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# The Effect of adding First Order Aberrations To Third Order Aberrations on Point Spread Function for Triangular Aperture

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**ABSTRACT**

In this work, Point Spread Function (PSF) for triangular aperture was studied, in case of ideal optical system, also in case of presence first order aberrations (focal error and tilt aberration) and third order aberrations (spherical ,coma, and astigmatism aberrations) individually, and for the same values of aberration factor (W=0.1λ …..0.5 λ ) for angle of rotation (ψ =0).

First order aberrations were added to third order aberrations, to obtain the best balance and to reduce the curve width of the (PSF), and to increase the Strehl ratio, like adding a tilt aberration to the coma aberration and adding focal error to spherical aberration and astigmatism aberration.

There are some aberrations effects impact negatively on point spread function, such as focal error with coma aberration and tilt aberration with spherical aberration.

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## تأثير إضافة زيوغ الرتبة الأولى إلى زيوغ الرتبة الثالثة على دالة الانتشار النقطية لفتحة مثلثة

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**الكلمات المفتاحية:**

دالة الانتشار النقطية
زيوغ الرتبة الاولى
زيوغ الرتبة الثالثة
الفتحة المثلثة .

**الـــخُـــلاصـــة**

تم في هذا البحث دراسة دالة الانتشار النقطية لفتحة مثلثة، لنظام بصري محدد بالحيود ، وكذلك في حالة وجود زيوغ الرتبة الاولى (الخطأ البؤري وزيغ الامالة) وزيوغ الرتبة الثالثة (الزيغ الكروي،الزيغ المذنبي وزيغ الاستكماتزم) بصورة منفردة، ولنفس القيم من معامل الزيغ (w=0.1 λ....0.5 λ)، ولزاوية الدوران (ψ=0).

تم اضافة زيوغ الرتبه الاولى الى زيوغ الرتبة الثالثة، للحصول على افضل توازن وتقليل عرض المنحني للدالة، و زيادة نسبة ستريل مثل اضافة زيغ الإمالة الى الزيغ المذنبي، وإضافة الخطأ البؤري الى الزيغ الكروي وزيغ الاستكماتزم. ان هناك بعض الزيوغ تؤثر تأثيرا سلبيا على دالة الانتشار النقطية مثل الخطأ البؤري مع الزيغ المذنبي وزيغ الامالة مع الزيغ الكروي.

## 1. INTRODUCTION

Many optical designers used aberration curves to explain the state of correction of an optical system , because these curves grant a designer important details about the relative contributions of individual aberrations to lens Action [1]. The rays that emitted from a point object do not focus at one point, with the result that the picture is blurred. The departures of real pictures from the ideal model are called aberrations [٢ ].

In this work, we will study the adding first order aberration with third order aberration to improve image quality, which improve the performance of the optical system.

Many researchers studied balance of aberrations on different apertures. In 1997, Ali.H. Abdul. Munaim, used different shapes of apertures (including triangular aperture), new formulae for the variance were derived and the optimum balance aberration coefficients were calculated. [3]. In 2011, Rafael. Navarro. et.al, studied the wave front passing through different pupil stop shapes (annular, semicircular, elliptical and triangular) [4]. In same year, Azhr. A.Raheem. et.al, studied the effect of aperture shape on the formed image for the system contains Gaussian filter, the aperture is used in different shapes (circle ,square, triangle) [5].While in ٢012, Azhr. A.Raheem, discussed effect of the multiple synthetic aperture on linear spread function using optical system contain high degrees of aberrations [6]. In 2019, Farah.M. Faisal Hussein and Sundus .Y. Hasan, studied the optimum balance of aberrations on point spread function for annular eccentric circular aperture [7].

## 2. The Equation of Point Spread Function (PSF)

The point spread function can be defined as the intensity distribution at the image plane of a Specific optical system [8], the pupil function can be written as the following [9]

$$f(x,y) = \tau(x,y)e^{ikw(x,y)} \qquad (1)$$

$\tau(x,y)$: represents the real amplitude function distributed in exit pupil and it is called "pupil transparency" often equals one unit if the illumination is uniform,

$e^{ikw(x,y)}$: the wave front of aberration function, $w(x,y)$ : the aberration factor.

We can express the complex amplitude in the point $(u, v)$ in the image plane by using Fourier transform to pupil function [8,9,10]

$$F(u,v) = \frac{1}{A}\int_y\int_x f(x,y)\, e^{2\pi i(ux+vy)}dxdy \qquad (2)$$

Where $F(u,v)$ : represent complex amplitude in point *(u ,v)* , (A) represent the exit pupil area,

$f(x,y)$: pupil function

The intensity in the Point Spread Function is calculated by multiplying the complex amplitude by the complex conjugate.

$$G(u,v) = |F(u,v)|^2 = F(u,v).F(u,v)^* \qquad (3)$$

$$PSF = G(u,v) = n.f\left|\int_y\int_x f(x,y)\, e^{2\pi i(ux+vy)}dxdy\right|^2 \qquad (4)$$

Where $(n.f)$ : normalization constant.

Assume that $z$=2$\pi u$ and 2$\pi v$=$m$ ,the equation (4) becomes:

$$PSF = G(z,m) = n.f\left|\int_y\int_x f(x,y)e^{i(zx+my)}\, dxdy\right|^2 \qquad (5)$$

Because of The intensity distribution on the two axes( $z, m$): are symmetric, we can take it to one axis only, let $(\mathrm{m} = 0)$; therefore, the equation (5) becomes as follows:

$$PSF = G(z) =$$
$$n.f \left| \int_y \int_x f(x,y) e^{i(zx)} \, dxdy \right|^2 \quad (6)$$

## 3.Triangular Aperture

The boundaries of the triangular aperture is defined by the equilateral triangular pupil aperture with the normalizing area ( π ) , the length of horizontal side $(2x)$ and The distance from the middle of the base to the head is (2y). Thus, the area of the triangle, as follows:

$$A = 2xy = \pi$$

From this figure, we find that,

$$\tan\theta = \frac{2y}{x}$$

$$\tan 60 = \sqrt{3} \;\rightarrow\; y = \frac{\sqrt{3}}{2}\, x$$

$$\because A = 2xy = \pi \rightarrow x = \frac{\pi}{2y}$$

$$\therefore\; y = \pm 1.16634$$

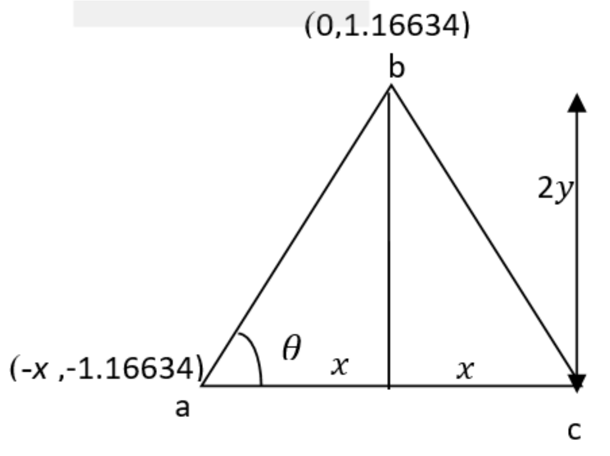



Fig. (1) Triangular aperture

The equation for the line (ab) is,

$$x = \frac{y - 1.16634}{\sqrt{3}}$$

When applying the limits of integration to eq.(6) will be:

$$PSF = n.f \left| \int_{-1.16634}^{1.16634} \int_{-\frac{y-1.16634}{\sqrt{3}}}^{\frac{y-1.16634}{\sqrt{3}}} f(x,y) e^{i(zx)} dxdy \right|^2 \quad (7)$$

## 4. The Equation of Aberration

The wave front aberration function can be written as a series of terms [11]

$$W = W(\sigma, r, \Phi) = \sum_{klm} W_{klm} \sigma^k r^l \cos^m \Phi \quad (8)$$

Where $W_{klm}$ is the aberration factor, σ is principal ray height from optical axis in exit pupil, r is the radial distance of wave in exit pupil, and Φ is the angle between r and x axis, while k, l, and m represent powers of σ, r, and Φ respectively. equation (5) can be written as [ 12,13]

$$W = W(\sigma^2, r^2, \sigma r \cos\Phi) = W_{000} + W_{020} r^2$$
$$+ W_{111} \sigma r \cos\Phi + W_{040} r^4 + W_{131} \sigma r^3 \cos\Phi$$
$$+ W_{222} \sigma^2 r^2 \cos^2\Phi + W_{220} \sigma^2 r^2$$
$$+ W_{311} \sigma^3 r \cos\Phi \quad (9)$$

The first term should be zero since the reference wave and the front wave are coherent on the optical axis [١4], the second and third terms represent the (focal error and tilt aberration), while last five terms represent the (spherical ,coma, and astigmatism aberrations) respectively .

When we used the Cartesian coordinate: $(x, y)$: $x = r \sin\Phi$ , $y = r \cos\Phi$ , And the Cartesian coordinates are rotated by an angle ψ, aberration function take the form [15]:

$$W(x,y) = W_{020}(x^2 + y^2) \quad \text{(Focal Error )}$$

$$W(x_1,y_1) = W_{111}\,(x\sin\psi + y\cos\psi)$$

(Tilt Aberration )

$$W(x,y) = W_{040}(x^2 + y^2)^2 \quad \text{(Spherical Aberration)}$$

$$W(x_1,y_1) = W_{131}(x^2 + y^2)\,(x\sin\psi + y\cos\psi)$$

(Coma Aberration)

$$W = W_{222}\,(x^2\sin^2\psi + y^2\cos^2\psi + xy\sin 2\psi)$$

(Astigmatism Aberration) (10)

## ٥. Results And Discussion

Equation (7) represents point spread function for triangular aperture, was programmed by using MATLAB code.

Three cases were studied in this work, the first is aberration free system, the second with present different types of aberrations (first and third order aberrations) with the aberration factor (W=0.1λ , 0.2 λ,0.3 λ,0.4 λ,0.5 λ ) and angle of rotation (□=0), and the last case is balancing first order aberrations with third order aberrations, as follows:

**5.1.PSF with First Order Aberrations (Focal Error and Tilt Aberration).**

According to the figures in this research ,the maximum value of central intensity (Strehl ratio) of point spread function for triangular aperture equal(1) because of the normalization with ideal optical system.

Fig. (2) shows the effect of focal error on (PSF), and its obvious that the central peak is decreased to (0.854) when ($W_{20}$= 0.1 λ), and it continued decreasing till it reached a value of (0.114) when ($W_{20}$= 0.5 λ). Fig. (3) represents the effect of tilt aberration ($W_{11}$) on the point spread function of a triangle aperture, and its observe that the central peak is decreased to (0.887) when ($W_{11}$= 0.1 λ), and it still decreasing till it reached a value of (0.058) when ($W_{11}$= 0.5 λ). Tilt aberration leads to reducing the half width curves, but, secondary peaks appeared when ( $W_{11}$=0.4 λ ,0.5 λ).

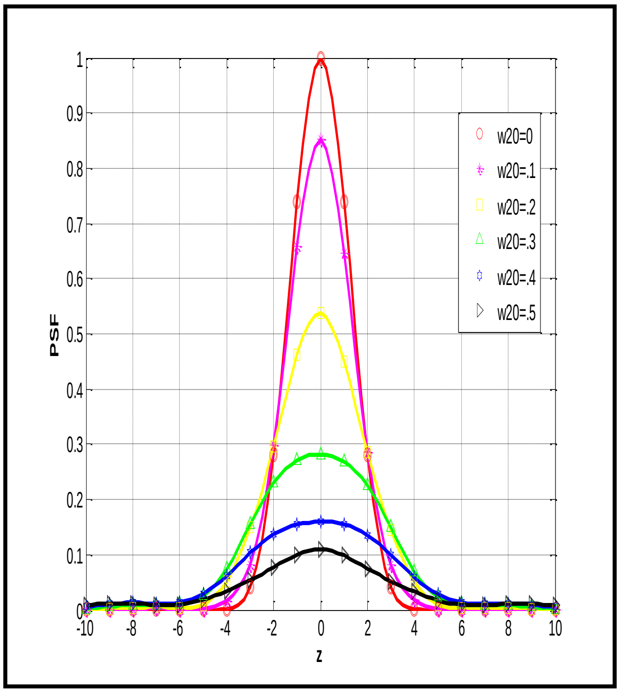



Fig.( 2) : PSF for triangle aperture of optical system, different values of $W_{20}$ (focus error).

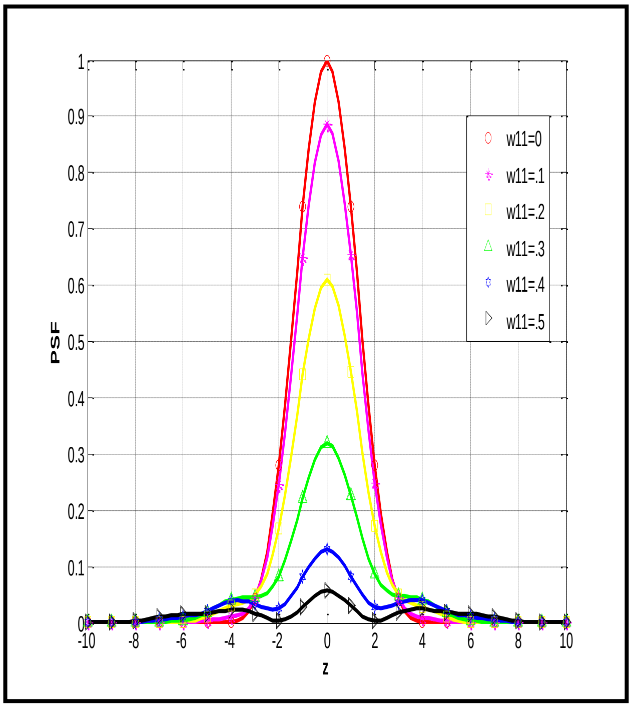



Fig.(3): PSF for triangle aperture of optical system, different values of $W_{11}$ (tilt aberration).

**5.2. PSF with Third Order Aberrations (Spherical ,Coma And Astigmatism Aberrations)**

Fig. (4) indicates the effect of spherical aberration ( $W_{40}$) on the point spread function of a triangular aperture , its appear that the central peak is decreased to (0.598) when ($W_{40}$= 0.1 λ), and it continued decreasing till it reached a value of (0.205) when ($W_{40}$= 0.5 λ). Fig. (5) represents the effect of coma aberration ($W_{31}$) on the point spread function of a triangle aperture, and its explain that the central peak is decreased to (0.793) when ($W_{31}$= 0.1 λ), and it continued decreasing till it reached a value of (0.134) when ($W_{31}$= 0.5 λ). Fig. (6) represents the effect of astigmatism aberration ($W_{22}$) on the point spread function of a triangle aperture, and its express that the central peak is decreased to (0.937) when ($W_{22}$= 0.1 λ), and it continued decreasing till it reached a value of (0.173) when ($W_{22}$= 0.5 λ).

Fig. (7) shows the point spread function with every single aberration of first and third order with value of (W= 0.1 λ), and is clear that the less, effecting one is the astigmatism aberration where the half width curve has the less value, and central .peak is the heightest one.

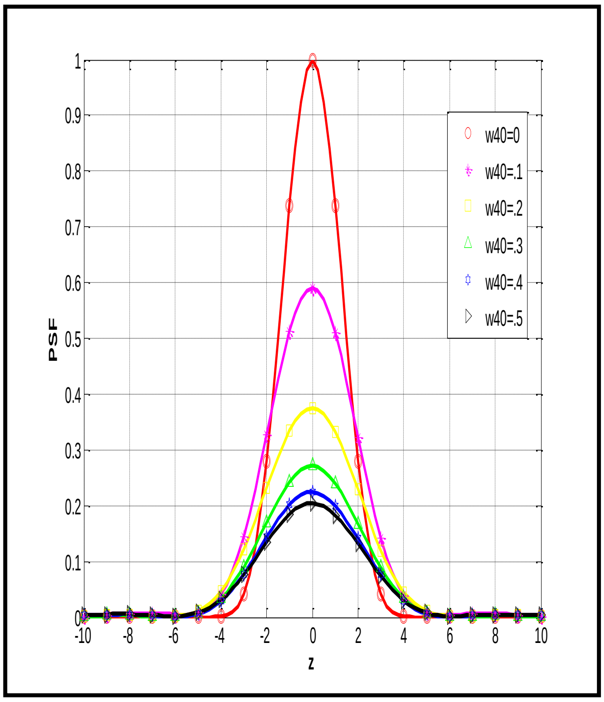



Fig.( 4): PSF for triangle aperture of optical system, different values of $W_{40}$ (spherical aberration)

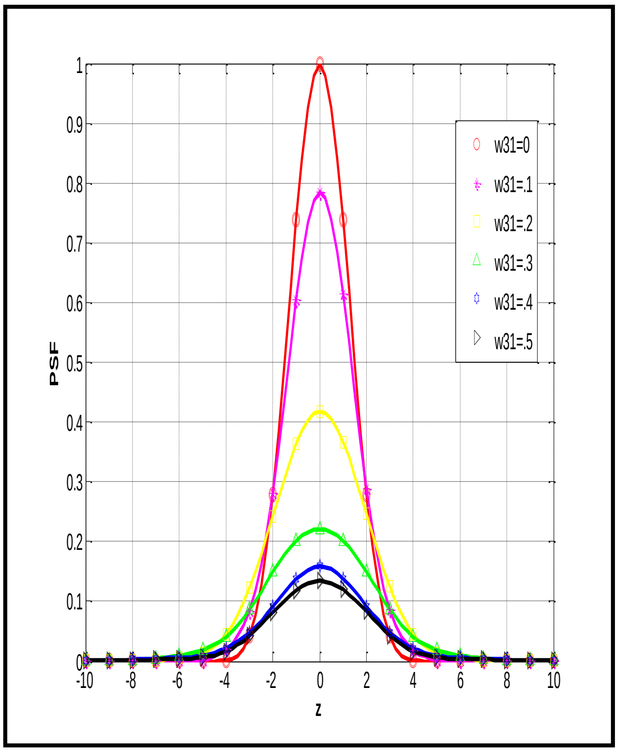



Fig.(5): PSF for triangle aperture of optical system, different values of $W_{31}$ (coma aberration)

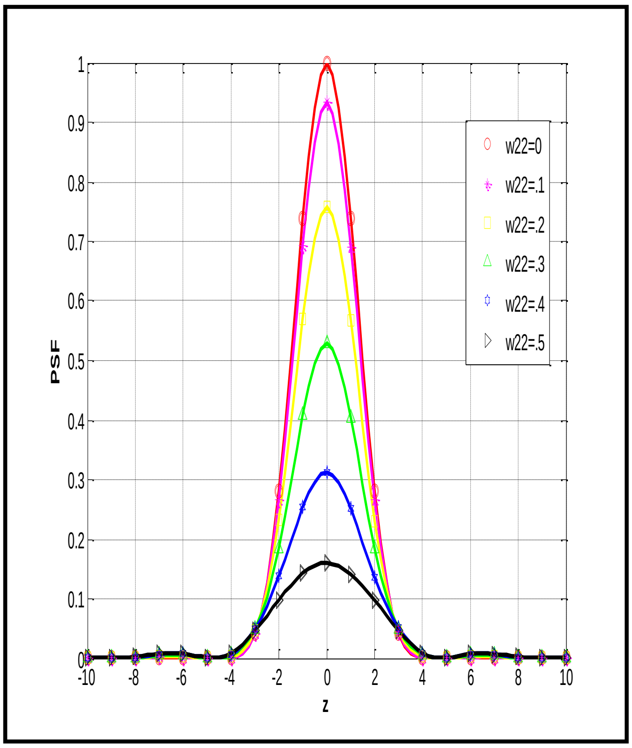



Fig.(6): PSF for triangle aperture of optical system, different values of $W_{22}$ (astigmatism aberration)

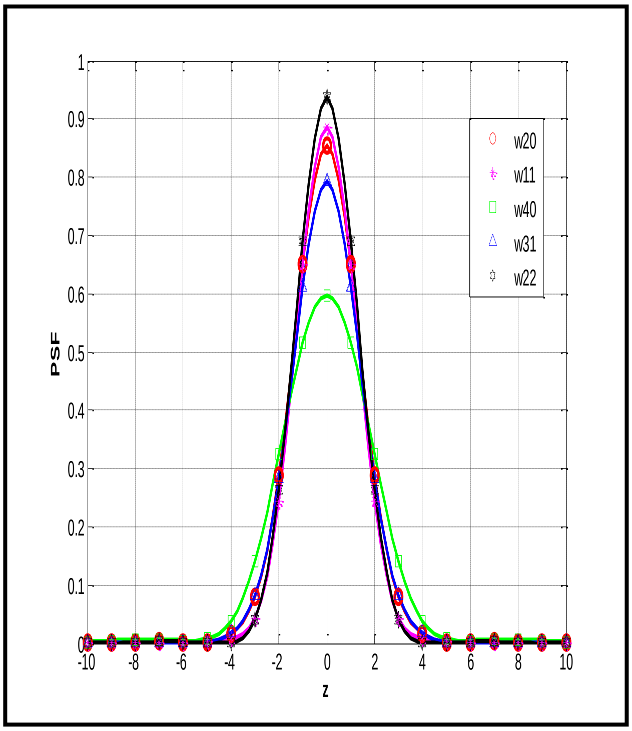



Fig.(7): PSF for triangle aperture of optical system, every single aberration when ($W_{20}$, $W_{11}$, $W_{40}$, $W_{31}$, $W_{22}$,=0.1 λ)

### 5.3. Balancing Aberrations

**a. presence Focal Error ($W_{20}$) with ($W_{40}$, $W_{31}$ and $W_{22}$)**

when adding focal error to different types of third order aberrations. we got figures (8 to 10).

Fig. (8) shows that Strehl ratio will be better when adding $W_{20}$ to $W_{40}$ as compared with those of fig. (4) for the presence of $W_{40}$ only. Thats mean that $W_{20}$ can balance spherical aberration and reduce its effect on optical system. The same thing when adding $W_{20}$ to $W_{22}$ as obvious in fig. (9) when compared with fig. (6), while fig. (10) shows that focal error doesn't balancing with coma aberration, and in reverse its presence will increase the amount of aberration i.e. decrease that strehl ratio.

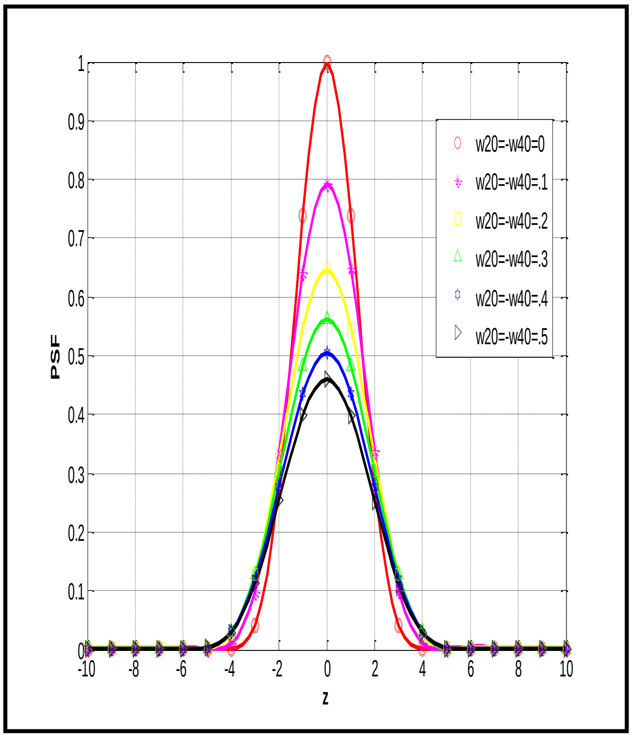



Fig. (8): PSF for triangle aperture of optical system, different values of balanced aberrations ($W_{20}$= $-W_{40}$)

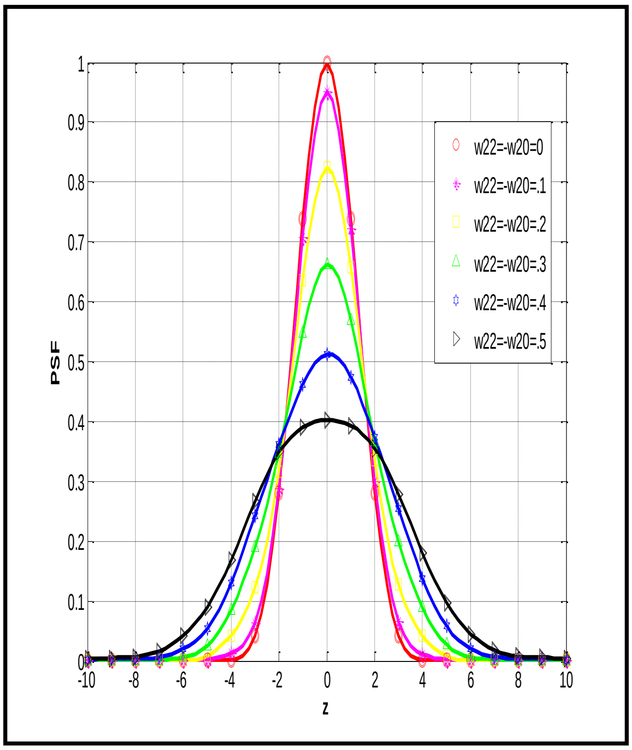
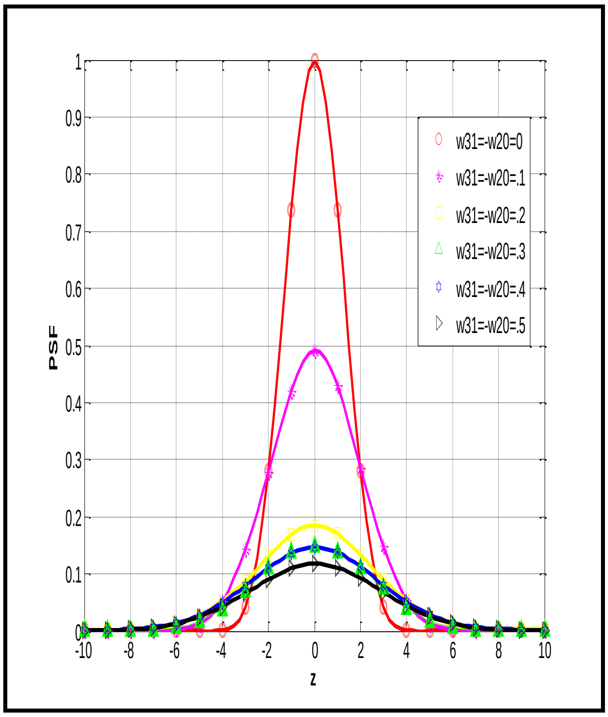



Fig.(9): PSF for triangle aperture of optical system, different values of balanced aberrations ($W_{22}$= $-W_{20}$)




\Fig( 10): PSF for triangle aperture of optical system, different values of balanced aberrations ($W_{31}$= $-W_{20}$)

**b. presence Tilt Aberration ($W_{11}$) with ($W_{40}$, $W_{31}$ and $W_{22}$)**

when adding tilt aberration to different types of third order aberrations. we got figures (11 to 13).

Fig. (11) shows that Strehl ratio will be better when adding $W_{11}$ to $W_{31}$ as compared with those of fig. (5) for the presence of $W_{31}$ only. Thats mean that $W_{11}$ can balance coma aberration and reduce its effect on optical system. while fig. (12 ) shows that tilt aberration doesn't balancing with spherical aberration, and in reverse its presence will increase the amount of aberration i.e. decrease that strehl ratio. The same thing when adding $W_{11}$ to $W_{22}$, but the secondary peaks are appeared clearly when ( $W_{22}= -W_{11}= 0.3\ \lambda$). as in fig. (13).

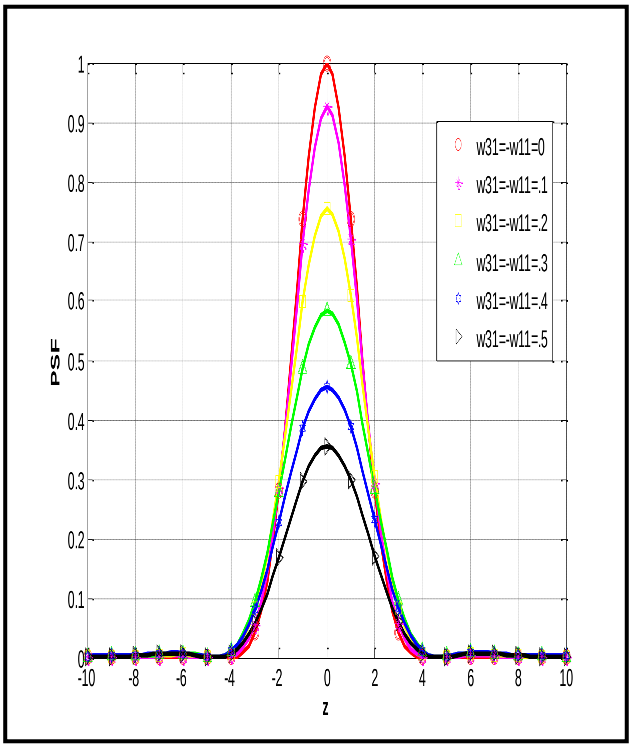



Fig.( 11): PSF for triangle aperture of optical system, different values of balanced aberrations ($W_{31}= -W_{11}$)

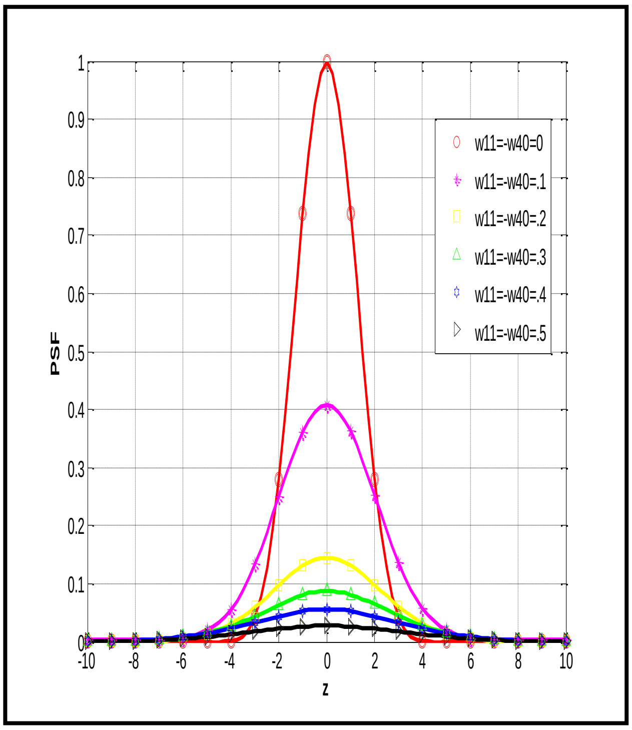



Fig.( 12): PSF for triangle aperture of optical system, different values of balanced aberrations ($W_{11}= -W_{40}$)

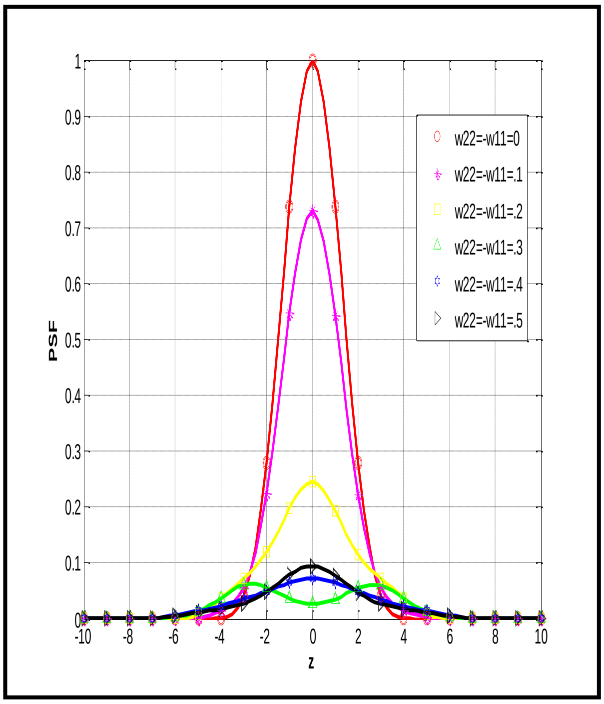



Fig.(13): PSF for triangle aperture of optical system, different values of balanced aberrations ($W_{22}= -W_{11}$)

**6- Conclusions**

1- Astigmatism aberration is the less effecting on the point spread function, Compared to other aberrations Comparison

2-When adding focal error to third order aberrations, focal error can balance spherical and astigmatism aberration, while focal error can't balancing with coma aberration.

3- When adding tilt aberration to third order Aberrations, tilt aberration can balance coma aberration, while tilt aberration can't balancing with spherical and astigmatism aberration.

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