

Theoretical Calculations of the Diffusion and Friction Coefficients for Neutral Atoms that Moving in a Laser Field

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ABSTRACT

This research aimed to study the friction and diffusion coefficients of neutral atoms that moving and interact with a laser field .The Mathcad 14 program used to calculate the results. The theoretical model was solved and shown the effect of different optical parameters on the diffusion and friction of neutral atoms .The different values of ellipticity (ϵ) were used to find the friction and diffusion coefficients ,for each value of ellipticity we found that the relationship between the natural width of excited level (γ) and the difference (δ) between the frequency of laser field (ω) and the atomic resonance (ω_0) has strong effect on the friction coefficient of atoms .The effect of Rabi frequency (Ω) on the diffusion coefficient has been investigated .In this research the results showed that each of the parameters δ and Ω has effect on the diffusion of atom and the Rabi frequency has more influence. The dependence of the friction and diffusion coefficients on the natural width of the excited level was studied. The measuring of the friction and diffusion coefficients of atom very important to control the movement and then localize it, this have many applications in physics fields.

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الحسابات النظرية لمعاملات الانتشار والاحتكاك للذرات المتعادلة المتحركة في مجال ليزري

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الكلمات المفتاحية:

معامل الانتشار
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الذرات المتعادلة
تموقع الذرة
ضغط الضوء

الخلاصة

يهدف هذا البحث إلى دراسة معاملات الاحتكاك والانتشار للذرات المتعادلة التي تتحرك وتتفاعل مع مجال ليزري . استخدم برنامج mathcad14 للحصول على النتائج. النموذج الرياضي تم حله وظهر تأثير معاملات بصرية مختلفة على انتشار واحتكاك الذرات المتعادلة . قيم مختلفة للإهليجية (ϵ) قد استخدمت لإيجاد معامل الاحتكاك ومعامل الانتشار، لكل قيمة من الإهليجية وجدنا بان العلاقة بين العرض الطبيعي للمستوى المهيج (γ) والفرق (δ) بين

تردد المجال الليزري (ω) والرنين الذري (ω_0) يمتلك تأثير قوي على معامل احتكاك الذرات. تأثير تردد رابي (Ω) على معامل انتشار الذرات تم التحقق منه. في هذا البحث النتائج اوضحت ان كل من المعاملات δ و Ω يمتلكون تأثير على انتشار الذرة وان تردد رابي يمتلك التأثير الاكبر. اعتماد معاملات الاحتكاك والانتشار على العرض الطبيعي للمستوى المهيج تمت دراسته. ان قياس معاملات الاحتكاك والانتشار للذرة مهم جدا للسيطرة على الحركة وتموقعها، وهذا له تطبيقات عديدة في مجالات الفيزياء.

1. INTRODUCTION

Catch atoms or ions in a trap and using laser for cooling have a great developed in the field of atomic physics, and scientific projects in this field obtained many of Nobel prizes through the past 20 years [1-3]. Many uses in physics have been continue to benefit from this development, including interferometry of the atom[4-6], the process of printing from a flat surface [7-8], spectroscopy in laser[9], the processes which treat quantum datum[10], and in chemical field when use it for measuring the reactions of matter[11]. In 1997 the achievements in the field of laser cooling and atomic trapping have been awarded the Nobel Prize (C. Cohen-Tannuji, S. Chu, and W. Phillips) [12, 13, 14]. The force of light pressure was a fundamental consequence of the interaction of light waves with matter. Initially the hypothesis that light can exert pressure was expressed many years ago. As early as the 17th century I. Kepler put forward the hypothesis that the cause of the comet's tail deflection is the force of light pressure caused by the sun. Despite the fact that light pressure is not the main factor for this astronomical phenomenon, this hypothesis stimulated theoretical work to understand the causes of light pressure. The electromagnetic theory proposed by Maxwell in 1873 allowed him to express the power of light pressure through the energy of an electromagnetic field in a unit volume. So it was shown that the power of light pressure from the sun or other heat source is relatively small. However, at the beginning of this century P.N. Lebedev carried out the first experimental measurement of the force

with which light presses on a thin metal plate[15]. Soon a similar experiment was carried out by Nichols and Hull [16]. However a significant step in understanding the nature of light pressure made it possible to make the subsequent development of quantum theory. Einstein showed that a quantum of light – a photon with energy $h\nu$ has an impulse $p = h\nu/c = h/\lambda$ where h is Planck's constant, c , ν , λ is the speed, frequency and wavelength of light [17]. In this case the basis of the mechanism of light pressure is the exchange of momentum in the processes of absorption and emission of photons. In problems of the interaction of light with atoms with respect to other particles its resonance nature of interaction. The resonance cross section of photon scattering on an atom exceeds the Thomson cross section on an electron by 15 - 17 orders of magnitude. In this case the rate of resonant scattering of photons (for alkali metals) is 10^7 photons per second which corresponds to atomic acceleration of 10^5 g. The first experimental study of the effects of recoil when photons are scattered by atoms was started by Frish in 1933 [18]. For these purposes a well-collimated hot beam of atoms Na was used irradiated from the side with a sodium lamp. In the experiment a small deviation of the beam of atoms in the direction from the lamp was observed which was explained by a small saturation of the atoms by the lamp used. Later with the advent of coherent light sources with a large spectral density of radiation - lasers, work on the atomic motion in resonant light fields shifted to a new round of evolution. It can be said that a new intensively developing field of

science has arisen connected with the mechanical action of resonant radiation on atoms.

Using the resonant nature of the interaction of atoms with light, one can selectively influence certain atoms and molecules. This circumstance allows one to efficiently control atomic and molecular beams including their deceleration, selective deflection and scattering to significant angles, focusing and collimation. The mechanical action of resonance radiation on atoms and in particular the motion of atoms in a standing wave have been adequately studied in the framework of the simplest model of a two-level atom [19, 20, 21, 22, 23]. This description allowed us to understand the physical mechanisms and the nature of the forces acting on an atom in a light field. Forces by their nature are divided into the forces of spontaneous light pressure and the forces of induced light pressure (dipole or gradient force). The force of spontaneous light pressure arises in the process of absorption and spontaneous scattering of a photon. In the process of absorption of a photon an atom receives a momentum $\hbar k$ and due to the equally probable directionality of spontaneously emitted photons, the atom experiences a constant action of the force of light pressure. Also in a uniform field ellipticity leads to a significant difference in the kinetic coefficients (force, coefficient of friction and diffusion), the problem of the motion of atoms in no uniformly polarized fields is a very specific example of the fact that for the complete description of kinetic processes it is necessary to take into account the degeneracy of atomic levels from the projection of angular momentum[24]. The atoms have two kinds of grades of independence: first kind called internal grade of independence like arrangement of electrons or shape of polarization, second kind called outer grade of independence that are basically represent the momentum and the location of the middle of cluster. The optical pumping have a great role

for the internal grades[25-26]. The great role in the outer grades of independence comes from the force that occur between the radiation and the atom[27].

2. Theory of diffusion and friction coefficients of neutral atoms

The interaction between atom and the laser field was shown by figure(1). Moving atom has a velocity v in the ground state ingest a photon has momentum \hbar/λ , after ingesting the photon the atom will be in the excited state and its velocity decreased by $\hbar k/2\pi m$ then the atom returned to ground state after losing energy ($\hbar\nu$).

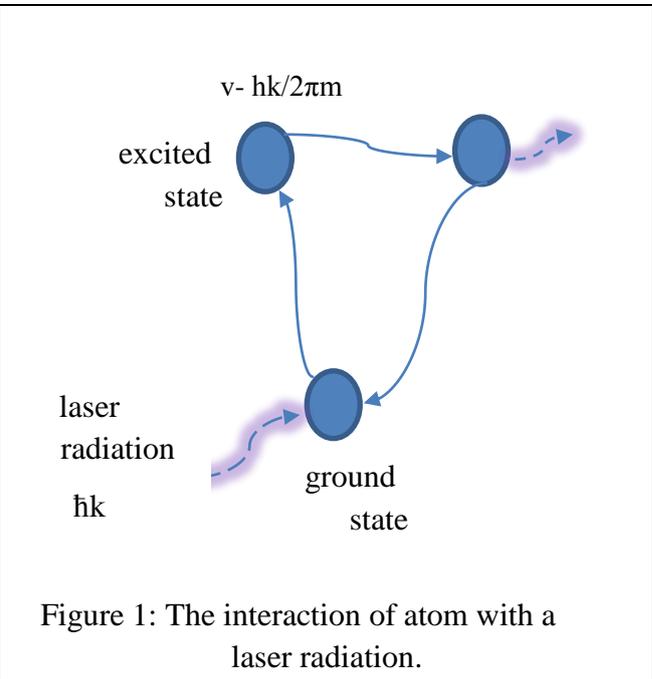


Figure 1: The interaction of atom with a laser radiation.

The monochromatic field that interacted with atoms was given by following formula[24]:

$$E(\mathbf{r}, t) = E(\mathbf{r}) \exp(-i\omega t) + E^*(\mathbf{r}) \exp(+i\omega t) \quad (1)$$

Complex vector field amplitude has the form

$$E(\mathbf{r}) = E \exp(i\phi) \mathbf{e} \quad (2), \text{Where } E \text{ real amplitude, } \phi \text{ represent field phase}$$

$$\mathbf{e} = \sum_{\sigma=0,\pm 1} e^{\sigma} \mathbf{e}_{\sigma} \quad (3)$$

Unit($\mathbf{e}^* \cdot \mathbf{e}$) = 1, e^{σ} -its contravariant components in a periodic base

Figure2. shows the vector of elliptical polarization . In this figure we noticed that $\mathcal{E}=0$

corresponds to linear polarization, and $\mathcal{E} = \pm \pi/4$ right and left circular polarization. There is another choice of a coordinate system in which the vector \mathbf{e} is the sum of at most two cyclic components $\mathbf{e}^{+1}, \mathbf{e}^0$ or $\mathbf{e}^{-1}, \mathbf{e}^0$ [28].

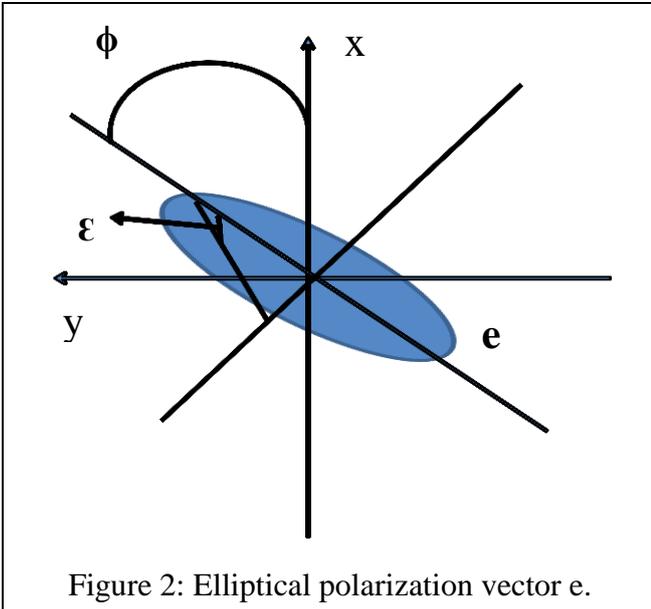


Figure 2: Elliptical polarization vector \mathbf{e} .

In the field of a standing wave formed by counter propagating light waves, atoms move in a potential formed by force F , and dissipative processes are determined by friction ζ and diffusion D coefficients[24]:

$$k_B T = \frac{\langle P \rangle^2}{M} = - \frac{\langle D \rangle}{\langle \zeta \rangle} \quad (4)$$

where k_B -Boltzmann constant , T -temperature , P -momentum and M -mass of atom respectively. The relationship between the electric field(E) and ellipticity of the light field(\mathcal{E}) is defined by formula:

$$\cos^2(2\mathcal{E}) = \frac{|\mathbf{E} \cdot \mathbf{E}^*|^2}{(\mathbf{E} \cdot \mathbf{E}^*)^2}$$

The force in a standing wave field with elliptical polarization is given by following expression[24]:

$$F = \frac{\hbar k \delta}{2} \frac{I_{\mathcal{E}} \sin(2kz)}{1 + I_{\mathcal{E}} \cos^2(kz)} \quad (5),$$

Where k –wave number , δ - the difference between the frequency of laser field(ω) and the atomic resonance (ω_0) and \mathcal{E} represent the ellipticity of the light field.

The optical potential has the following formula :

The optical potential has the following formula :

$$U = \frac{\hbar \delta}{2} \ln(1 + I_{\mathcal{E}} \cos^2(kz)) \quad (6)$$

Effective saturation parameter depends on the ellipticity of the light field was given by equation:

$$I_{\mathcal{E}} = 8I_0 \cos^2(2\mathcal{E})/3 \quad (7)$$

Where I_0 -saturation parameter per one wave

$$I_0 = \frac{|\Omega|^2}{(\delta^2 + \gamma^2/4)} \quad (8),$$

where Ω - Rabi frequency and γ is the natural width of excited level.

The relationship between the distribution function of atoms(f) and diffusion coefficient (D) is given by following equation

$$\frac{\partial f(r, p, t)}{\partial t} = - \frac{\partial}{\partial r} Mf - \frac{\partial}{\partial P} Mf + \sum_{ij} \frac{\partial^2}{\partial p_i \partial p_j} D_{ij} f \quad (9)$$

Where the distribution function was given by:

$$f(r, p, t) = \iint d\hat{r} d\hat{p} \Pi(r, p, t/\hat{r}, \hat{p}, \hat{t}) f(\hat{r}, \hat{p}, \hat{t}) \quad (10)$$

$$\frac{\delta \bar{r}}{\delta t} = \frac{p}{m}, \quad \frac{\delta \bar{p}}{\delta t} = -\gamma p = \frac{dU}{dr}, \quad (11)$$

$$\frac{\delta \bar{r}^2}{\delta t} = 0 = \frac{\delta \bar{r} \delta \bar{p}}{\delta t} \quad \frac{\delta \bar{p}^2}{\delta t} = 2D \quad (12)$$

From the above equations and equation(9) we found that:

$$D = \frac{\frac{\partial f(r, p, t)}{\partial t} + \frac{p}{M} \frac{\partial}{\partial r} f(r, p, t) - \frac{\partial}{\partial p} \left[\gamma p + \frac{dU}{dr} \right] f(r, p, t)}{\frac{\partial^2 f(r, p, t)}{\partial p^2}}$$

The equation of the diffusion coefficient is given by following expression:

$$D = \frac{\hbar^2 k^2 \gamma I_\epsilon \sin^2(kz)}{4} \left[1 + 4 \frac{\delta^2}{\gamma^2} \frac{I_\epsilon \cos^2(kz) (I_\epsilon^2 \cos^4(kz) - \gamma^2 (\frac{\gamma^2}{4} + \delta^2))^{-1}}{(1 + I_\epsilon \cos^2(kz))^3} \right] + \frac{\hbar^2 k^2 \gamma I_\epsilon \sin^2(kz) \sin^2(2\epsilon)}{\gamma (1 + I_\epsilon \cos^2(kz))^3} \left[3\delta^2 + I_\epsilon \cos^2(kz) (\frac{\gamma^2}{4} + \delta^2) (1 + I_\epsilon \cos^2(kz))^2 \right]$$

The light pressure force that acted on atom at point r has the following expression :

$$F_i(r, p) = F_{i(r)} + \sum_j \zeta_{ij}(r) v_j \quad (13)$$

The above equation represent the relationship between the friction coefficient ζ and the light pressure force.

$$F_r(v) = F_r(v = 0) + \zeta_r v + \dots$$

Then the equation for the friction coefficient is given by formula:

$$\zeta = \frac{\hbar k^2 \delta \gamma I_\epsilon \sin^2(kz)}{(\delta^2 + \frac{\gamma^2}{4})(1 + I_\epsilon \cos^2(kz))^3} \left[1 - I_\epsilon \cos^2(kz) - \left(\delta^2 + \frac{\gamma^2}{4} \right) \frac{2I_\epsilon^2 \cos^4(kz)}{\gamma^2} + \left(\frac{\gamma^2}{4} + \delta^2 \right) \frac{6 \sin^2(2\epsilon)}{\gamma^2} \right]$$

In the above equations the parameter γ represent the natural width of excited level , δ - the difference between the frequency of laser field and the atomic resonance , ϵ - the ellipticity of the light field, k -the wave number and I_ϵ - effective saturation parameter.

3. Results and discussion

Figure 3 shows the spatial dependence of the diffusion coefficient with different values of ellipticity ϵ by using various values of Rabi frequency Ω and different values of quantity δ that represent the difference between the frequency of laser field and the atomic resonance. Figure 3a showed that the diffusion coefficient D equal to maximum value 88 when the ellipticity ϵ equal to $\pi/8$ for Rabi frequency Ω equals to three times of the natural width of excited level γ and the value of δ was 5MHz .When the value of ϵ equals to $\pi/6$ for the same

value of Ω and δ the greatest value of D decreased to 65 then goes to equal 5 for $\epsilon=0$ when the quantities Ω and δ have the same values. When the value of δ increased to 50 as shown in figure 3b and the value of Ω equal 3γ , we noticed that the greatest value of D equals to 90 when ϵ was $\pi/8$ and then this value decreased to 67 for $\epsilon=\pi/6$. From figure 3a and figure 3b we can see there is a slight change in the diffusion coefficient D when δ increased by 10 times. Figure 3b explained that when ellipticity ϵ was 0, the diffusion coefficient equals to zero. It was easily to see from figure 3c that the diffusion coefficient D increased tremendously when the reading of Rabi frequency increased. Figure 3c showed when $\Omega=20\gamma$, the largest value of the diffusion coefficient D was 3800 for $\epsilon=\pi/8$ and decreased to 2800 when ϵ was $\pi/6$, this peak reached to minimum value(1125) when ϵ was zero. Figure 3d showed if the value of Rabi frequency was increased and it becomes 40γ the largest value of the diffusion coefficient D increased to 14200 when the value of ellipticity was $\pi/8$ and then we can see that the reading of the diffusion coefficient D reduced to 11100 for $\epsilon=\pi/6$. For ellipticity equal to zero the peak of the diffusion coefficient was decreased to 4700 .

Figure 4 illustrates spatial dependence of the friction coefficient with different values of ellipticity and different values of quantity that represent the difference between the frequency of laser field and the atomic resonance due to the natural width of excited level. Figure 4 shows that the friction coefficient ζ decreased with increasing the value δ due to the value of γ . Figure 4a showed that the largest value of the friction coefficient ζ was 0.08 when the value of δ equal to half of the natural width of excited level and this result was obtained for ellipticity ϵ equal to $\pi/8$. For the same value ($\delta=1/2\gamma$) the peak of the friction coefficient ζ decreased to 0.049 when ellipticity ϵ was $\pi/4$ and then this value decreased to 0.019 for ϵ equal to 0. If the difference between the frequency of laser field

and the atomic resonance was 20 times smaller than the natural width of excited level then the peak of friction coefficient ζ is increased and become equals to 0.1 ($\mathcal{E}=\pi/8$) and that was showed in figure 4b. For ellipticity equal to $\pi/4$, the maximum value of friction coefficient was 0.07 and then this reading decreased to 0.039 when the ellipticity \mathcal{E} equals to zero. Figure 5a showed that when the difference between the frequency of laser field and the atomic resonance have the same value of the natural width of excited level for ellipticity equal to $\pi/8$, the peak of friction coefficient ζ was 0.069 and when the reading of ellipticity \mathcal{E} equals to $\pi/4$, the largest value of ζ was 0.039, this value reduced to 0.005 for \mathcal{E} equals to zero. For δ equals to three times of the natural width of excited level γ figure 5b explained that the largest value of friction coefficient ζ was 0.059 ($\mathcal{E}=\pi/8$) and this result becomes 0.03 when the ellipticity equals to $\pi/4$. The maximum value of friction coefficient ζ decreased to zero for the ellipticity equal to zero.

The depending of the friction coefficient ζ and the diffusion coefficient D on the natural width of the excited level γ was shown by figure 6 and figure 7. Figure 6 explained how changed the friction coefficient ζ with γ such that for small values of γ (1-20) the changing in friction coefficient was high and then start to be low when the reading of γ increased until to be stable for γ ranged from 60 to 100. Figure 7 showed that the diffusion coefficient D was very high in the region of the excited level γ (1-8) and his changing was stable for the natural width of the excited level ranged from 8 to 100. Figure 8 illustrates spatial dependence of the diffusion coefficient with different values of the natural width of the excited level. In figure 8 we showed that even when the value of the ellipticity equal to zero and small value of γ we can get the high value of D by choosing suitable values for others optical parameters.

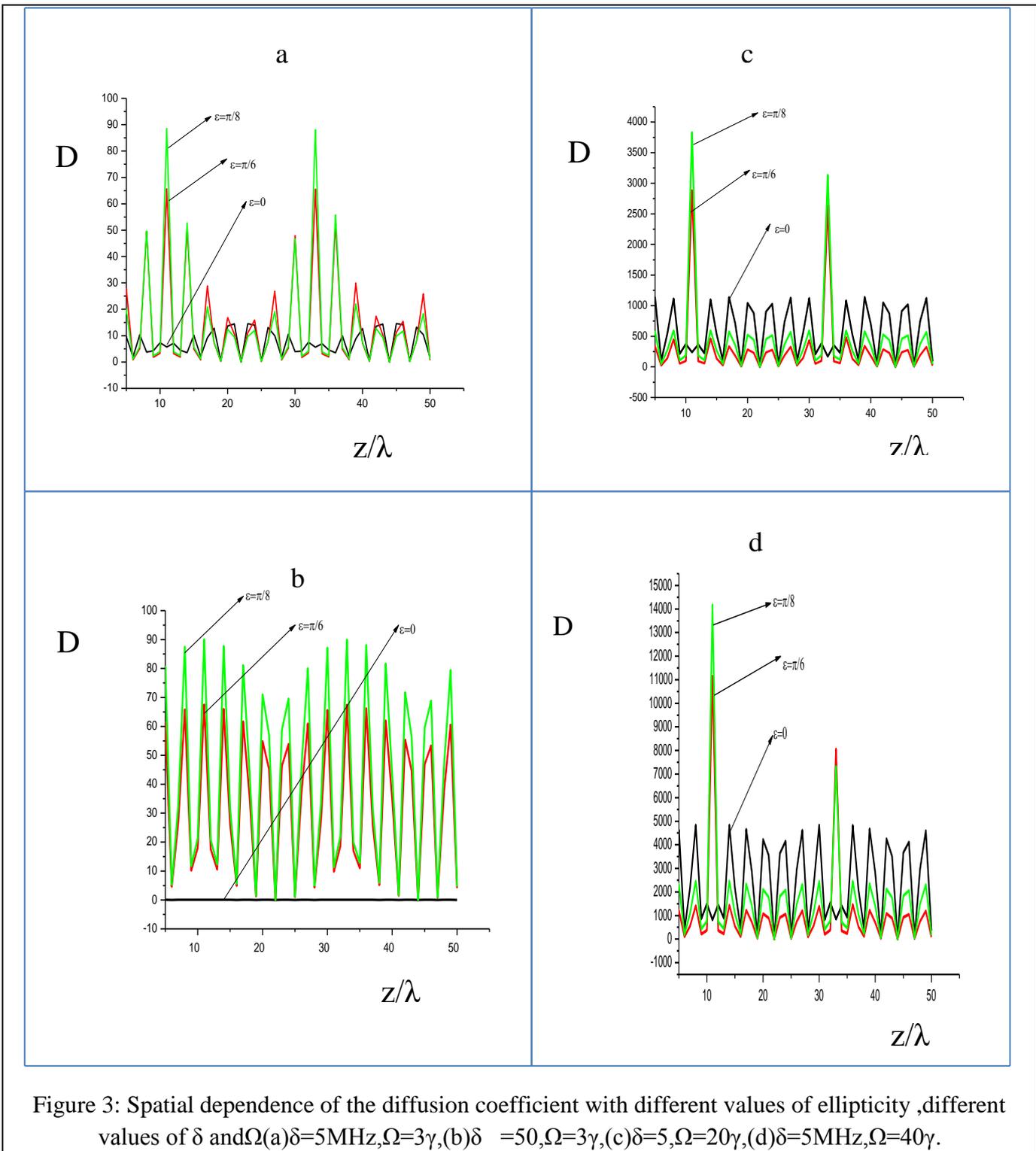


Figure 3: Spatial dependence of the diffusion coefficient with different values of ellipticity ,different values of δ and Ω (a) $\delta=5\text{MHz},\Omega=3\gamma$,(b) $\delta =50,\Omega=3\gamma$,(c) $\delta=5,\Omega=20\gamma$,(d) $\delta=5\text{MHz},\Omega=40\gamma$.

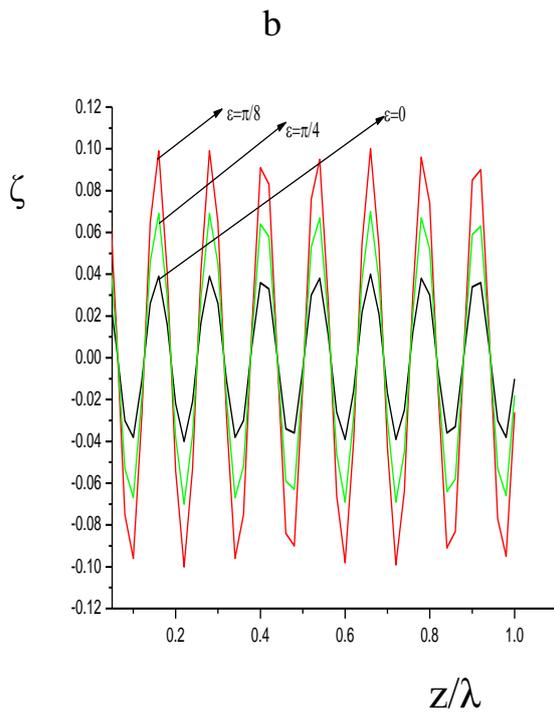
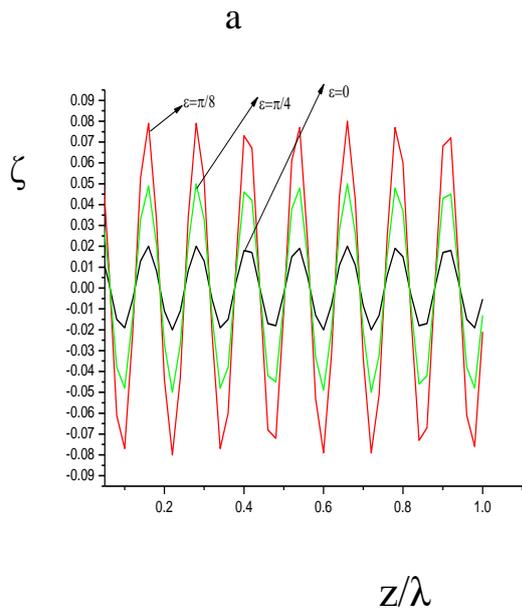


Figure 4: Spatial dependence of the friction coefficient with different values of ellipticity and different values of δ (a) $\delta = 1/2\gamma$, (b) $\delta = \gamma/20$.

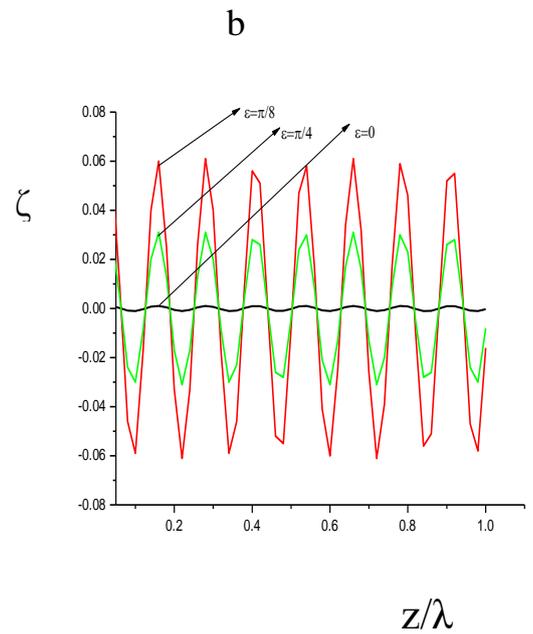
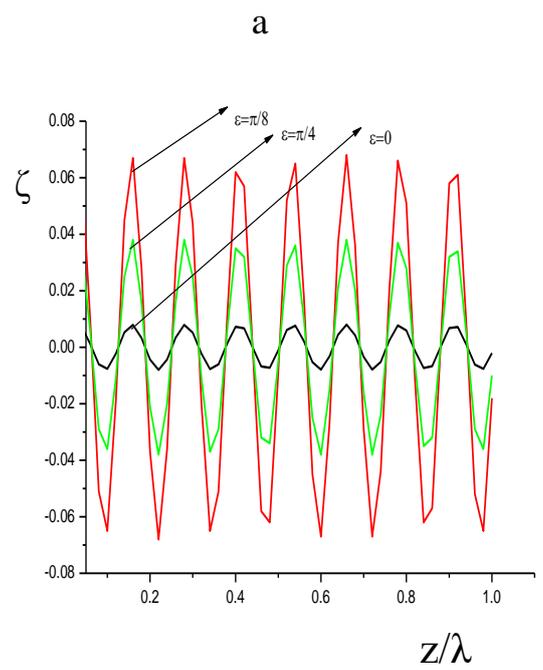


Figure 5: Spatial dependence of the friction coefficient with different values of ellipticity and different values of δ (a) $\delta = \gamma$, (b) $\delta = 3\gamma$.

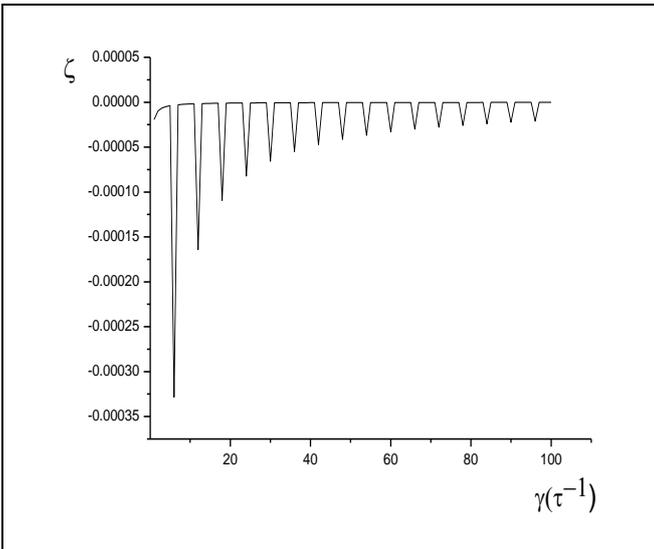


Figure 6: Dependence of the friction coefficient on the natural width of the excited level.

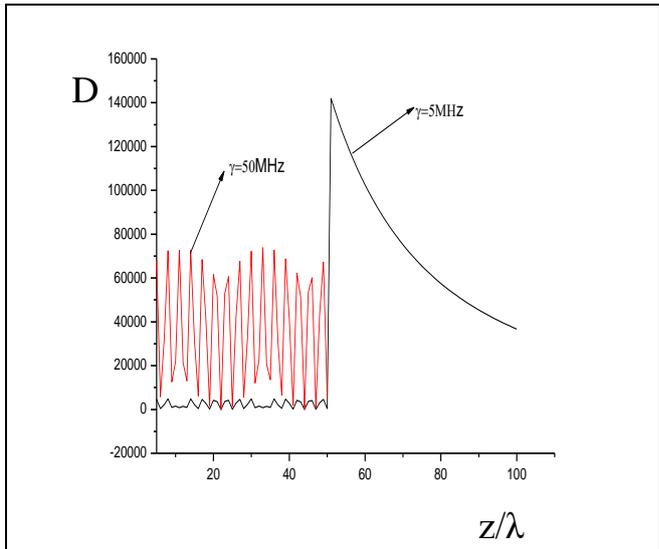


Figure 8: Spatial dependence of the diffusion coefficient with different values of the natural width of the excited level ($\gamma=50\text{MHz}$ for $\mathcal{E}=\pi/8, \gamma=5$ for $\mathcal{E}=0$).

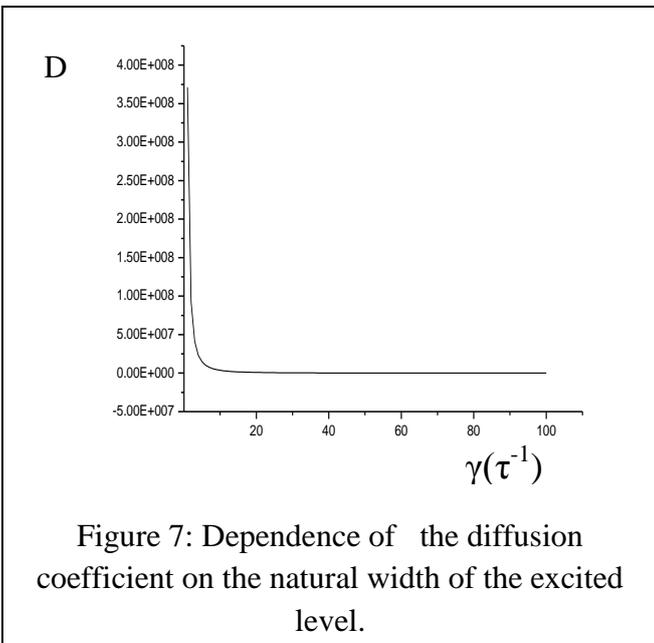


Figure 7: Dependence of the diffusion coefficient on the natural width of the excited level.

4. Conclusion

The result have shown that the diffusion coefficient for neutral atom that moving and interact with a laser field increased with decreasing the ellipticity of the electromagnetic field if the atom has the same Rabi frequency and same difference between the frequency of laser field and the atomic resonance. Rabi frequency affect strongly on the diffusion coefficient of atom .The increasing of value of Rabi frequency leads to increase in the diffusion coefficient and the velocity of atom can be increase . The reducing of Rabi frequency leads to decrease in the diffusion of atom so the movement decelerate, while the optical parameter δ has a slight influence on the diffusion coefficient then we can control the movement of atom by choosing certain value of Rabi frequency. For friction coefficient the opposite occurred ,the difference between the frequency of laser field and the atomic resonance has more effect than the Rabi frequency .The control of friction coefficient achieved by choosing certain optical parameter that represent the difference(δ) between the frequency of laser field and the atomic resonance. Even small value of

the natural width of the excited level with ellipticity equals to zero can give a high reading for the diffusion of neutral atom by using suitable values of other optical parameters. The changing of the diffusion and friction coefficients with the natural width of the excited level occurred in the negative and positive regions.

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