Calculation of Solar Radiation Pressure Effect and Sun , Moon Attraction at High Earth Satellite

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Abstract

The effects of the solar radiation pressure and the attraction of the sun and the moon at high Earth orbit satellite have been investigated . Computer simulation for the equation of motion and velocity with perturbations is designed by Matlab a 7.4 where using Jacobian matrix method to increase the accuracy .

حساب تأثير ضغط الإشعاع الشمسي وجاذبية الشمس والقمر على الأقمار الاصطناعية عالية المدار

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الخلاصة

تم في هذا البحث دراسة تأثير ضغط الإشعاع الشمسي وجاذبية الشمس والقمر على الأقمار الاصطناعية عالية المدار ،حيث تم محاكاة سرعة وموقع الأقمار الاصطناعية واضطراباتها باستخدام لغة المـاتلاب وباستخدام طريقة مصفوفة جاكوبيان لغرض زيادة الدقة.

1-Introduction

There are two kinds of perturbations of satellite which effect position and velocity of satellite then the lifetime of satellite , gravitational and non-gravitational. Gravitational perturbations include the spherical harmonics , Earth tide ,ocean tide effect and effect of sun and moon attraction. Non-gravitational perturbations include atmospheric drag force , solar radiation pressure , magnetic forces etc. Rodolpho V. Moraes studied the joint effects of direct solar radiation pressure and atmospheric drag on the orbit of an artificial Earth satellite [1], D. Vokrouhlicky and his group studied the perturbation effect of the force due to direct solar radiation pressure on the dynamics of artificial satellites during penumbra [2], Some practical issues of interest such as the practical solar radiation pressure model formation flight of more than two satellites attitude control considerations sailing near earth space was performed by Zhong S. Wang [3] .The rotational dynamic of a small solar system body subject to solar radiation torques was investigated by Daniel J. and Sepidehsadat [4].The relation between area to mass ratio of satellite was presented by R. Musci, and his group. They assumed variation from 1 to 20 m^2/kg for geosynchronous Earth orbits GEO [5].

In the present , numerical simulation of the equation of motion of two body problem under the effects of solar radiation pressure and attraction of sun and moon at the high Earth orbit satellite by using Matlab a7.4 of the Jacobian matrix which was supplemented

to improve the accuracy of the numerical solution .

2-Cowell`s Method

The application of Cowell's method is simply to write the equations of motion of the object, including all the perturbations and then to integrate them step-by-step numerically. For the twobody problem with perturbations, the equation would be [6]

$$\ddot{\vec{r}} + \frac{u}{r^3}\vec{r} = \vec{a}_p \tag{1}$$

Equation (1) is a second order differential equation. Numerically, we can change this second order differential

equation into first order differential equation as in equation

$$.\dot{\vec{r}} = \vec{v} \qquad \dot{\vec{v}} = \vec{a}_p - \frac{u}{r^3}\vec{r} \tag{2}$$

where r and v are the radius and velocity of a satellite with respect to the larger central body. Equation (2) will have to be broken down into the vector components.

$$\dot{x} = v_x \qquad v_x = a_{px} - \frac{u}{x^3}x \tag{3}$$

$$\dot{y} = v_y \qquad v_y = a_{py} - \frac{u}{y^3}y \tag{4}$$

$$\dot{z} = v_z \qquad v_z = a_{pz} - \frac{u}{z^3}z \tag{5}$$

3-Solar Radiation Pressure

Solar radiation pressure is force acting on the satellite's surface caused by the sunlight. The force acting directly on the satellite is proportional to the effective satellite surface area, to the reflectivity of the surface and to the solar flux; it is inversely proportional to the velocity of light, the force result from solar pressure radiation is [7,8,9]

$$\vec{f}_{solar} = m\gamma P_S C_r r_{sun}^2 \frac{S}{m} \frac{\vec{r} - r_{sun}^2}{\left|\vec{r} - \vec{r}_{sun}\right|^3} \tag{6}$$

where

m= mass of satellite

γ =shadow factor

$\quad \gamma = 1$ for complete sun light

$\quad \gamma = 0$ for umbra phase

$\quad 0 < \gamma < 1$ for penumbra phase

C_r =surface reflectivity (has values from 1 to 2)

S= area / cross section of satellite

r_{sun}=is the geocentric distance of the sun

$\vec{r}$ and $\vec{r}_{sun}$= are the geocentric vectors of the satellite and sun

P_S = is the luminosity of sun which can be calculated from [10]

$$p_S = \frac{E}{c} \tag{7}$$

where

E= Solar constant (nominal 1358 W/m²)

c =in vacuum speed of light.

4-Attractions of Sun and Moon

The sun and moon causes periodic variations in all keplerian elements, but secular perturbations only to the right ascension of ascending node and argument of perigee[11]

$$\dot{\Omega}_{sun} = -0.00154 \frac{\cos i}{n} \tag{8}$$

$$\dot{\omega}_{sun} = 0.00077 \frac{5\cos^2 i - 1}{n} \tag{9}$$

$$\dot{\Omega}_{moon} = -0.00338 \frac{\cos i}{n} \tag{10}$$

$$\dot{\omega}_{moon} = 0.00169 \frac{5\cos^2 i - 1}{n} \tag{11}$$

where n is the number of revolutions per day, i inclination , $\dot{\Omega}_{sun}, \dot{\Omega}_{moon}$ rate of right ascension of ascending node for sun and moon at sequences and $\dot{\omega}_{sun}, \dot{\omega}_{moon}$ rate of argument of perigee for sun and moon at sequences are given in degrees/sec.

5-Jacobian Matrix

The Jacobian matrix is the matrix of all first-order partial derivatives of a vector-valued function .That is ,the Jacobian of a function describes the

orientation of a tangent plane to the function at a given point[12] . Let $x \in R^n$ and $y = f(x) \in R^n$ be a differentiable vector-valued function of x.

In this case ,can take the following form [13]

$$J = \begin{pmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} & \cdots & \dfrac{\partial f_n}{\partial x_n} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} & \cdots & \dfrac{\partial f_2}{\partial x_n} \end{pmatrix} = \left(\dfrac{\partial f_i}{\partial x_j} \right)_{i,j=1,2,.n} \quad (12)$$

6. Discussion of Simulation Results

A computer simulation has been developed to the equation of orbital motion of two body problems with perturbations due to the effects of the solar radiation pressure and attractions of the sun and moon using Matlab a 7.4 .The solar radiation pressure perturbation was calculated for 50 orbital cycles .

Figure (1) explains the effect of the solar radiation pressure on the position vector , it is clear that the position vector behaves normally in the beginning , but after a many revolutions the position vector decreases and this decreasing appears clearly when the number of the orbital cycles increases ,on the other hand figure (2) explains that without the effect of the solar radiation pressure on the position vector the last stays as it is .
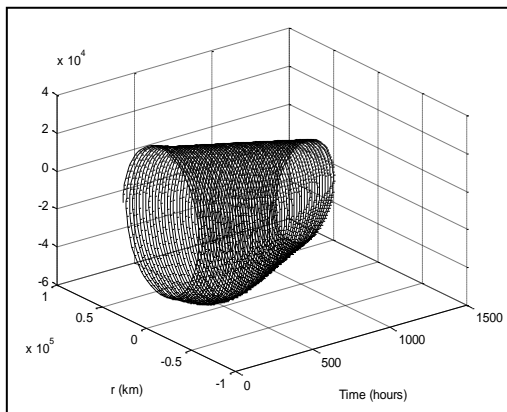

Fig.(1)Shows the relationship between position vector with time under solar radiation pressure effect
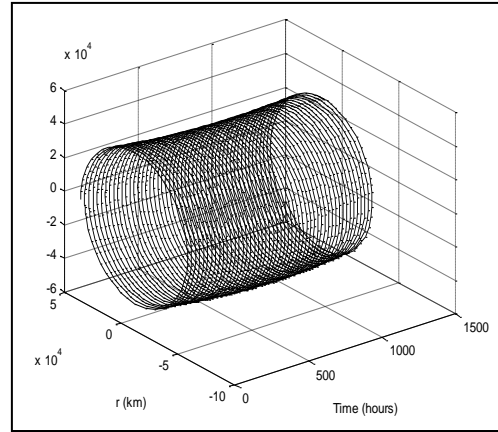

Fig.(2) Shows the relationship between position vector with time without solar radiation pressure effect

The variation of orbital velocity along with time under the effect of solar radiation pressure is shown in figure (3). This figure defines that the velocity vector increasing step-by-step when revolution number increases and this is clear when compared with figure (4) which shows the velocity vector without the effect of the solar radiation pressure where it is still constant without any change.
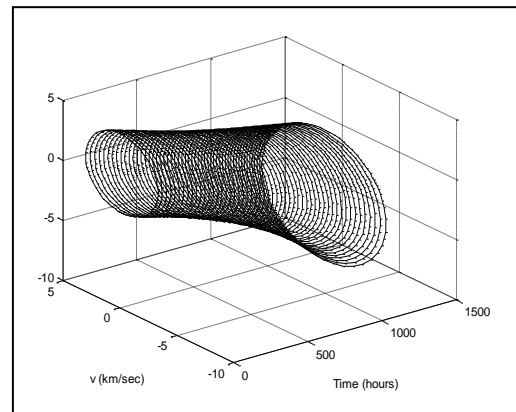

Fig.(3) Shows the variation of orbital velocity with time under the effect solar radiation pressure
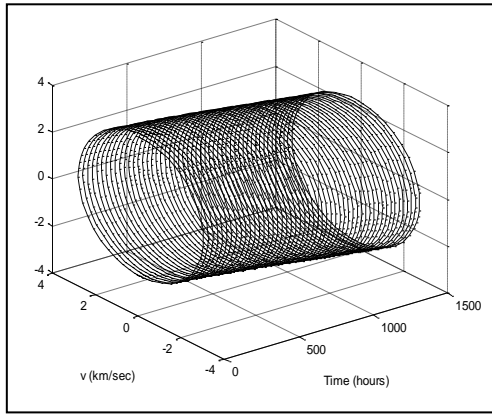
Fig.(4) Shows the velocity vector with time in the absence of solar radiation pressure effect
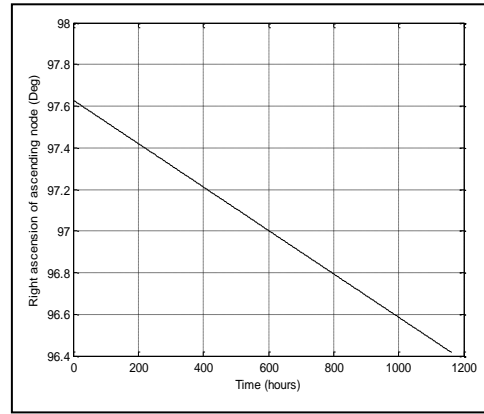

Fig.(5) Shows the effect of sun attraction on right ascension of ascending node

The effect of attractions of the sun and moon   are shown in figures (5),(6),(7) and (8) at the right ascension of ascending node and the argument of perigee. It can be seen from these figures that there is a small difference between the attractions of the sun and moon on the right ascension of ascending node and the argument of perigee actions .

It can be concluded from these figures that ,the solar radiation pressure impacts on the position and velocity of the orbit of the satellite and subsequently causes a reduction of the position of the orbit and steps-up   the velocity of the orbit , then reduces the lifetime of the satellite .It is possible to decrease this kind of perturbation by controlling the   area-to-mass ratio of the satellite depending on equation (6) .
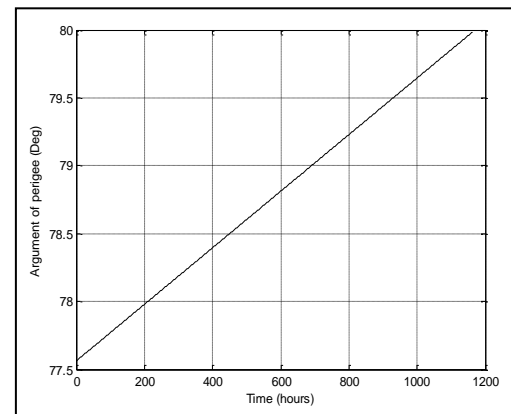

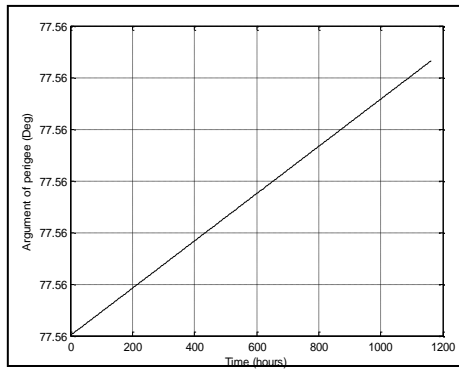Fig.(6) Shows the effect of sun attraction on argument of perigee

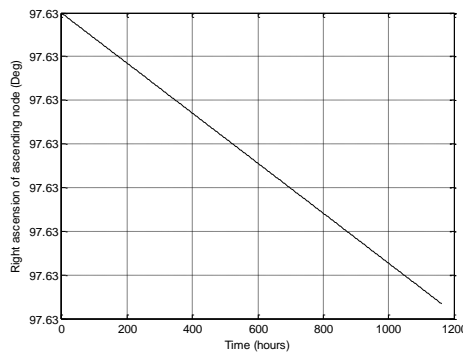Fig.(7) Show the effect of moon attraction on right ascension of ascending node

Fig.(8) Show the effect of moon attraction on argument of perigee

References

1- Rodolpho V. Moraes, Combined Solar Radiation Pressure and Drag Effects on the Orbits of Artificial Satellite ,Celestial Mechanics ,25,p281-292,(1981).

2-D. Vokrouhlicky and group,Solar Radiation Pressure Perturbations for Earth Satellites,Astronomy and astroph sics,280,p295-312.(1993).

3-Zhong S. Wang, Dynamics Analysis of Satellite Formation Flight Using Solar Radiation Pressure, Ph.D, Department of Aerospace Engineering and Engineering Mechanics, College of Engineering, University of Cincinnati,China,(2001).

4-Daniel J. and Sepidehsadat M., Rotational Dynamics of a Solar System Body Under Solar Radiation Torques, Celest.Mech.Dyn.Ast.,**101**,February p69-103,(2008).

5- R. Musci, and group , Evolution of the Orbital Elements for Objects with High Area-to-Mass Ratios in Geostationary Transfer Orbits,Advances in Space Research 41 , 1071–1076, (2008).

6-M.Adnan,R. Razali,and M. Said,A Study of Perturbation Effect on Satellite Orbit Using Cowell`s Method,School of Aerospsce Engineering ,Engineering Campus, University Science Malaysia, Malaysia.

7- Guochang Xu,Orbits Springer-Verlag Berlin Heidelberg. Germany, (2008).

8-Guochang Xu, GPS Theory, Algorithms and Applications, Springer-Verlagz Berlin,Germany, (2007).

9-Gerhard Beutler, Methods of Celestial Mechanics, **II**,Application to Planetary System,Geodynamics and Satellite Geodesy,Springererlag BerlinHeidelberg, Germany,(2005).

10-Robert Wolf,Satellite Orbit and Ephemeris Determination using Inter Satellite Links, Ph.D College of Engineering University of Bundeswehr Germany, (2000).

11-Kristian Svartveit,AttitudedDetermin-ation of the NCUBE Satellite, M.Sc. thesis , Department of Engineering Cybernetics ,Institude for technique Cybernetics,Norges teknisk naturvitens kapelige universitet,(2003).

12-www.Wikipedia.com.

13-Alan Edelman & N. Raj , Random Matrix Theory,Acta. Numerica ,p1-65 , (2005).