with emitting or absorbing amount of energy known as the Q -value of the reaction [6]. For simplicity, we can consider the two colliding nuclei as rigid spherical objects that interact with the potential barrier which is composed of the Coulomb and the nuclear potentials [7, 8]. Experimentally, there are many other factors should be taken into account in studying fusion reactions such as the relative motion of the internal degrees of freedom of the colliding nuclei, nuclear deformation, and the particle transfer because they have a great influence on the reaction cross section [9, 10]. The nuclear fusion associated with different mechanisms that need to a unique nuclear potential to be understood [11]. The probability of tunneling depends on many factors such as the change of the intrinsic synthesis during the reaction and to the influence of the open channels [12, 13]. The nuclear fusion process provides us with the information that we need to understand the nuclear synthesis and the astrophysical nucleogenesis [14]. Recently, F. A. Majeed and Abdul-Hussien [15] had employed the semiclassical approach to study the effect of the breakup channel using CDCC method on the

fusion on the fusion reaction cross section σ_{fus} , and the fusion barrier distribution D_{fus} for ${}^{6,8}\text{H}$ halo nuclei. F.A.M et al. [16] had performed had performed semiclassical coupled-channels calculations in heavy-ion fusion reaction for the systems ${}^{40}\text{Ar}+{}^{110}\text{Pd}$ and ${}^{132}\text{Sn}+{}^{48}\text{Ca}$, they proved that he semiclassical approach including the coupling between the elastic channel and the continuum proves to be very successful in describing the total fusion reaction cross section σ_{fus} and the fusion barrier distribution D_{fus} below and above the Coulomb barrier for medium and heavy systems. The aim of the present is to employ a semiclassical approach by adopting Alder and Winther (AW) [17] theory originally used to treat the Coulomb excitation of nuclei. The semiclassical approach has been implemented and coded using FORTRAN programming language codename (SCF) is written and developed by L. F. Canto et al., [18]. Which has been used for the calculations of the total fusion cross section σ_{fus} (mb), fusion barrier distribution D_{fus} (mb/MeV) and fusion probability P_{fus} . The results from the present study will be compared with the quantum mechanical calculations using the FORTRAN code (CC)[19] and with the experimental data for the three systems ${}^{11}\text{B}+{}^{237}\text{Np}$, ${}^{15}\text{N}+{}^{54}\text{Fe}$ and ${}^{58}\text{Ni}+{}^{54}\text{Fe}$.

2. Theoretical Framework

The strong coupling between the entrance channel and the breakup channel will help to solve the complexity of the weakly bound nuclei fusion reaction semi-classically by assuming a Rutherford trajectory for the relative motion (r) that can be significantly improved in the (Wentzel, Kramers and Brillouin approximation) WKB approximation between the colliding nuclei, and number of intrinsic coordinates (ξ) which are associated to eigenfunction set as [20-22].

$$\langle \dot{v} | v \rangle = \int dE \varphi_{\dot{v}}^*(E) \varphi_v(E) = \delta_{\dot{v}v}(1)$$

Where v is the asymptotic relative velocity that given from the relation between the wave number k and the reduced mass μ as $v = \hbar k / \mu$, and an intrinsic Hamiltonian

$$h|\varphi_v\rangle = \varepsilon_v|\varphi_v\rangle, \tag{2}$$

where ε_v is the energy of the internal motion, then the Hamiltonian of the system can be written as [22],

$$H = T + h(\mathcal{E}) + V(\mathcal{E}, r) \tag{3}$$

where H is the intrinsic Hamiltonian, $T = -\hbar^2 \nabla^2 / 2\mu$ is the kinetic energy operator of the relative motion between the projectile and target nuclei, and $V(\mathcal{E}, r)$ is the interaction potential [23-25]. Alder-Winther (AW) method represent the base of the semiclassical approach, which simplify studying Coulomb excitations of fusion reactions to be called (CDCC) method that include the excitations of the breakup channel [26-29]. The quantum mechanical treatment was represented by solving the time-dependent Schrödinger equation. The internal wave function of the excited nucleus have been taken to treat the dynamics in the intrinsic space with the Hamiltonian as [29-30],

$$\begin{aligned} H(\mathcal{E}, t) &= h(\mathcal{E}) + V(\mathcal{E}, r(t)) \\ &= h(\mathcal{E}) + V(\mathcal{E}, t) \end{aligned} \tag{4}$$

$$H(\mathcal{E}, t)\Psi(\mathcal{E}, t) = i\hbar\partial\Psi(\mathcal{E}, t)/\partial t \tag{5}$$

Expanding the wave function $\Psi(\mathcal{E}, t)$ in terms of a properly truncated set of eigenfunctions of h , from the above equation gives the set of coupled differential equations,

$$i\hbar\dot{a}_v(t) = \sum_v \langle \dot{v} | V(\mathcal{E}, t) | v \rangle e^{i\hbar(\varepsilon_v - \varepsilon_v)t/\hbar} a_v(t)$$

$$\dot{v} \text{ or } v = 0, 1, \dots, N \tag{6}$$

the potentials of the channel coupling are [30],

$$\begin{aligned} &\langle \dot{v} | V(\mathcal{E}, t) | v \rangle \\ &= \int d\mathcal{E} \Psi_{\dot{v}}^*(\mathcal{E}) V(\mathcal{E}, t) \Psi_v(\mathcal{E}) \end{aligned} \tag{7}$$

where, $\Psi_{\dot{v}}^*(\Psi_v)$ is the eigenfunction of h with eigenvalue $\varepsilon_{\dot{v}}(\varepsilon_v)$. The projectile before the collision ($t \rightarrow -\infty$) was in its ground state, then we can solve the above equation with initial conditions; $a_v(l, t \rightarrow -\infty) = \delta_{v0}$. The final population of channel v in a collision with angular momentum l is $P_l(v) = |a_v(l, t \rightarrow -\infty)|^2$ and the cross section is given by [24, 25],

$$\sigma_v = \frac{\pi}{\kappa^2} \sum_l (2l + 1) P_l(v) \tag{8}$$

and,

$$\begin{aligned} &P_l^F(v) \\ &= \frac{4\kappa}{E} \int dr |u_l(\kappa_0, r)|^2 W_v^F(r) \end{aligned} \tag{9}$$

$P_l^F(v)$ and $u_l(\kappa_0, r)$ is the fusion probability and the radial wave function respectively for the l^{th} -partial-wave in channel v , W_v^F is the absolute value of the optical potential imaginary part in this channel arising from fusion. The fusion cross section in multi-channel scattering is [25, 31],

$$\sigma_F^{(v)} = \frac{\pi}{\kappa^2} \sum_l (2l + 1) P_l^F(v), \tag{10}$$

with

$$P_l^F(v) \simeq \bar{P}_l^{(v)} T_l^{(v)}(E_v). \tag{11}$$

So, we can evaluate the fusion cross by assuming the probability that the system is in channel v at the point of closest approach on the classical trajectory, $T_l^{(v)}(E_v)$ is the probability that a particle with reduced mass $M_v = m_0 A_P A_T / (A_P + A_T)$ and energy $E_v = E - \varepsilon_v$ in that channel. The complete fusion cross section can be found semi-classically by representing the breakup channel by a single effective channel as, [25, 26, 31] by;

$$\bar{P}_l^{(0)} \equiv P_l^{(surv)} = |a_0(t_{c.d})|^2 \tag{12}$$

where the amplitude a_0 is evaluated along a trajectory and the factor $P_l^{(surv)}$ is usually called survival probability. Therefore [31],

$$\sigma_{C.F} = \frac{\pi}{k^2} \sum_l (2l + 1) P_l^{(sur)} T_l^{(0)}(E), \quad (13)$$

3. Fusion barrier distribution

The importance of studying the reaction dynamics of colliding nuclei at energies near the Coulomb barrier lead the investigators to the barrier distribution function. For the barrier heights B and the corresponding weights D (B) from fusion cross section can be determined as the sum of the fusion cross section from distributed fusion barriers in form[28, 32]

$$\sigma(E) = \int_0^{-\infty} \sigma^0(E; B) D(B) dB \quad (14)$$

We can find the barrier position and the corresponding weight by taking the derivative of the barrier transmission probability with respect to energy. From the classical formula of fusion cross section[25, 33],

$$\sigma_{fus}^{(0)}(E) \approx \pi R_B^2 \left(1 - \frac{V_B}{E}\right) \quad (15)$$

the barrier distribution can be evaluated as [15, 31, 35]

$$D_{fus}(E) = \frac{1}{\pi R_B^2} \frac{d^2(E\sigma)}{dE^2} = \frac{1}{\pi R_B^2} \frac{d^2(Q)}{dE^2} \quad (16)$$

and the theoretical representation of the barrier distribution can be written as [35],

$$\frac{d^2(Q)_2}{dE^2} = \frac{(Q)_3 + (Q)_1 - 2(Q)_2}{\Delta E^2} \quad (17)$$

the second derivative at energy E was associated by a statistical error δ_c which is approximately given by[32],

$$\delta_c \cong$$

$$\frac{E^2}{dE^2} \sqrt{(\rho_{fus})_1^2 + 4(\rho_{fus})_2^2 + (\rho_{fus})_3^2} \quad (18)$$

where ρ are the uncertainties of the absolute cross section. The approximated Wong formula can be used to obtain experimental D_{fus} , where [25, 33],

$$\sigma_W = R_B^2 \frac{\hbar\omega}{2E} \ln\left(1 + \exp\left\{\frac{2\pi(E - V)}{\hbar\omega}\right\}\right) \quad (19)$$

4. Results and Discussion

In this section, the theoretical results obtained for fusion cross σ_{fus} , the fusion barrier distribution D_{fus} and the reaction probability P_{fus} using the semiclassical approach for the systems $^{11}\text{B}+^{237}\text{Np}$, $^{15}\text{N}+^{54}\text{Fe}$ and $^{58}\text{Ni}+^{54}\text{Fe}$ are explained. The semiclassical calculations for the σ_{fus} , D_{fus} and P_{fus} are compared with the experimental data and with the full quantum mechanical calculations using the CC code, the Akyüz-Winther potential parameters used in the present calculations are displayed in Table 1.

Table 1: Parameters used in the Akyüz-Winther potential for real and imaginary parts.						
System	$V_0(\text{MeV})$	$r_0(\text{fm})$	$a_0(\text{fm})$	$W(\text{MeV})$	$r_i(\text{fm})$	$a_i(\text{fm})$
$^{11}\text{B}+^{237}\text{Np}$	-126.9	1.200	0.500	-38.8	0.925	0.775
$^{15}\text{N}+^{54}\text{Fe}$	-80.0	1.200	0.500	-24.3	0.928	0.772
$^{58}\text{Ni}+^{54}\text{Fe}$	-82	1.101	.896	-21.4	.967	.733

4.1. The $^{11}\text{B}+^{237}\text{Np}$ System

The calculations of the fusion cross σ_{fus} and the fusion barrier distribution D_{fus} in Figure 1 panel (a) and panel (b), respectively, for the system $^{11}\text{B}+^{237}\text{Np}$ system. The dashed blue and red curves represent the semiclassical and full quantum mechanical calculations without coupling, respectively. The solid blue and red curves are the calculations including the

coupling effects for the semiclassical and full quantum mechanical calculations, respectively. Panel (a) shows the comparison

between our semiclassical and full quantum mechanical calculations with the corresponding experimental data (solid green circles). The experimental data for this system are obtained from Ref. [35].

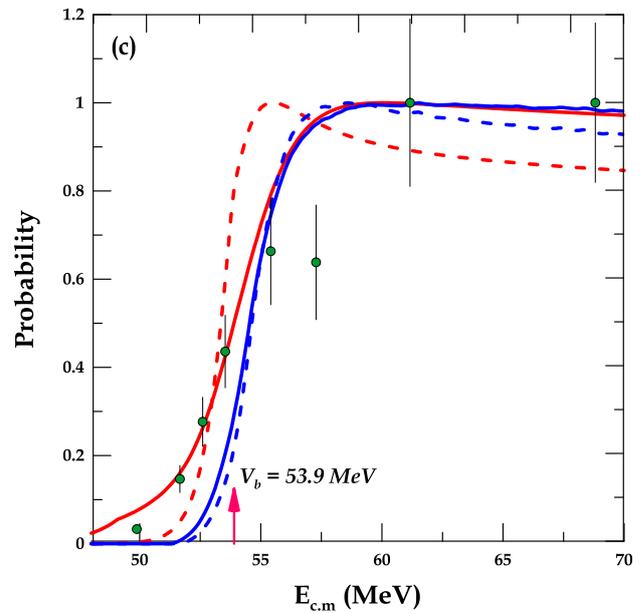
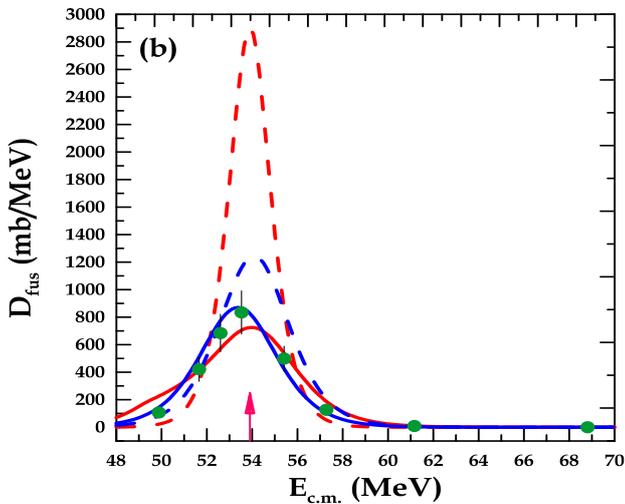
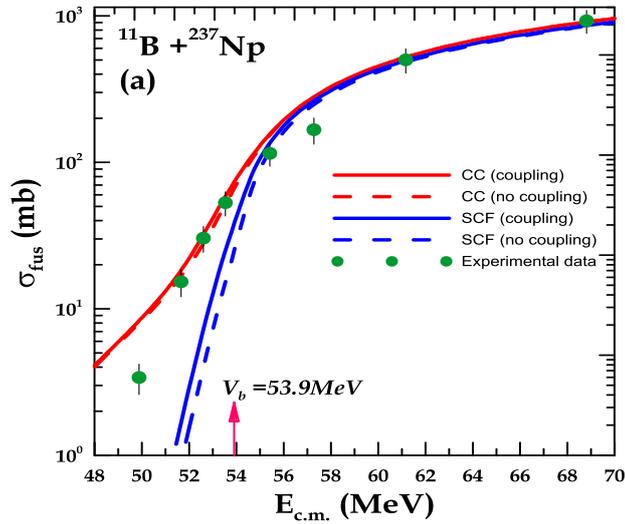


Figure 1: The comparison between semiclassical (blue curves) and full quantum mechanical (red curves) with the experimental data (green filled circles) for $^{11}\text{B}+^{237}\text{Np}$ system. Panel (a) for the total fusion cross section σ_{fus} (mb), and Panel (b) for the fusion barrier distribution D_{fus} (mb/MeV). The arrow on the x-axis indicate the position of the Coulomb barrier V_b .

Table 2: The obtained chi-square values from comparison between theory and experiment for the $^{11}\text{B}+^{237}\text{Np}$ system for the total fusion cross section σ_{fus} and the fusion barrier distribution D_{fus} and the probability above and below

V_b .

system	CC				SCF			
	No coupling		coupling		No coupling		coupling	
$^{11}\text{B}+^{237}\text{Np}$	Below V_b	Above V_b	Below V_b	Above V_b	Below V_b	Above V_b	Below V_b	Above V_b
σ_{fus}	.005661	.092984	.006336	.094280	.803889	.074661	.362523	.082183
D_{fus}	.182582	.134752	.016851	.003532	.015782	.018413	.008578	.004406
Prob.	.000058	.000033	.000005	.000020	.264439	.000021	.026640	.000018

A) Below v_b

The lowest obtained value for the chi-square is found to be $\chi^2 = 0.005661$ for the total fusion cross section σ_{fus} in the case of no-coupling which corresponds to the full quantum mechanical calculations is in the best agreement with the experimental data as shown in Table 2. The best obtained value of chi-square for the fusion barrier distribution D_{fus} calculations is $\chi^2 = .008578$ which corresponds to the semiclassical calculations including coupling effects as shown in Table 2. The best calculated chi-square value obtained is $\chi^2 = 0.000005$, as shown in Table 2, which corresponds to the full quantum mechanical calculations with channel coupling are in the best agreement with experimental data for the fusion probability P_{fus} .

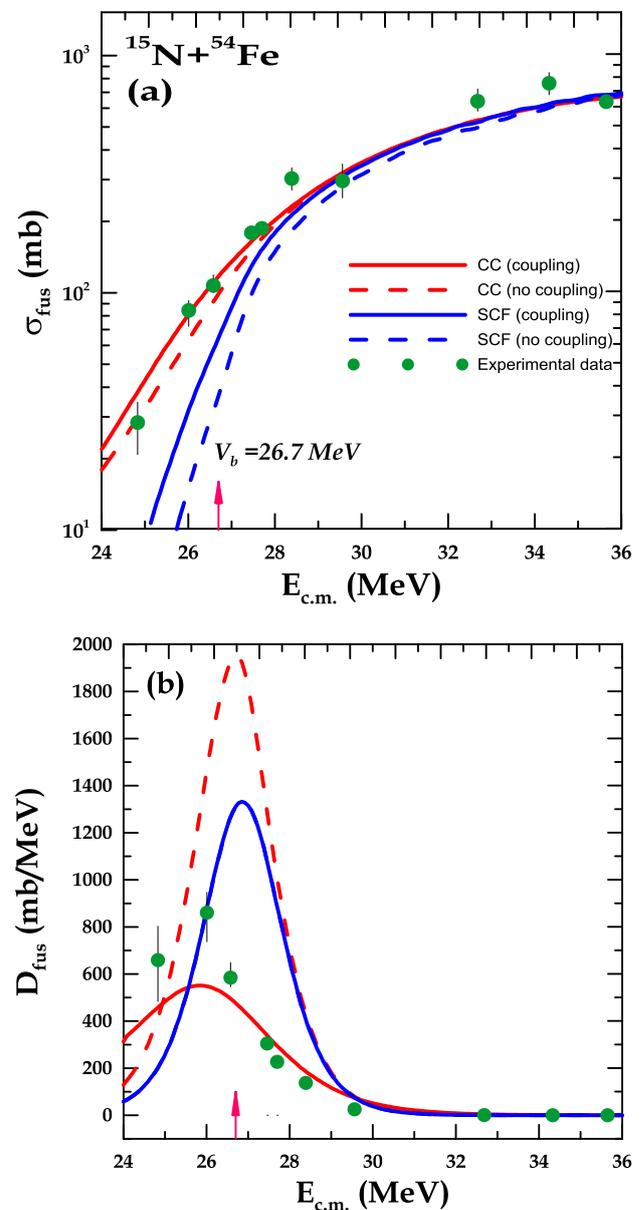
B) Above v_b

The chi-square values obtained for this system are shown in Table 2 for total fusion cross section σ_{fus} , fusion barrier distribution D_{fus} and fusion probability P_{fus} , respectively. The lowest value found for semiclassical calculation including no-coupling as $\chi^2 = 0.074661$ for σ_{fus} . The best calculated chi-square value obtained is $\chi^2 = 0.003532$ which corresponds to the full quantum mechanical calculation including coupled channel are in the best agreement with the experimental data for the fusion barrier distribution D_{fus} , while chi-square value is $\chi^2 = 0.000018$ for P_{fus} , which corresponds to the semiclassical calculations including coupling are in the best agreement with experimental data.

4.2. The $^{15}\text{N}+^{54}\text{Fe}$ system

Figure 2 panel (a) and (b) presents the comparison between our theoretical calculations for the total fusion reaction cross section σ_{fus} and the fusion reaction barrier distribution D_{fus} using both semiclassical and quantum mechanical calculations with the corresponding experimental data for the system $^{15}\text{N}+^{54}\text{Fe}$ the

experimental data for this system are obtained from Ref [36]. The dashed blue and red curves represent the semiclassical and full quantum mechanical calculations without coupling, respectively. The solid blue and red curves are the calculations including the coupling effects for the semiclassical and full quantum mechanical calculations, respectively. The calculated chi-square values for the total fusion cross section, and fusion barrier distribution for both semiclassical and quantum mechanical coupled channel compared with their corresponding experimental data for CC and SCF codes for $^{15}\text{N}+^{54}\text{Fe}$ system are:



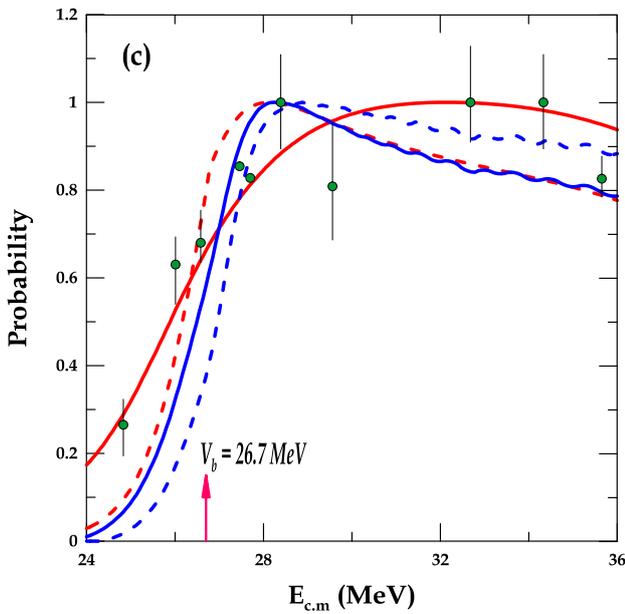


Figure 2: The comparison between semiclassical (blue curves) and full quantum mechanical (red curves) with the experimental data (green filled circles) for $^{15}\text{N}+^{54}\text{Fe}$ system. Panel (a) for the total fusion cross section σ_{fus} (mb), and Panel (b) for the fusion barrier distribution D_{fus} (mb/MeV).

Table 3: The obtained chi-square values from comparison between theory and experiment for the $^{15}\text{N}+^{54}\text{Fe}$ system for the total fusion cross section σ_{fus} and the fusion barrier distribution D_{fus} and the probability above and below the Coulomb barrier V_b .

system	CC				SCF			
	No coupling		coupling		No coupling		coupling	
$^{15}\text{N}+^{54}\text{Fe}$	Below V_b	Above V_b	Below V_b	Above V_b	Below V_b	Above V_b	Below V_b	Above V_b
	σ_{fus}	.001192	.011880	.000500	.011117	.078011	.029838	.019595
D_{fus}	.137258	.221503	.031837	.047839	.144815	.186630	.144815	.186630
Prob.	.042711	.016887	.004408	.009791	.516565	.010116	.094209	.015382

A) Below v_b

The best values of chi-square for the calculated total fusion cross section, fusion barrier distribution and fusion probability in case of coupled channel for full quantum mechanical calculations compared to the experimental data is found to be $\chi^2 = 0.000500$, $\chi^2 = 0.031837$ and $\chi^2 = 0.004408$ for σ_{fus} , D_{fus} and P_{fus} , respectively, as tabulated in Table 3.

B) Above V_b

The best values of chi-square for the calculated total fusion cross section, fusion

barrier distribution and fusion probability in case of coupled channel for full quantum mechanical calculations compared to the experimental data is found to be $\chi^2 = 0.011117$, $\chi^2 = 0.047839$ and $\chi^2 = 0.009791$ for σ_{fus} , D_{fus} and P_{fus} , respectively, as tabulated in Table 3.

4.3. The $^{58}\text{Ni} + ^{54}\text{Fe}$ System

The calculations of the fusion cross section σ_{fus} and fusion barrier distribution D_{fus} is presented in Fig. 3 panel (a) and panel (b), respectively for the system $^{58}\text{Ni} + ^{54}\text{Fe}$. The dashed blue and red curves represent the semiclassical and full quantum mechanical calculations without coupling, respectively. The

solid blue and red curves are the calculations including the coupling effects for the semiclassical and full quantum mechanical calculations, respectively. Figure 3 panel (a) shows the comparison between our semiclassical and full quantum mechanical calculations with the respective experimental data (solid green circles). The experimental data for this system obtained from Ref. [37]. The calculated chi-square values for the total fusion cross section, and fusion barrier distribution for both semiclassical and quantum mechanical coupled channel compared with the corresponding experimental data [37].

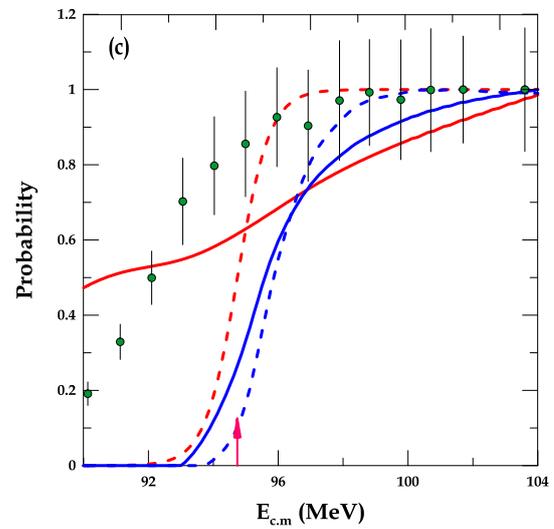
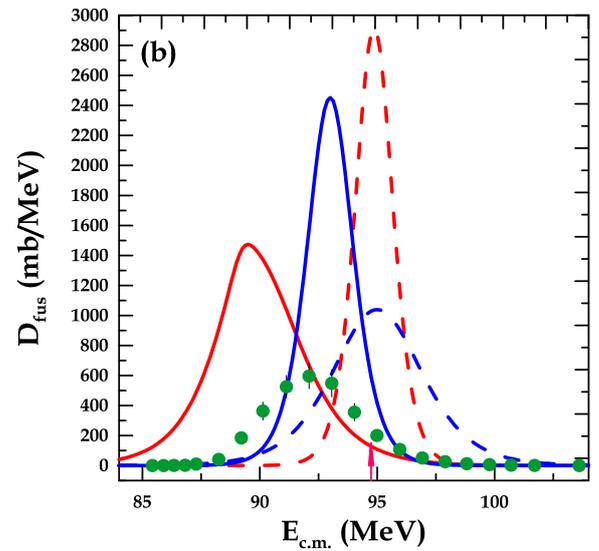
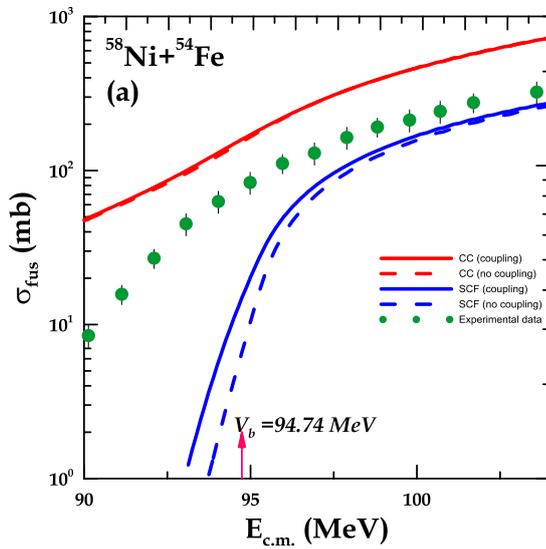


Figure 3: The comparison between semiclassical (blue curves) and full quantum mechanical (red curves) with the experimental data (green filled circles) for $^{58}\text{Ni} + ^{54}\text{Fe}$ system. Panel (a) for the total fusion cross section σ_{fus} (mb), and Panel (b) for the fusion barrier distribution D_{fus} (mb/MeV).

Table 4: The obtained chi-square values from comparison between theory and experiment for the $^{58}\text{Ni} + ^{54}\text{Fe}$ system for the total fusion cross section σ_{fus} and the fusion barrier distribution D_{fus} and the probability above and below the Coulomb barrier V_b .

system	CC				SCF			
	No coupling		coupling		No coupling		coupling	
	Below V_b	Above V_b	Below V_b	Above V_b	Below V_b	Above V_b	Below V_b	Above V_b
$^{58}\text{Ni} + ^{54}\text{Fe}$								
σ_{fus}	.030729	.021461	.030728	.021749	.041775	.063125	.075323	.026282
D_{fus}	.109532	.079426	.000991	.003225	.000578	.048014	.000978	.014908
Prob.	.004177	.003789	.000004	.007805	.178731	.054642	.114991	.020545

A) Below V_b

The chi-square values obtained for this system are shown in Table 4 for total fusion cross section σ_{fus} , fusion barrier distribution D_{fus} and fusion probability P_{fus} , respectively. The lowest value found for full quantum mechanical including coupling as $\chi^2 = .030728$ for σ_{fus} . The best calculated chi-square value obtained is $\chi^2 = .000578$ which corresponds to the semiclassical calculation including no-coupled channel are in the best agreement with the experimental data for the fusion barrier distribution D_{fus} , while chi-square value is $\chi^2 = .000004$ for P_{fus} , which corresponds to the full quantum mechanical calculations including coupling are in the best agreement with experimental data.

B) Above V_b

The calculated chi-square value is found to be $\chi^2 = .021461$ for CC code as listed in Table 4, in the case of no-coupled channel calculations, which corresponds to the full quantum mechanical calculations including coupling are in the better agreement with the experimental data for the total fusion cross section σ_{fus} . The best obtained value of chi-square for the fusion barrier distribution calculations is $\chi^2 = .003225$ for CC code

and with the corresponding to the full quantum mechanical calculations with coupled

channel. The minimum value of chi-square is $\chi^2 = .003789$ for the fusion probability which corresponds to the full quantum mechanical calculations without including channel coupling are in the best agreement with the corresponding experimental data.

5. Conclusion

The semiclassical and quantum mechanical calculations for the total fusion reaction σ_{fus} , the fusion barrier distribution D_{fus} and the fusion probability P_{fus} calculations below and around Coulomb barrier for the systems $^{11}\text{B} + ^{237}\text{Np}$, $^{15}\text{N} + ^{54}\text{Fe}$ and $^{58}\text{Ni} + ^{54}\text{Fe}$. We conclude that the breakup channel is very important to be taken into consideration to describe the total fusion reaction σ_{fus} , the fusion barrier distribution D_{fus} and fusion probability P_{fus} for light and medium systems. A comparison of our semiclassical calculations and full quantum mechanical calculations with the corresponding experimental data shows good agreement, above and below the Coulomb barrier. Our results for the light systems are in more agreement with the experimental data from those for the medium one because the breakup and the relative motion have more effect on the light systems than on the medium ones.

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