

A Simple and General Analytical Expression for Calculating Crosstalk in Multiwaveguide Directional Couplers Using Finite Differences Method

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ABSTRACT

In optical communication systems, the main aim is to control and limit the unwanted Crosstalk (CT) effects on wavelength divisions multiplexing (WDM) channel, which degrade the transmission performance. To do this, a simple approximate formula was presented for calculating the CT of multiwaveguide directional coupler (DC) using Finite Differences (FD) method. Through this formula, the input power can be controlled to be entered from one specified channel, which is required in some applications. Then, the evolution of the power has been determined without using Beam Propagation Method (BPM). The purpose of this paper is to calculate the CT for arbitrary input channel. Result obtained from this formula are in a complete agreement with the previous works for two and three waveguide directional couplers.

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علاقة تحليلية عامة وبسيطة لحساب القدرة العابرة في الموجات المترابطة المتعددة الأذرع باستعمال طريقة الفروقات المحددة

منصور حنظل منصور

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الخلاصة

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ان الهدف الأساسي في أنظمة الاتصالات البصرية هو السيطرة على تأثيرات القدرة العابرة Crosstalk غير المرغوب بها وتحجيمها وخاصة تلك الموجودة في قنوات أجهزة المجمعات البصرية والتي تقلل من كفاءة هذه الأجهزة في نقل القدرة لأنها تعتبر قدرة ضائعة. لغرض دراسة هذه القدرة الضائعة في قنوات الاتصالات البصرية فاننا قدّمنا صيغة تحليلية لحسابها في الموجات المقترنة او المترابطة Directional couplers المتعددة القنوات باستعمال طريقة الفروقات المحددة التقريبية. ان هذه الصيغة التحليلية من خلالها أصبح بالإمكان السيطرة على القدرة الداخلة باختيار وتحديد قناة معينة للدخال القدرة. لذلك فان الصيغة المقدمة في هذا البحث تعطينا توزيع وتغير القدرة وانتشارها على القنوات المتعددة بدون استعمال طريقة BPM. أظهرت النتائج المستحصلة من هذه الصيغة تطابقها مع نتائج بحوث سابقة لموجات مقترنة ثنائية وثلاثية.

1. INTRODUCTION

Directional couplers are, in general, composed of two parallel waveguides (WG) placed close enough to give rise to optical coupling. DCs are considered as an elementary parts for optical switching/modulation, optical power splitter/combiners [1] and electro-optic multiplexers [2]. In this regards, many researchers studied DC [3-6], many of them reported the way of minimizing CT caused by the coupling effect in tapered regions of the reversed delta beta parallel DC switches by using phase control [7]. By contrast, others presented a novel kind of terahertz DC which can achieve a lower absorption loss based on photonic crystals [8] and others considered the CT as an optical scatterings caused by devices defects [9]. Some CT can be treated as a noise sources on the impulse signals which have the same effect regardless of the signal strength. If the signal strength drops too much, the effect of noise increases [10]. It is also reported on CT of 2DC and 3DC with outer fed, when an incident wave is splitted into each normal mode unevenly [11]. When signals from one channel are crossed over another channel, they become noisy in the other channel. This can lead to serious effects on the signal-to-noise ratio and

hence on the error rate of the system. However, the most important problems in optical communication systems are unwanted CT, which is one of the factors that results in increasing the bit error rate. Crosstalk is quoted as the loss in dB between the input level of the signal and its (unwanted) signal strength in the adjacent channel. Also, It is possible to allow significantly more crosstalk in a modulator than in switch (all depending on the application) [12]. Channel crosstalk considerations are analogous to the polarization splitter design described in [13].

As our knowledge, there is no general mathematical expression in the literatures for determining the CT in multichannel DC, hence, in this paper, an analytical expression for calculating CT in multiwaveguide directional couplers (MWG DC) via controlling the power to be input from a chosen channel was derived.

2. EXPERIMENTAL

Fig.1 illustrates a typical integrated multi-waveguide directional coupler (MWG DC). The refractive index profile of cladding and guiding regions are n_1 and n_2 , respectively with waveguide width d and separation S .

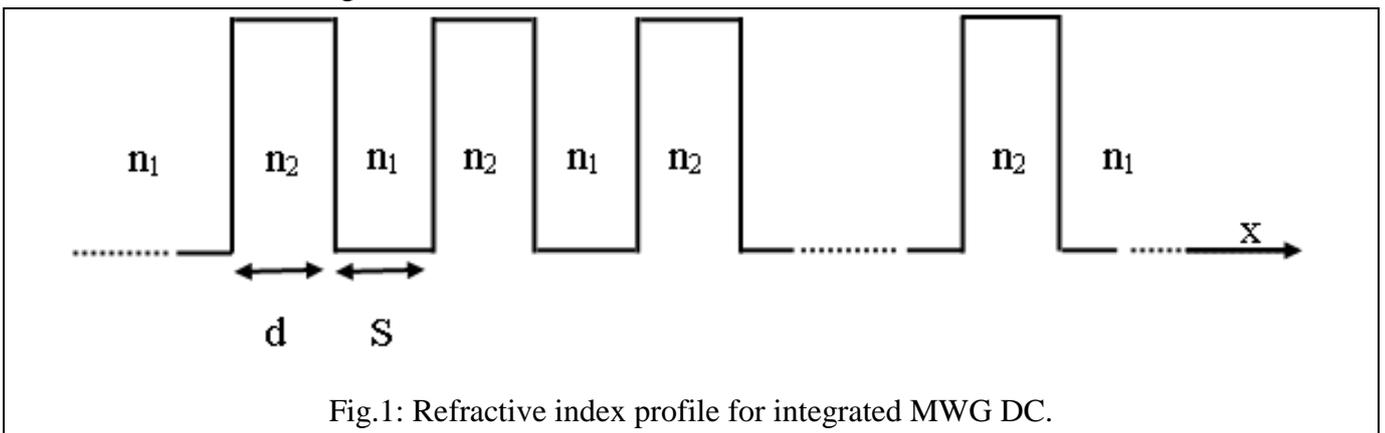


Fig.1: Refractive index profile for integrated MWG DC.

For simplicity, we will analyze 2WG DC with incident of field E_1 on WG1 and E_2 on WG2. The incident power are distributed between the two even and odd normal modes ψ_1 and ψ_1 ,

respectively. Therefore, the local fields can be expressed in terms of $\psi_1(x)$ and $\psi_2(x)$ as follows:

$$E_1(x) = a_{11}\psi_1(x) + a_{12}\psi_2(x) \quad (1a)$$

$$E_2(x) = a_{21}\psi_1(x) + a_{22}\psi_2(x) \quad (1b)$$

Where ψ_1 and ψ_2 are orthonormal functions with coefficients a_{ij} represent the contribution of the normal mode ψ_j in the local field E_i .

Equations (1) can be written in a matrix form

$$[E] = [a][\psi] \tag{2}$$

Or

$$E_i = \sum_{j=1}^2 a_{ij} \psi_j, \quad i = 1, 2 \tag{3}$$

Multiplying Eq. (3) by ψ_k and integrating over x , using the orthonormal properties of the normal modes yields:

$$a_{ik} = \int E_i \psi_k dx \tag{4}$$

Then, the total field evolution for input power from WG i for 2DC can be written as:

$$E_{T_i}(z) = \sum_{j=1}^2 a_{ij} \psi_j e^{-j\beta_j z}, \quad \text{for } i = 1, 2 \tag{5}$$

Where β_j is the propagation constant of the normal mode j . The CT_1 due to the first input

WG after propagation a distance equal to the coupling length ($z = L_c$) is:

$$CT_1 \equiv \left| \int E_1 E_{T_1}(z = L_c) dx \right|^2 \tag{6}$$

And CT_2 due to the second input channel is

$$CT_2 \equiv \left| \int E_2 E_{T_2}(z = L_c) dx \right|^2 \tag{7}$$

This leads to a CT expression in terms of a_{ij} 's

$$CT_1 = \left| a_{11}^2 e^{-j\beta_1 z} + a_{12}^2 e^{-j\beta_2 z} \right|^2 \tag{8}$$

$$CT_2 = \left| a_{21}^2 e^{-j\beta_1 z} + a_{22}^2 e^{-j\beta_2 z} \right|^2 \tag{9}$$

For 3WG DC, local modes can be written as a superposition of normal modes ψ_1, ψ_2 and ψ_3 as follows

$$E_1 = a_{11}\psi_1 + a_{12}\psi_2 + a_{13}\psi_3 \tag{10a}$$

$$E_2 = a_{21}\psi_1 + a_{22}\psi_2 + a_{23}\psi_3 \tag{10b}$$

$$E_3 = a_{31}\psi_1 + a_{32}\psi_2 + a_{33}\psi_3 \tag{10c}$$

For 3WG DC, one can find the field evolution as follows

$$E_{T_1}(z) = a_{11}\psi_1 e^{-j\beta_1 z} + a_{12}\psi_2 e^{-j\beta_2 z} + a_{13}\psi_3 e^{-j\beta_3 z} \tag{11a}$$

$$E_{T_2}(z) = a_{21}\psi_1 e^{-j\beta_1 z} + a_{22}\psi_2 e^{-j\beta_2 z} + a_{23}\psi_3 e^{-j\beta_3 z} \tag{11b}$$

$$E_{T_3}(z) = a_{31}\psi_1 e^{-j\beta_1 z} + a_{32}\psi_2 e^{-j\beta_2 z} + a_{33}\psi_3 e^{-j\beta_3 z} \tag{11c}$$

If the input in the first, second and third channel respectively, the corresponding CT are:

$$CT_1 \equiv \left| \int E_1 E_{T_1}(z = L_c) dx \right|^2 \tag{12a}$$

$$CT_2 \equiv \left| \int E_2 E_{T_2}(z = L_c/2) dx \right|^2 \tag{12b}$$

$$CT_3 \equiv \left| \int E_3 E_{T_3}(z = L_c) dx \right|^2 \tag{12c}$$

Where CT_3 is due to the third input channel, and L_c is the coupling length. If the power was launched from channel two, we used $z = L_c/2$ in calculating CT_2 since the power is

$$CT_1 = \left| a_{11}^2 e^{-j\beta_1 z} + a_{12}^2 e^{-j\beta_2 z} + a_{13}^2 e^{-j\beta_3 z} \right|^2, z = L_c \tag{13a}$$

$$CT_2 = \left| a_{21}^2 e^{-j\beta_1 z} + a_{22}^2 e^{-j\beta_2 z} + a_{23}^2 e^{-j\beta_3 z} \right|^2, z = L_c/2 \tag{13b}$$

$$CT_3 = \left| a_{31}^2 e^{-j\beta_1 z} + a_{32}^2 e^{-j\beta_2 z} + a_{33}^2 e^{-j\beta_3 z} \right|^2, z = L_c \tag{13c}$$

MWG DC for a chosen i^{th} input channel after travelling a distant L can be expressed through the

following formula:

$$CT_i = \left| \sum_{j=1}^N a_{ij}^2 \psi_j e^{-j\beta_j L} \right|^2, \text{ for } i=1,2,3\dots \tag{14}$$

$$\text{where } L = \begin{cases} \frac{L_c}{2}, & i = \frac{N+1}{2} \text{ if } N \text{ is odd,} \\ L_c, & \text{otherwise.} \end{cases}$$

The evolution of total electric field due to the input power from channel i is

$$E_{T_i}(z) = \sum_{j=1}^N a_{ij} \psi_j e^{-j\beta_j z}, \quad i = 1, 2, \dots, N. \tag{15}$$

totally coupled to the adjacent channels after propagating this distant. Substituting Eqs. (10) and (11) into Eq.(12) leads to a CT expression in terms of the coefficients a_{ij} as follows:

In conclusion, a simple and general analytical expression for determining CT in

Where N is the total number of channels or WG's, L_c is given by:

$$L_c = \frac{(N-1)\pi}{\beta_1 - \beta_N}, \quad \text{and } \beta \text{ 's are the}$$

propagation constant for the corresponding normal modes.

3. RESULTS AND DISCUSSION

In this section, numerical results of the CT is presented in order to ensure the validity of the proposed expression which expressed by Eq. (14). A MATLAB codes was developed to simulate numerically MWG DC's using FDM. Local and normal mode field profiles and their corresponding propagation constants are determined for a dielectric slab and MWG DC's, respectively. Selection of the input power to the entered from the required WG was done by using Eq. (15).

The application of this procedure is used to describe the evolution of the mode intensity profile along its propagating direction for multi planar WG DC with $n_1=2.20$, $n_2=2.2025$, $S=6 \mu m$ and $d=5 \mu m$ as shown in figures (2-5). The above WG DC parameters are selected to support only a single mode at $\lambda = 1.3 \mu m$. Moreover, single-mode fibers were designed to have zero dispersion at this wavelength.

We used FDM to calculate the eigenvalues for TE polarization $\lambda = 1.3 \mu m$, which is commonly used in telecommunications, of the following structures: for 2WG DC, the structure support two normal modes with effective refractive indices of 2.20142351 and 2.20130385, and support three normal modes with 2.20148022, 2.20145835 and 2.20129317 for 3WG DC, four modes with 2.20151453, 2.20145835, 2.20137929 and 2.20130436 for 4WG DC and five modes with 2.20152060, 2.20148057, 2.20142045, 2.20135214 and 2.20129503 for 5WG DC.

Figures (2-5) show that the light wave was coupled from one channel to another with ability of controlling the input power to be entered from appropriate channel which is important in calculating CT.

In order to ensure the validity of the general expression Eq. (14), CT in MWG DC was investigated as an example, as shown in figures

(6-9). Results showed that CT can be controlled by adjusting L_c and the gab separation. The coupling length L_c was decreased rapidly because of strong coupling between the waveguide cores as the gab separation was decreased by several micrometers. Conversely, the wider separation gab, s , gets the smaller CT which is desirable in multiplexers but it required larger DC length to complete power translation between different channels.

Kim, et. al., [1] used FD method and BPM to study CT for just two and three WG DC. Therefore, our Results are compared them for 2 and 3WG DC and showed an excellent agreement.

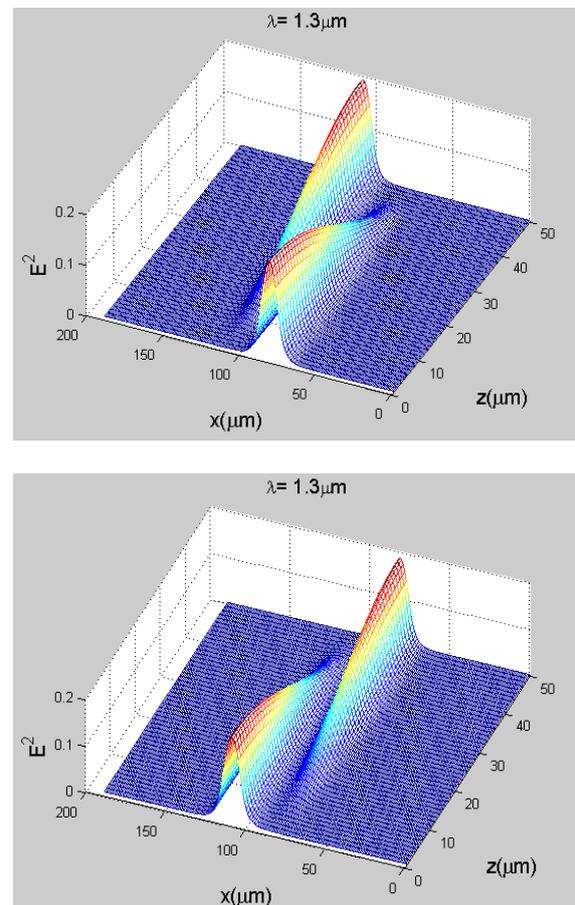


Fig.2: Power distribution in two WG DC if the input is from (a) First WG and (b) second WG.

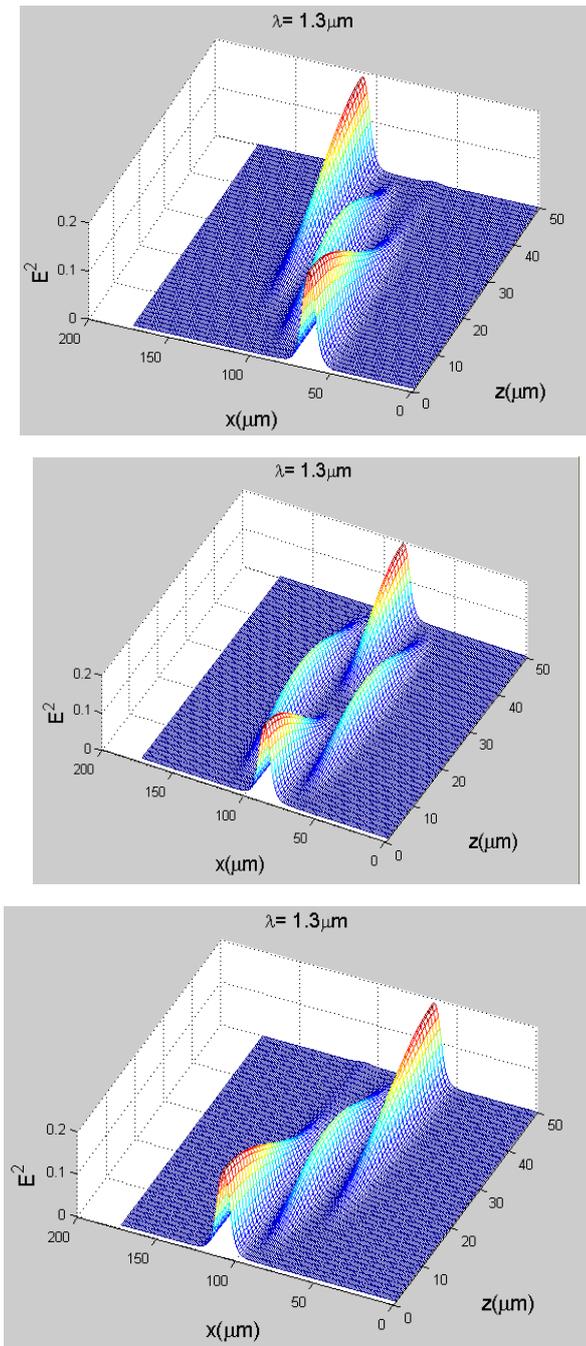


Fig.3: Power distribution in three WG DC if the input is from (a) First WG, (b) second WG and (c) Third WG.

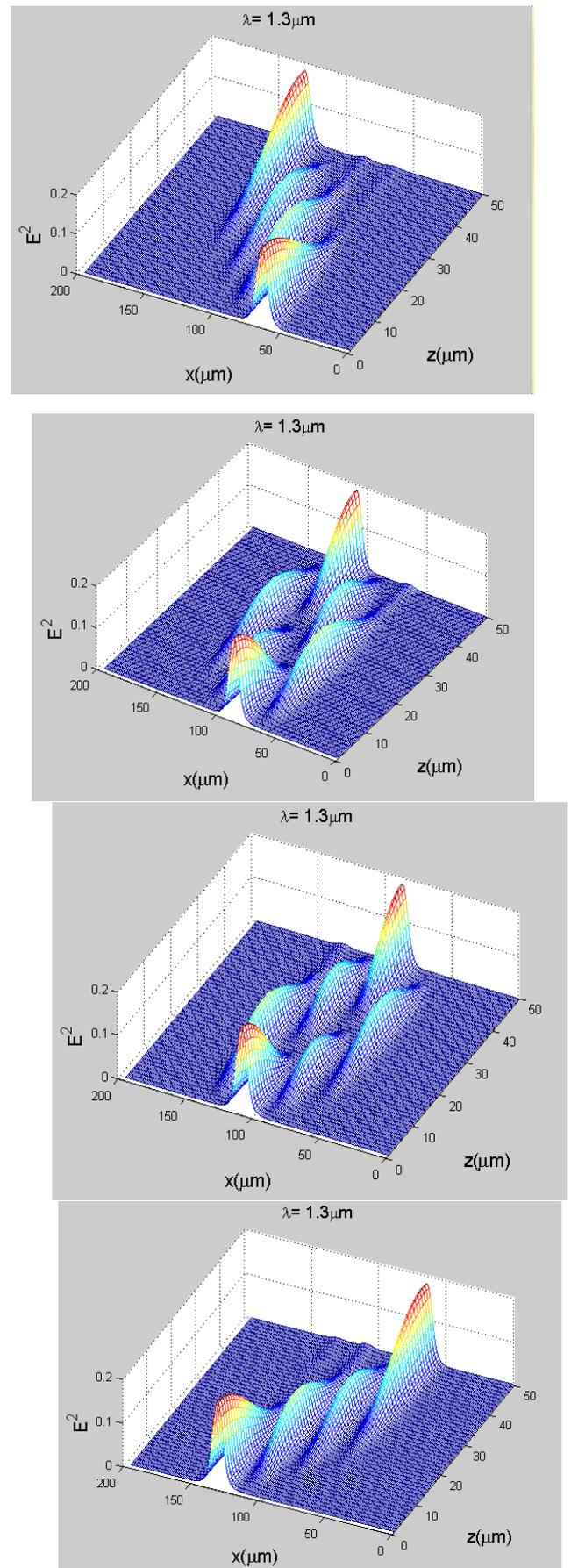


Fig. 4: Power distribution in four WG DC if the input is from (a) First WG, (b) second WG, (c) Third WG and (d) Fourth WG.

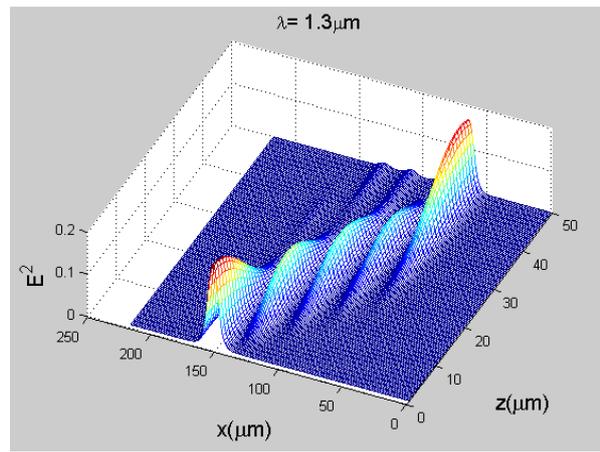
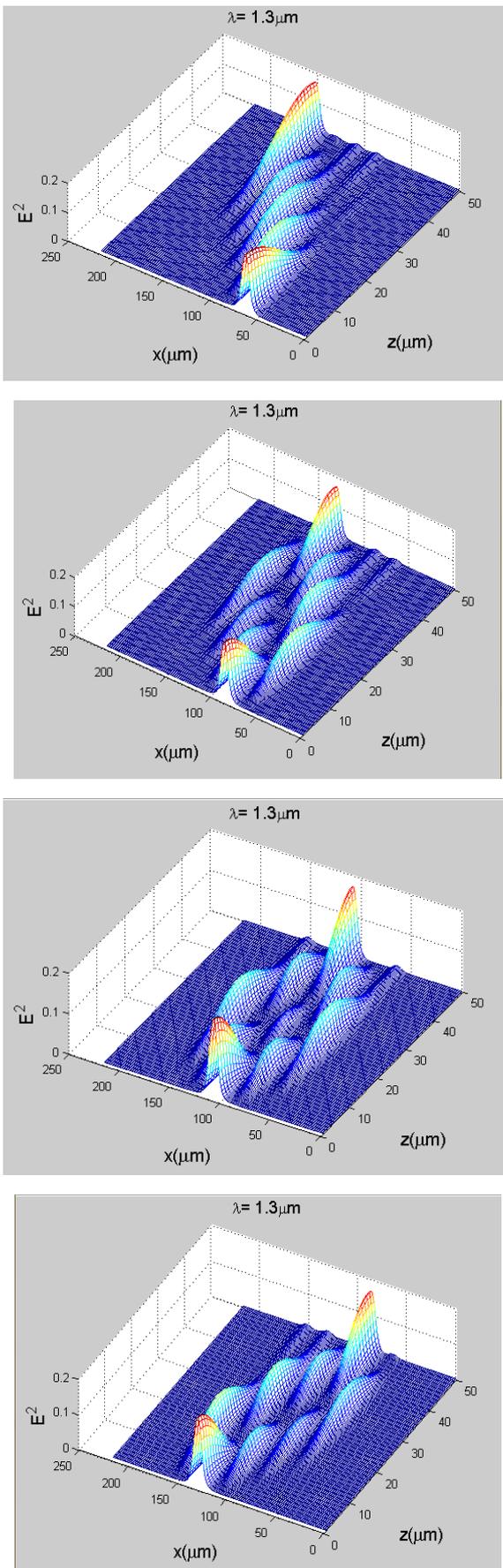


Fig.5 Power distribution in five WG DC if the input is from (a) First WG, (b) second WG, (c) Third WG, (d) Fourth WG and (e) Fifth WG

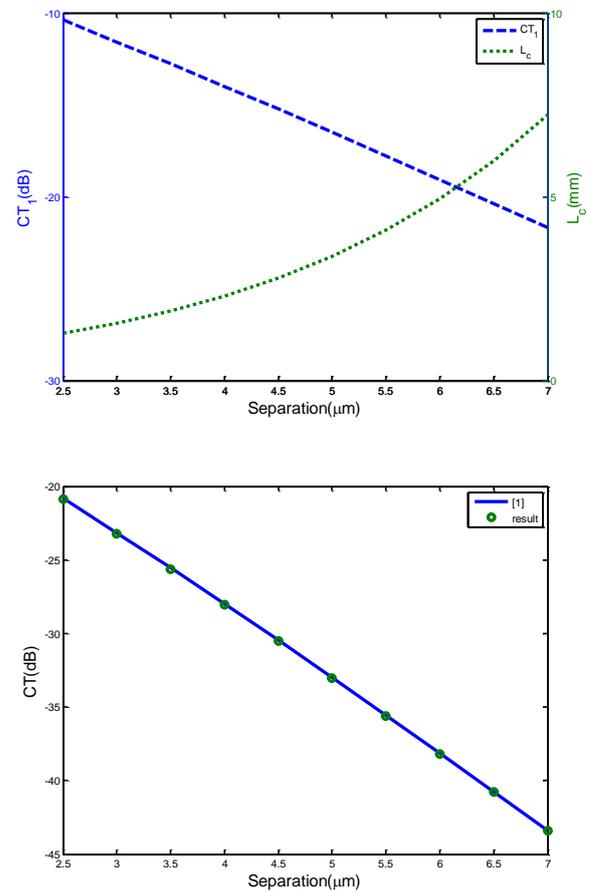


Fig. 6 (a) Coupling length and CT in 2WG DC as a function of S and (b) CT comparison with Kim et. al. [1].

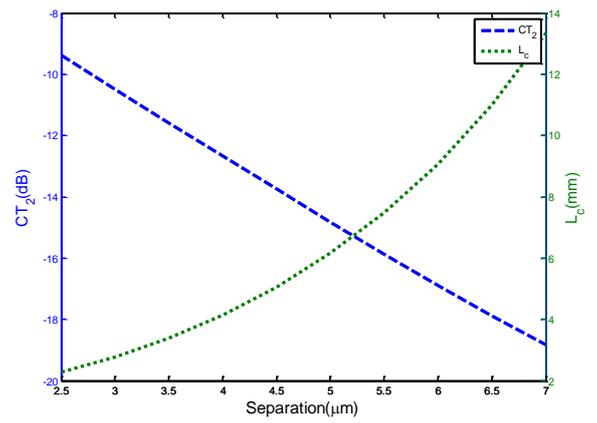
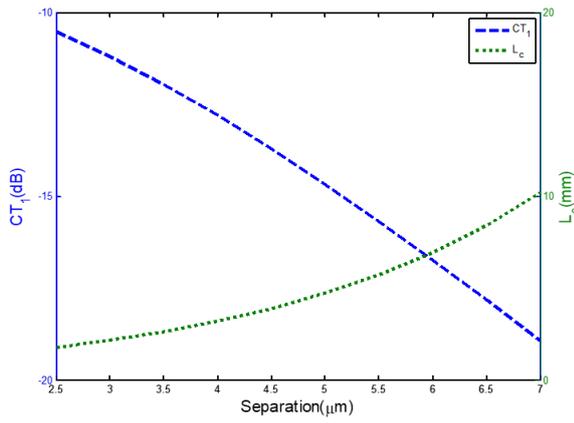


Fig.8 Coupling length and CT in 4WG DC as a function of S for input power from (a) First WG and (b) Second WG.

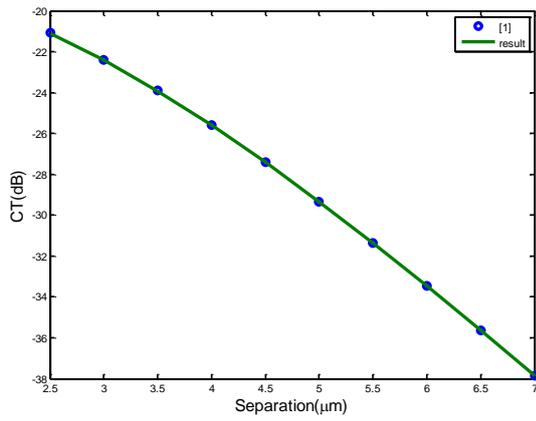
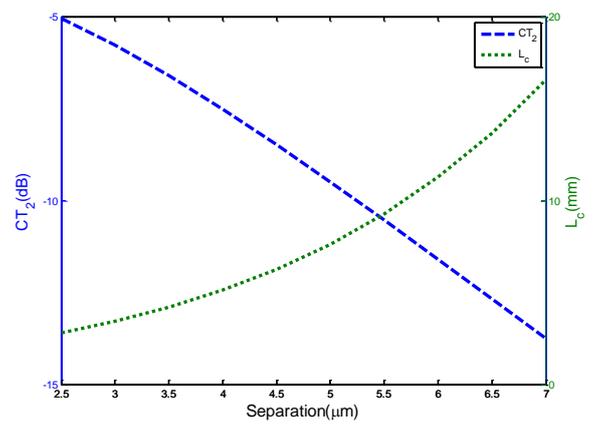
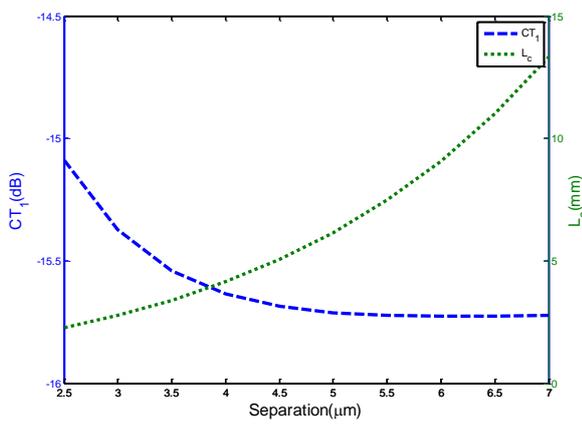
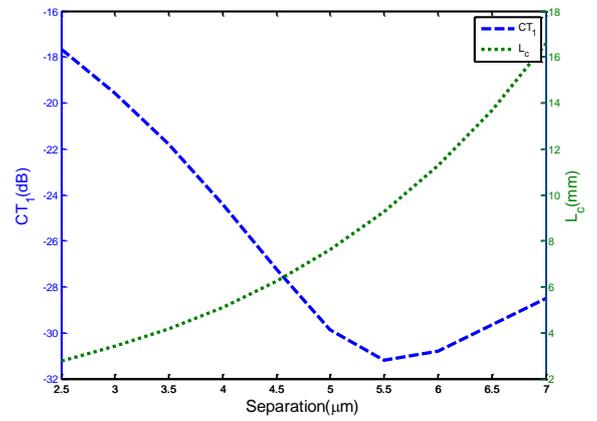


Fig. 7 (a) Coupling length and CT in 3WG DC as a function of S for input power from First WG and (b) CT comparison with Kim et al. [1].



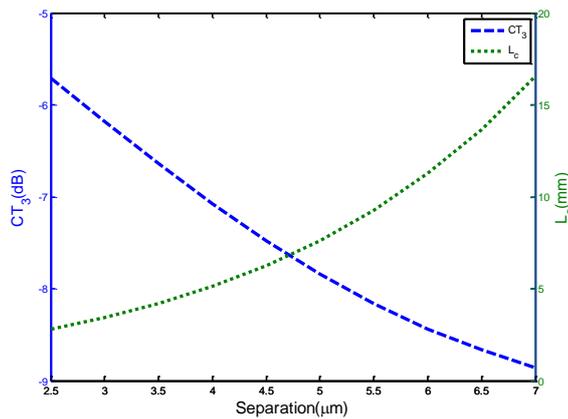


Fig.9: Coupling length and CT in 5DC as a function of S for input power from (a) First WG, (b) Second WG and (c) Third WG

4. REFERENCES

The paper presented a general analytical expression for calculating CT with different input channel for MWG DC. Simulation results of this analytical expression are based on the FDM and when compared with the available literatures, it gives an excellent agreement. Results showed that the proposed procedure is significantly accurate and useful especially for MWG DC, dense wavelength division multiplexers based on DCs, optical switches and Mach-Zehnder interferometers

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