Study the effect of Gaussian Filter on Optical System that contains an Obscured Circular Synthetic Apertures

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Abstract:

In this research, the effect of Gaussian spatial filter which has coefficient of normative deviation (a) for different values (a=0.25,0.5,0.75,1) on point spread function (PSF) was studied by using MathCAD program, it has been derived point spread function for the ideal optical system or contain focus error (W_{20}=0.25λ,0.5λ,0.75 λ),for set from circular synthetic apertures (N=4,8,12), the obscured ratio (ε= 0.25,0.5,0.75). The results showed that the high values of coefficient of normative deviation for Gaussian filter leads to decrease the intensity in the formed image by existence of obscured synthetic apertures. The increased apertures number, increase the intensity by the existence of Gaussian filter.

Keywords: Point Spread Function , Gaussian Filter, Synthetic Apertures, Obscured Apertures.

دراسة تأثير المرشح الكاوسي على نظام بصري يحتوي على الفتحات الدائرية المعاقة المركبة

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قسم الفيزياء / كلية العلوم / قسم الفيزياء

الـخلاصة:

جرى في هذا البحث دراسة تاثير المرشح الكاوسي لقيم مختلفة من معامل الانحراف المعياري (a=0.25,0.5,0.75,1) على دالة الانتشار النقطية (PSF) باستخدام برنامج MathCAD إذ إشتقت معادلة دالة الانتشار النقطية لنظام بصري محدد بالحيود وبوجود خطا بوري (W_{20}=0.25λ, 0.5λ, 0.75 λ) لصف من الفتحات الدائرية المركبة (N=4, 8, 12) بدلا من الفتحة المنفردة وبنسب اعاقة مختلفة (ε= 0.25, 0.5, 0.75) .

بينت النتائج ان القيم العليا من معامل الانحراف المعياري للمرشح الكاوسي تؤدي إلى نقصان الشدة في الصورة المتكونة بوجود الفتحات المركبة المعاقة وبزيادة عدد الفتحات المركبة المعاقة تزداد الشدة بوجود المرشح الكاوسي.

الكلمات المفتاحية: دالة الانتشار النقطية، المرشح الكاوسي، الفتحات المركبة، الفتحات المعاقة.

1. Introduction

There are several factors that affect the evaluation of the image quality which is formed by the optical system. From these important factors, shape of aperture and measured spread function (Point, Line, and Edge) which represents descriptions of the intensity distribution in image plane for an object (Point, Line and Edge, etc.) [1,2].

An optical system consists of a number of lenses placed apart, and have a common principal axis; The image formed by such a coaxial optical system is good and almost free of aberrations [3].

The point spread function (PSF) is the intensity of a Light source which is defined by the quantity of photons per unit of area [4].

Diffraction plays an important role in the optical system, where the phenomenon of binding light waves around corners and their spreading into the geometric shadow of an object is called diffraction [5].

The applications and advantages of the optical system of synthetic apertures prompted researches to study this optical systems. Some problems appeared in the optical system like (apertures distribution methods, aperture manufacturing methods, high materialistic costs, high weight and moment of inertia). These problems can be resolved usually to give some of the losses in optical performance. These losses are of interest applications with high usefulness [6].

" Synthetic Aperture " can be defined as a structure to separate optical systems of large individual aperture function sometimes called " Mosaic" or "Segmented Mirror". Synthetic aperture is an image system for independent optical systems which are together sharing the image domain [7].

Synthetic aperture techniques are commonly used to obtain high resolution from data acquired using low resolution sensors. These techniques are commonly used in modern sonar and radar systems, being designated by Synthetic Aperture Sonar (SAS) and by Synthetic Aperture Radar (SAR) systems, respectively. This kind of system is presently used in civil and military applications [8].

The properties of the optical system can be described by a point spread function. Point spread function is related to the phase at these exit pupil apertures through a Fourier transform [9].

Deriving the equation of Point Spread Function for Optical System with existence of Gaussian Filter

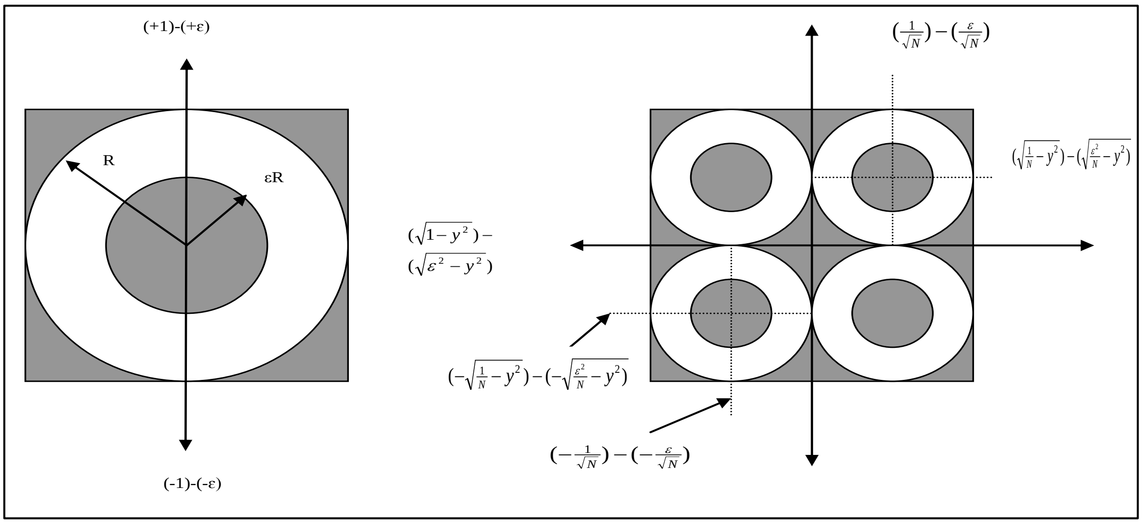

Figure (1) the Integral Boundary for Array of Obscured Circular Synthetic Apertures

(a) Obscured Individual Circular Aperture

(b) (Obscured Synthetic Aperture N=4, When N is the Number of Obscured Aperture

The intensity in point spread function for array of obscured circular synthetic

Apertures, can be calculated by the relation [10] ,[11].

$$PSF = |F|^2 = |F_1 - F_2|^2$$

………………(1)

Where

F_1: is the complex amplitude function for array of circular apertures.

F_2: is the complex amplitude function for array of obscured circular apertures.

We can express the complex amplitude in the point (u,v) in the image plane by using Fourier transform to pupil function

……..(2) $f(x',y') = T(x',y')e^{ikw(x'.y')}$

Where

$T(x',y')$: represents the real amplitude distribution in exit pupil and which is called " pupil transparency " or " transmission function " and often choose equal one unit. The research is by substituting equation Gaussian filter to study the effect on point spread function .

The equation Gaussian filter is [12]

$$T(x,y) = \frac{1}{\sqrt{2\pi a}} e^{-(\frac{x^2+y^2}{2a^2})}$$

Where

a: represents coefficient of normative deviation for Gaussian filter function, the Gaussian filter is put in rays path for specifie form, the ray inside or outside from optical system on Gaussian bundle figure. Coefficient of normative deviation represents the value of the point capsizing : of in curve downhill for filter function.

$e^{ikw(x',y')}$: Wave front of aberration function.

$w(x',y')$: Aberration factor

(x',y') : Exit pupil coordinates

[3,6,13 and 14].

We can express point spread function for circular aperture in its integral form, by putting the exit pupil coordinates of form (x,y) instead of (x',y') , for simplicity [15,16 and 17].

$$F_1(u',v') = n.f \int_y \int_x f(x,y) e^{i2\pi(u'x+v'y)} dxdy$$

…………………………..(3)

and for N from obscured circular apertures, we get:

$$y = y' + y_j \quad \because x = x' + x_j$$

…………… (4)

$$F_1(u',v') = n.f \sum_{j=1}^{N} \int_y \int_x f(x,y) e^{i2\pi[u'(x'+x_j)+v'(y'+y_j)]} dxdy$$

………… .(5)

$$F_1(u',v') = n.f \sum_{j=1}^{N} \int_y \int_x f(x,y) e^{i2\pi(u'x'+v'y')} .e^{i2\pi(u'x_j+v'y_j)} dxdy$$

…………… (6)

$$F_1(u',v') = n.f \int_y \int_x f(x,y) e^{i2\pi(u'x'+v'y')} dxdy . \sum_{j=1}^{N} e^{i2\pi(u'x_j+v'y_j)}$$
…………… (7)

By substituting the equation (2) in equation (7), we obtain:

$$F_1(u',v') = n.f \int_y \int_x \tau(x,y) e^{ikw(x,y)} . e^{i2\pi(u'x'+v'y')} dxdy . \sum_{j=1}^{N} e^{i2\pi(u'x_j+v'y_j)}$$
…………… (8)

Let $T(x,y) = \frac{1}{\sqrt{2\pi a}} e^{-(\frac{x^2+y^2}{2a^2})}$

$$F_1(u',v') = n.f \int_y \int_x \frac{1}{2\pi a} e^{-(\frac{x^2+y^2}{2a^2})} . e^{ikw(x,y)} . e^{i2\pi(u'x'+v'y')} dxdy . \sum_{j=1}^{N} e^{i2\pi(u'x_j+v'y_j)}$$
…..……… (9)

Where $k = \frac{2\pi}{\lambda}$

$$F_1(u',v') = n.f \int_y \int_x \frac{1}{2\pi a} e^{-(\frac{x^2+y^2}{2a^2})} e^{i2\pi[w(x,y)+(u'x'+v'y')]} dxdy . \sum_{j=1}^{N} e^{i2\pi(u'x_j+v'y_j)}$$
………….(10)

By substituting the integral boundary for figure (1) the area of synthetic aperture includes

$$F_1(u',v') = n.f \int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} \frac{1}{2\pi a} e^{-(\frac{x^2+y^2}{2a^2})} e^{i2\pi[w(x,y)+(u'x'+v'y')]} dxdy . \sum_{j=1}^{N} e^{i2\pi(u'x_j+v'y_j)}$$
…………… (11)

where $e^{i\theta} = \cos\theta + i\sin\theta$

$$F_1(u',v') = n.f \left[\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} (\frac{1}{2\pi a} e^{-(\frac{x^2+y^2}{2a^2})}).[\cos\{2\pi[w(x,y)+u'x'+v'y']\} + i\sin\{2\pi[w(x,y)+u'x'+v'y']\}] dxdy \right]$$
$$.\left[\sum_{j=1}^{N} \cos\{2\pi(u'x_j+v'y_j)\} + i\sin\{2\pi(u'x_j+v'y_j)\} \right]$$
…………. (12)

Let $z' = 2\pi u'$ and $m' = 2\pi v'$ become:

$$F_1(z',m') = n.f \left[\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} [\frac{1}{2\pi a} e^{-(\frac{x^2+y^2}{2a^2})}].[\cos\{2\pi w(x,y)+z'x'+m'y'\} + i\sin\{2\pi w(x,y)+z'x'+m'y'\}] dxdy \right]$$
$$.\left[\sum_{j=1}^{N} \cos\{z'x_j+m'y_j\} + i\sin\{z'x_j+m'y_j\} \right]$$
…………… (13)

The intensity distribution on the two axes (z',m') is symmetric, so; we can reduce it to one axis only; let $(m'=0)$, then equation (13) will take a new form .

$$F_1(z') = n.f \left[\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} [\frac{1}{2\pi a} e^{-(\frac{x^2+y^2}{2a^2})}].[\cos\{2\pi w(x,y)+z'x'\} + i\sin\{2\pi w(x,y)+z'x'\}] dxdy \right]$$
$$.\left[\sum_{j=1}^{N} [\cos(z'x_j) + i\sin(z'x_j)] \right]$$
…………. (14)

Equation (14) represents the complex amplitude for array of circular synthetic apertures.

The complex amplitude function for array of **obscured** circular apertures from equation (3) can be found as:

$$F_2(u',v') = n.f \int_y \int_x f(x',y') e^{i2\pi(u'x'+v'y')} dx'dy'$$
…………. (15)

By using the same steps and substituting the integral boundary from figure (1) for area of obscured circular apertures, we obtain.

$$F_2(z') = n.f \left[\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}} \int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}} [\frac{1}{2\pi a} e^{-(\frac{x^2+y^2}{2a^2})}].[\cos\{2\pi w(x',y')+z'x'\} + i\sin\{2\pi w(x',y')+z'x'\}] dx'dy' \right]$$
$$.\left[\sum_{j=1}^{N} [\cos(z'x_j) + i\sin(z'x_j)] \right]$$
……….. (16)

By using the physical conception of the equation (1), we can obtain the following:

$$PSF = n.f\left|\begin{array}{l}\left[\displaystyle\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}}[\frac{1}{2\pi a}e^{-(\frac{x^2+y^2}{2a^2})}].\left[\cos\{2\pi w(x,y)+z'x'\}+i\sin\{2\pi w(x,y)+z'x'\}\right]dxdy.\sum_{j=1}^{N}\left[\cos(z'x_j)+i\sin(z'x_j)\right]\right]\\ -\left[\displaystyle\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}}\int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}}[\frac{1}{2\pi a}e^{-(\frac{x^2+y^2}{2a^2})}].\left[\cos\{2\pi w(x',y')+z'x'\}+i\sin\{2w(x',y')+z'x'\}\right]dx'dy'.\sum_{j=1}^{N}\left[\cos(z'x_j)+i\sin(z'x_j)\right]\right]\end{array}\right|^2$$

………. (17) From the relation the equation above becomes:

$$|x+iy|^2 = x^2+y^2$$

$$PSF = n.f\left|\begin{array}{l}\displaystyle\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}}\frac{1}{2\pi a}e^{-(\frac{x^2+y^2}{2a^2})}.\cos\{2\pi w(x,y)+z'x'\}dxdy.\sum_{j=1}^{N}e^{iz'x_j}\\ -\displaystyle\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}}\int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}}\frac{1}{2\pi a}e^{-(\frac{x^2+y^2}{2a^2})}.\cos\{2\pi w(x',y')+z'x'\}dx'dy'.\sum_{j=1}^{N}e^{iz'x_j}\\ +\displaystyle\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}}\frac{1}{2\pi a}e^{-(\frac{x^2+y^2}{2a^2})}.\sin\{2\pi w(x,y)+z'x'\}dxdy.\sum_{j=1}^{N}e^{iz'x_j}\\ -\displaystyle\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}}\int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}}\frac{1}{2\pi a}e^{-(\frac{x^2+y^2}{2a^2})}.\sin\{2\pi w(x',y')+z'x'\}dx'dy'.\sum_{j=1}^{N}e^{iz'x_j}\end{array}\right|^2$$

………(18)

This equation represents the point spread function for array of obscured circular synthetic apertures.

By substituting the value of normalizing factor ($\frac{1}{(\pi-\varepsilon^2\pi)^2}$)[7], in equation (17), we obtain:

$$PSF = \frac{1}{(\pi-\varepsilon^2\pi)^2}\left|\begin{array}{l}\displaystyle\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}}\frac{1}{2\pi a}e^{-(\frac{x^2+y^2}{2a^2})}.\cos(z'x')dxdy.\sum_{j=1}^{N}e^{iz'x_j}\\ -\displaystyle\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}}\int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}}\frac{1}{2\pi a}e^{-(\frac{x^2+y^2}{2a^2})}.\cos(z'x')dx'dy'.\sum_{j=1}^{N}e^{iz'x_j}\\ +\displaystyle\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}}\frac{1}{2\pi a}e^{-(\frac{x^2+y^2}{2a^2})}.\sin(z'x')dxdy.\sum_{j=1}^{N}e^{iz'x_j}\\ -\displaystyle\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}}\int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}}\frac{1}{2\pi a}e^{-(\frac{x^2+y^2}{2a^2})}.\sin(z'x')dx'dy'.\sum_{j=1}^{N}e^{iz'x_j}\end{array}\right|$$

…………. (19)

Equation (19) represents the point spread function for array of obscured circular synthetic apertures with diffraction – limited system.

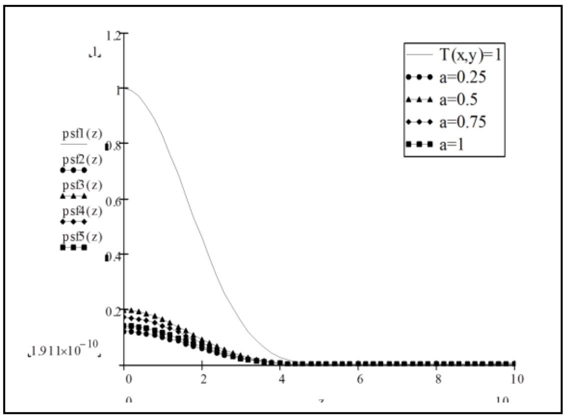

Figure (1) PSF for optical system contain (N=1, W_{20}=0, ε=0)

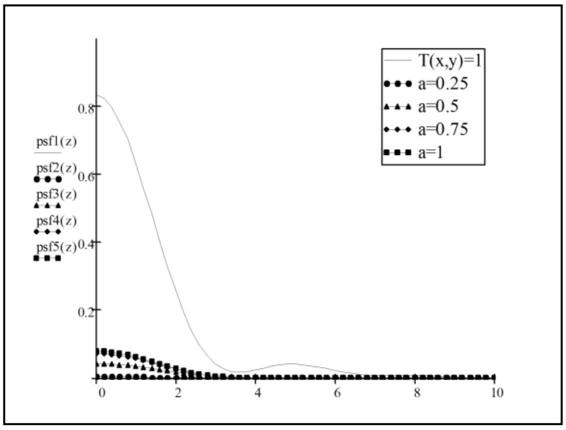

Figure (2) PSF for optical system contain

(N=1, W_{20}=0.25, ε=0.25)

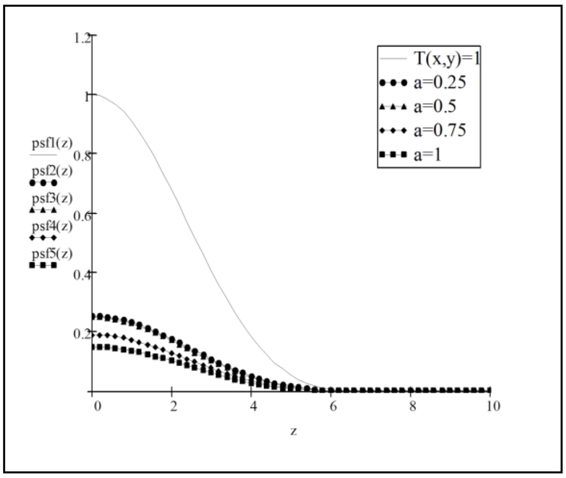

Figure (3) PSF for optical system contain

(N=4, W_{20}=0, ε=0)

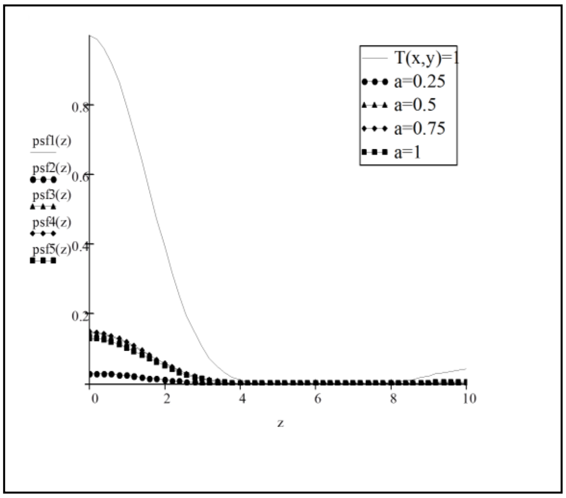

Figure (4) PSF for optical system contain

(N=4, W_{20}=0.25, ε=0.75)

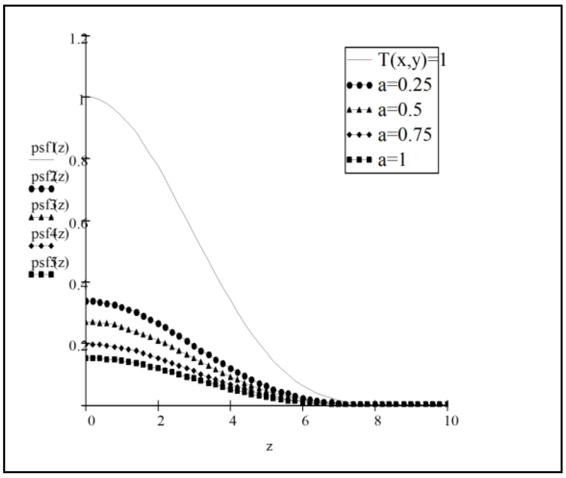

Figure (5) PSF for optical system contain
(N=8, W_{20}=0, ε=0)

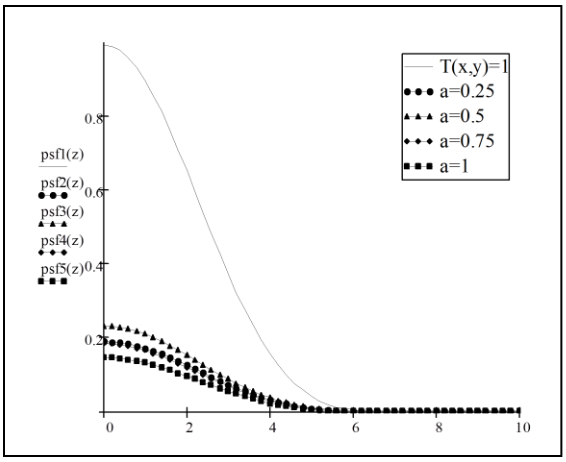

Figure (6) PSF for optical system contain

(N=8, W_{20}=0. 5, ε=0.5)

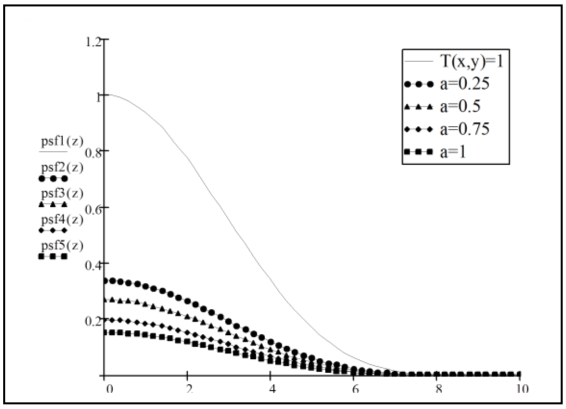

Figure (7) PSF for optical system contain

(N=12, W $_{20}$=0, ε=0)

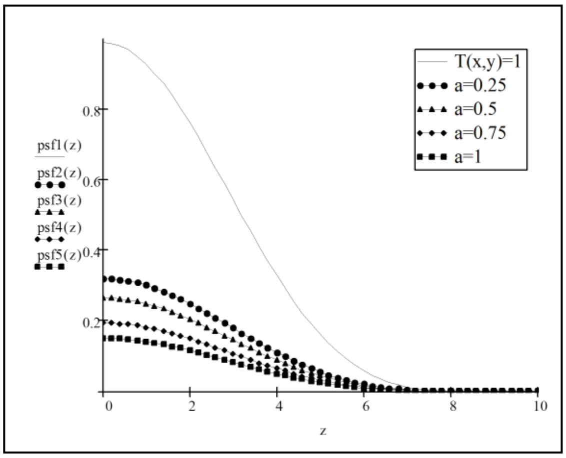

Figure (8) PSF for optical system contain

(N=12, W $_{20}$=0.75, ε=0.25)

2. Result and Discussion:

By Using MathCAD program to solve equation (19) to study the effect of normative deviation coefficient (a) for Gaussian filter for values (a=0.25, 0.5, 0.75, 1) on the point spread function to optical system or contained on different values from focus error ($W_{20}=0.25\lambda$, 0.5 λ and 0.75 λ).

Figures (1,3,5 and 7) represent the intensity distribution curves of point spread function for individual aperture and synthetic apertures (N=4,8 and 12) for diffraction limited system with no obscuration. While figures (2,4,6 and 8) represent the intensity distribution curves of point spread function for individual aperture and synthetic apertures (N=4,8 and 12) found focus error ($W_{20}=0.25\lambda$,0.5 λ and 0.75 λ) with different values of obscuration ratios (ε=0.25, 0.5 and 0.75).

From the figures drawing the point spread function and optical axis for ideal optical system contained on Gaussian filter for different values from coefficient of normative deviation. In conclusion, the point spread function no existence Gaussian filter is normalized function ,and maximum intensity value equal one.

The results showed that the high values of coefficient of normative deviation for Gaussian filter leads to decrease the intensity in the formed image by existence of obscured synthetic apertures and by the apertures number increase to increase the intensity by existence Gaussian filter.

The intensity increase in the optical system, which contains focus error is the good result for production of more clarity image and the optical system that depth of the field greater

From this we conclude that Gaussian filter positively effective on the optical system on the diffraction limited optical system in terms of noise and confusion reducing at the level of the image through the attenuation of secondary peaks and negatively effects on resolving power for optical system by the curve width increase for central intensity in the picture.

Therefore, it prefers not to use this technique in the ideals optical systems which requires use of high analysis capacity and the preferably to use in optical systems that required increase of image purity and the signal ratio increase to the noise in the picture.

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