The Interacting Boson Model-1 Applied to the even-even $^{106-116}$ Pd Isotopes

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Abstract

The low lying levels structure and electric quadrupole transitions of $^{106-116}$Pd nuclei have been studied by using the Interacting Boson Model-1 (IBM-1). The Potential surfaces of these nuclei are also studied and the shapes of them are analyzed. The calculated results are in good agreement with recent experimental data. The results obtained and the values of parameters used in this calculations indicated that Pd isotopes have a vibrational properties with small amount of rotational and pair properties. tendency towered some more rotational properties appears as the number of neutrons approaches mid-shell between 50-82 shells.

تطبيق نموذج البوزونات المتفاعلة الأول على النوى زوجية- زوجية لنظائر $^{106-116}$Pd

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الخلاصة:

تمت دراسة تشوه مستويات الطاقة الواطئة والانتقالات رباعية القطب الكهربائية لنوى $^{106-116}$ Pd باستخدام نموذج البوزونات المتفاعلة الأول IBM-1 . ودرست ايضا سطوح تساوي الجهد لهذه النوى. أظهرت النتائج توافقا جيدا مع القيم العملية الحديثة وأظهرت قيم المعاملات المستخدمة في هذه الحسابات ان هذه النظائر تمتلك صفات اهتزازية مع نسبة قليلة من الصفات الدورانية. ويزداد الميل نحو الصفات الدورانية باقتراب عدد النيوترونات من منتصف القشرة بين القشرتين 82-50.

1. Introduction

The IBM was introduced in 1974 by F. Iachello and A. Arima, it has been successfully applied to a wide range of nuclear collective phenomena[1-3]. A model of the atomic nucleus has to be able to describe nuclear properties such as spins and energies of the lowest levels, decay probabilities for the emission of gamma quantas, probabilities (spectroscopic factors) of transfer reactions, multipole moments and so forth.

The interacting boson model (IBM) is suitable for describing intermediate and heavy atomic nuclei.

Adjusting a small number of parameters, it reproduces the majority of the low-lying states of such nuclei.

The IBM is based on the well-known shell model and on geometrical collective models of the atomic nucleus. Despite its relatively simple structure, it has proved to be a powerful tool. In addition, it is of considerable theoretical interest since it shows the dynamical symmetries of several nuclei, which are made visible using Lie algebras

The essential idea is that the low energy collective degrees of freedom in nuclei can be described by proton and neutron bosons with spins of 0 and 2. These collective building blocks interact. Different choices of L=0 (s-boson) and L=2 (d-boson) energies and interaction strengths give rise to different types of collective spectra[4].

These bosons are interpreted as correlated pairs of protons and correlated pairs of neutrons in the valence shell. This interpretation places restriction on the boson number which is determined by counting the number of particle pairs (separately for protons and neutrons) if the shell is less than half filled. And by counting the number of hole pairs if the shell is more than half filled. If the bosons of neutrons and the bosons of neutrons were considered identical then the interacting boson model is in its simplest form which is called IBM-1[5]

In the work[6] Isacker and Puddu studied the low-lying states and electromagnetic transition rates of $^{100-124}$Pd nuclei using proton- neuron boson interacting boson model.

A consistent Q-formalism extended to IBM-1 Hamiltonian is applied to Pd isotopes by D. Bucurescu et. al. and they concluded that these nuclei were transitional between the vibrational SU(5) and the γ-unstable O(6) [7]. In1999, K. Zajak et. al. describe quadrupole excitation of even-even Pd isotopes within a microscopic approach based on the general collective Bohr model which include the effect of coupling with pairing vibrations [8]. Recently. Lalkovski and Isacker [9] have performed systematicanalysis of even-even N=66(112Pd, 110Ru, 108Mo and 106Zr) isotopes in IBM-1, satisfactorily reproduced the excitation energy levels and electric quadrupole transition probabilities of these isotopes.

2- The Model Operators

The Hamiltonian of IBM-1[4] used is

$$H=\varepsilon\ \hat{n}_d+a_0\hat{P}^\dagger\hat{P}+a_1\hat{L}^\dagger\hat{L}+a_2\hat{Q}^\dagger\hat{Q}+a_3\hat{T}_3^\dagger\hat{T}+a_4\hat{T}_4^\dagger\ \hat{T}_4^\dagger \quad ....(1)$$

Where ε is the boson energy, the parameters a_i's designate the strengths of the, pairing, angular momentum, quadrupole, octupole, and hexadecapole interaction between bosons respectively. Where

$$\hat{n}_d=(d^\dagger d),\ \ \hat{P}=\frac{1}{2}\left[\left(d^\dagger.d\right)-\left(s^\dagger.s\right)\right],\ \ \hat{L}=\sqrt{10}\left(d^\dagger\times d\right)^{(1)}$$

$$\hat{Q}=\left(d^\dagger\times s+s^\dagger\times d\right)^{(2)}+\chi(d^\dagger\times d)^{(2)}\ \ and\ \hat{T}_l=\left(d^\dagger\times d\right)^{(l)},l=3,4$$

The $(s.d)$ and $(s^\dagger.d^\dagger)$are the creation and annihilation operators of s and d. A successful nuclear model must yield a good description not only of the energy spectrum of the nucleus but also of its electromagnetic transitions. The electric quadrupole transition B(E2) operator in the IBM-1has the form.

$$\hat{T}^{E2}=\alpha_2[d^\dagger\times s+s^\dagger\times d]^{(2)}+\beta_2[d^\dagger\times d]^{(2)}...(2)$$

The parameters α_2 and β_2 are adjusted to fit the experimental data.

The classic limit of IBM-1 Hamiltonian can be obtained through the IBM coherent intrinsic state (boson condensate) introduced in references [10,11]

A general expression for the energy surface, stated in terms of the Hamiltonian of equation 1, is given by Van Isacker and Chen[4]

$$E(N,\beta,\gamma)=E_0+\frac{N(\varepsilon_s+\varepsilon_d\beta^2)}{(1+\beta^2)}+$$
$$\frac{N(N-1)}{(1+\beta^2)^2}(f_1\beta^4+f_2\beta^3\cos(3\gamma)+f_3\beta^2+f_4)...(3)$$

where the parameters f_i 's are simply related to the coefficients of Hamiltonian of equation 1 (see references [4,12]). β is the magnitude of the deviation from the spherical shape, and γ is the magnitude of the deviation from the symmetry axis. One notes that γ occurs only in the term cos(3γ), and thus, the energy surface has minima only at $\gamma = 0^o$ and 60^o. Simpler expression, which displays the essential dependence on β and γ has been given by

$$E^{I}(N;\beta,\gamma)=\varepsilon_d N\frac{\beta^2}{1+\beta^2}..U(5)..(4)$$

$$E^{II}(N;\beta,\gamma)=kN(N-1)\frac{1+\frac{3}{4}\beta^4-\sqrt{2}\beta^3\cos 3\gamma}{(1+\beta^2)^2}..SU(3)....(5)$$

$$E^{III}(N;\beta,\gamma)=k'N(N-1)\left[\frac{1-\beta^2}{1+\beta^2}\right]^2$$

$$...O(6)......................................(6)$$

Clearly in SU(5) the energy minimum for the ground states where $n_d = 0$ correspond to β= 0 , while in O(6) the minimum is at β = 1 , and in SU(3) at β= $\sqrt{2}$

3. Calculations and results

3.1 Energy levels

Calculations of energy levels for even-even $^{106-110,114,116}$Pd isotopes were performed with the whole Hamiltonian (eq.1) using IBM-1 computer code .
For ^{106}Pd, ^{108}Pd , ^{110}Pd , ^{114}Pd , and ^{116}Pd nuclei (Z=46) the number of proton bosons and neutron bosons and the total number of bosons are listed in table 1.

The parameters of equation (1) were calculated from the experimental schemes of these nuclei[13-17] and the analytical solutions for the three dynamical systems (see reference [4]). These parameters were tabulated in table (2).

Table (1) The number of proton bosons and neutron bosons and the total number of bosons for the even- even $^{106-110,114,116}$Pd isotopes.

Isotope	Proton bosons N_π	Neutron bosons N_ν	Tot. no. of bosons N
^{106}Pd	2 (hole pairs)	5 (particle pairs)	7
^{108}Pd	2 (hole pairs)	6(particle pairs)	8
^{110}Pd	2 (hole pairs)	7 (particle pairs)	9
^{114}Pd	2 (hole pairs)	7 (hole pairs)	9
^{116}Pd	2 (hole pairs)	6 (hole pairs)	8

Table (2) The parameters of the Hamiltonian
equation and E2 operators used for the describtion of the Pd isotopes.

Isotope	N	ε	a_0	a_1	a_2	a_3	a_4	E2SD	E2DD
^{106}Pd	7	0.448	0.025	0.0018	0.002	0.001	0.04	0.074	-0.09
^{108}Pd	8	0.32	0.02	0.015	0.003	0.001	0.04	0.0794	-0.09
^{110}Pd	9	0.26	0.008	0.004	0.006	0.001	0.04	0.0843	-0.09
^{114}Pd	9	0.28	0.011	0.01	0.003	0.008	0.055	0.0685	-0.09
^{116}Pd	8	0.31	0.02	0.01	0.0015	0.008	0.055	0.087	-0.095

The calculated and experimental and energy levels are exhibit in figures (1-6)

3.2 Electric quadrupole transition probability

Calculations of B(E2) values were performed by using computer code "FBEM"[8]. The parameters in E2 operator eq.(2) are determined by fitting the experimental (B(E2;$2_1^+ \rightarrow 0_1^+$) data, and the parameters arte listed in table 2. Where

$$\mathrm{E2}SD=\alpha_2,\mathrm{E2}DD=\sqrt{5}\beta_2$$

And $\beta_2=\frac{-0.7}{5}\alpha_2,-\sqrt{7/2}\alpha_2 \ and \ =0$ in SU(5), SU(3) and O(6) respectively[12].

Table (3) Comparison between present values of B(E2)(in unit e^2b^2) for even-even $^{106-10,114,116}$Pd isotopes (Theo.) and experimental ones (Exp.) [13-19]. The quadrupole moment of 2_1^+ state listed in last line.

Transition	^{106}Pd		^{108}Pd		^{110}Pd		^{114}Pd		^{116}Pd	
	Exp.	Theo.	Exp.	Theo.	Exp.	Theo.	Exp.	Theo.	Exp.	Theo.
$2_1^+\rightarrow 0_1^+$	0.133	0.136	0.152	0.153	0.172	0.172	0.068	0.068	0.11	0.111
$2_2^+\rightarrow 0_1^+$	0.013	0.018	0.012	0.012	-	0.012	-	0.0061	-	0.0076
$0_2^+\rightarrow 2_1^+$	0.004	0	0.006	0	-	0	-	0	-	0
$2_2^+\rightarrow 2_1^+$		0.167	-	0.193	-	0.218	-	0.1062	-	0.169
$4_1^+\rightarrow 2_1^+$	0.21	0.183	0.233	0.21	-	0.236	-	0.0948	-	0.154
$0_3^+\rightarrow 2_1^+$	-	0	<0.002	0	-	0	-	0.0005	-	0.00094
$2_3^+\rightarrow 2_2^+$	-	0	0.007	0	-	0	-	0.0395	-	0.0018
$2_3^+\rightarrow 0_1^+$	-	0	0	0	-	0	-	0	-	0
$2^+\rightarrow 2_1^+$	-	0.001	0.001	0.001	-	0.001	-	0.0001	-	0.0001
$4_2\rightarrow 2_3^+$	-	0.006	0.005	0.006	-	0.006	-	0.0096	-	0.0158
$Q2_1^+$	-0.51	-0.44	-0.58	-0.45	-0.72	-0.46	-	-0.094	-	-0.07

The three electromagnetic transition rates are plotted in fig.(6)where [4]

$$R = \frac{B(E2:4_1^+ \to 2_1^+)}{B(E2:2_1^+ \to 0_1^+)}$$

$$R' = \frac{B(E2:2_2^+ \to 2_1^+)}{B(E2:2_1^+ \to 0_1^+)}$$

$$R'' = \frac{B(E2:0_2^+ \to 2_1^+)}{B(E2:2_1^+ \to 0_1^+)}$$

that changes from

$$R = R' = R'' = 2[(N-1)/N] \text{ in U(5)}$$

to

$$R = \frac{10}{7}\frac{(N-1)(2N+5)}{2(2N+3)} \approx 1.4, R' = R'' = 0 \quad \text{in SU(3)}$$

and to

$$R = R' = \frac{10}{7}\frac{(N-1)(N+5)}{2(N+4)} \approx 1.4, R'' = 0 \quad \text{in O(6)}$$

3.3 The equipotential energy surfaces

The parameters of the energy surface were calculated by transforming the parameters of Hamiltonian of equation 1 by several equations (see reference [4]), and they are found to be as in table (5).

Table (4) Parameters of energy surface for the even- even $^{106-110,114,116}$Pd isotopes

Isotope	ε_s	ε_d	f_1	f_2	f_3	f_4
^{106}Pd	0.01	0.638	0.028	0.006	0.004	0.006
^{108}Pd	0.016	0.492	0.027	0.009	0.003	0.005
^{110}Pd	0.032	0.375	0.026	0.018	0.022	0.002
^{114}Pd	-0.015	0.449	0.031	0.0	0.018	0.003
^{116}Pd	-0.008	0.48	0.034	0.0	0.016	0.005

A mathlab program was written, see appendix 1, to draw the contour plots in the γ-β plane for even-even $^{106-110,114,116}$Pd, shown figs.(7-11) and because of the angle of (3γ) in equation 3, it is sufficient to take γ between 0^o and 60^o.

4.Discussion and Conclusions

The IBM-1 code is based on a simple Hamiltonian with 6 parameters which can be determined from experimental data.

The IBM-1 model is applied to the even-even Palladium $^{106-110,114,116}$Pd isotopes, but not ^{112}Pd because in this isotope the number of neutrons is a semi-magic number (N=66) and this case, as like as the magic numbers, is not preferred to be studied in this model and it is probably a spherical nuclei[12].

At first glance it is believed that $^{106-110,114,116}$Pd isotopes have γ-unstable O(6) characteristics because the ratios of E_4^+/E_2^+ and E_6^+/E_2^+ were near the values 2.5 and 4.5 (see table 5) which are the values for pure O(6). But when observing the experimental values of the energy levels of 4_1^+,2_2^+ and 0_2^+,(two phonon state)it is clear that these values were very close to each other for the five isotopes, and this feature is a characteristic of U(5) .

Table (5): the ratios of the energy of the level 4_1 and 6_1 to the level 2_1 for $^{106-110,114,116}$Pd isotopes

isotope	E_4^+/E_2^+	E_6^+/E_2^+
^{106}Pd	2.41	4.06
^{108}Pd	2.42	4.09
^{110}Pd	2.47	4.22
^{114}Pd	2.56	4.51
^{116}Pd	2.57	4.58

In this study , the whole Hamiltonian has been used in the IBM-1 program and the values of parameters as shown in table (1), it can be seen that the value of ε is near the value of 2_1^+ while the values of pairing a_0 and a_2 are small as compared to the value of ε so this means that these isotopes have the characteristics of U(5) with a small effect from SU(3) and O(6).

Figure (9) shows the ratios of a_0/ε and a_2/ε and it is clear that the effect of rotational limit is increased with A till it reached near the semi-magic number then it decreases. This behavior because the rotational characteristics and the deformation increased as the number of nucleons approaches the mid-shell. While the effect of O(6) approximately decreases with A till it reaches near the semi-magic number and then it increased and this increment because of the increasing of hole bosons . In fig.(7) one can observe that B(E2) ratios (R≈R'≈1.3 and R''=0) for all isotopes lie between the two limits U(5)[R=R'=R''≈1.7] and O(6) [R=R'≈1.4 and R''=0].

From $Q(2_1^+)$ values it can be concluded that $^{106-110}$Pd isotopes have slightly deformed increasing with A. This is obvious in drawing the energy surfaces for these isotopes, where the form of contour lines in ^{106}Pd (fig. 7) is similar to that of vibrational U(5) one, and the deformation becomes clear in contour lines of ^{108}Pd (fig. 8) , while in ^{110}Pd (fig. 9) the deformation is obvious, and this is due to the effect of rotation. Then circular surfaces returned for, $^{114-116}$Pd(fig.s 10,11) because the vibrational and pairing characteristics become dominant. So it can be imagine the location on the casten's triangle [11] as in fig.(13).

The harmonic behavior of the first three isotopes and then the difference behavior in the last two can be interpreted if it looks at the neutron distribution in the shells. The $^{106-110}$Pd all occupy the sub-level $2d_{5/2}$, while ^{114}Pd occupy $2d_{3/2}$ and ^{116}Pd occupy $3s_{1/2}$.

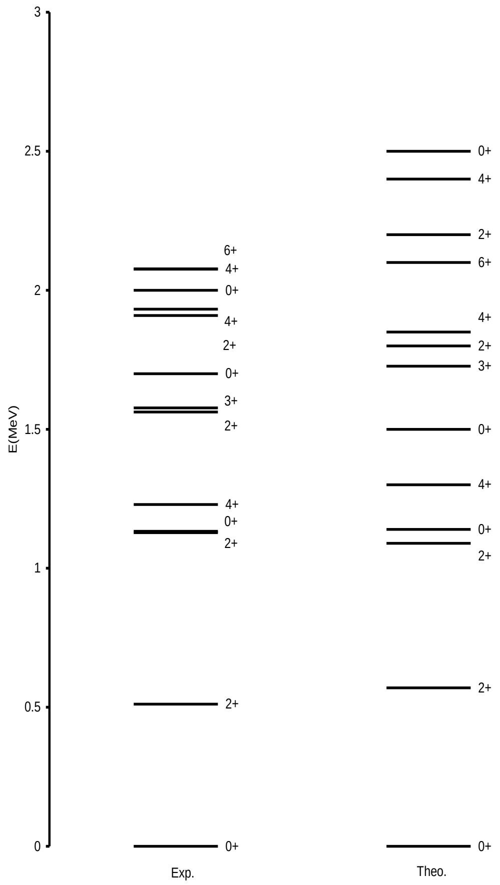

Fig. (1): A comparison between theoretical values of energy levels and the corresponding experimental one for ^{106}Pd[13]

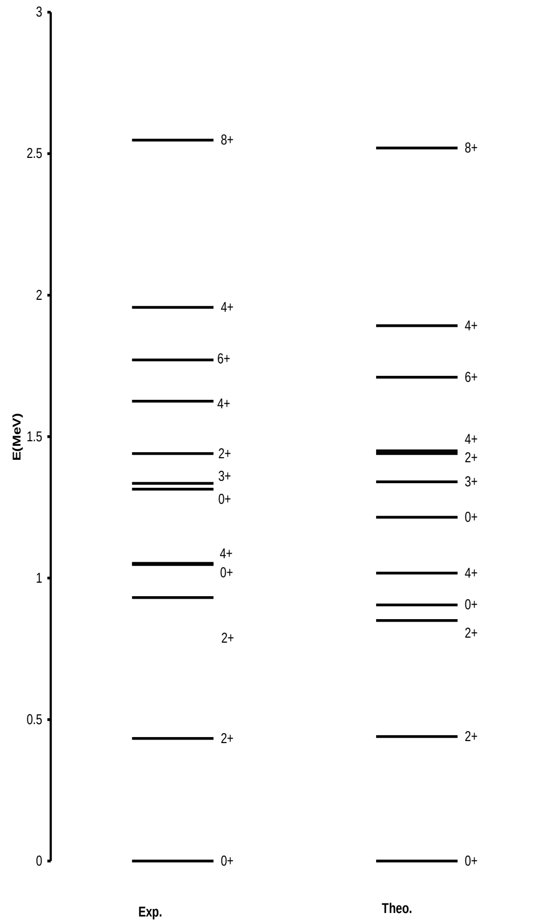

Fig. (2): A comparison between theoretical values of energy levels and the corresponding experimental one for ^{108}Pd[14]

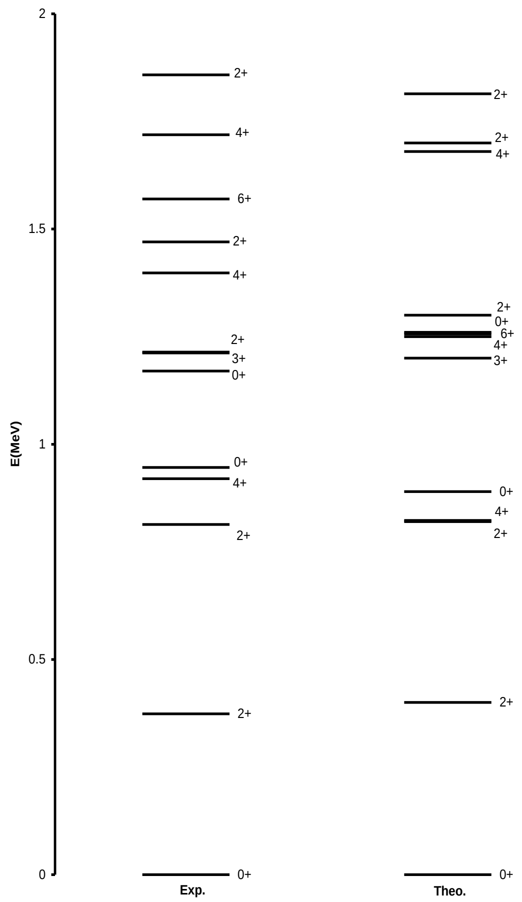

Fig. (3): A comparison between theoretical values of energy levels and the corresponding experimental one for ^{110}Pd[15]

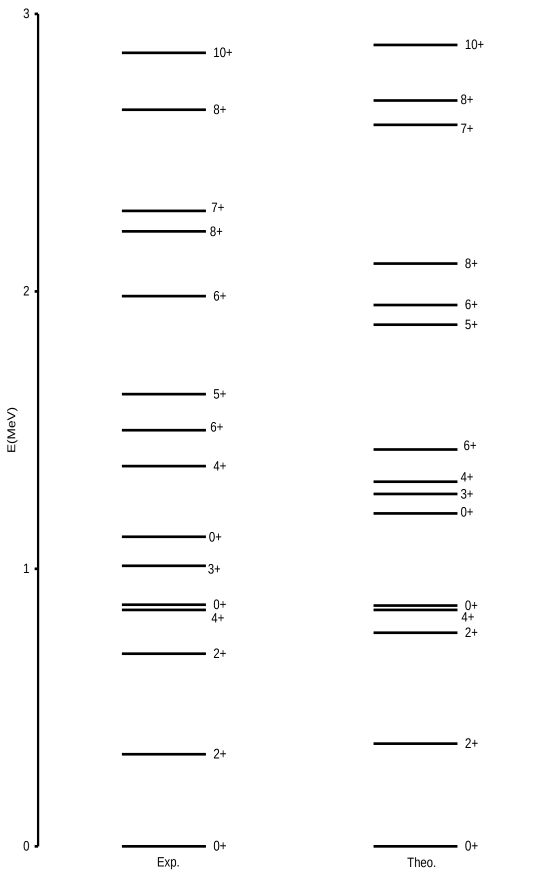

Fig. (4): A comparison between theoretical values of energy levels and the corresponding experimental one for ^{114}Pd[16]

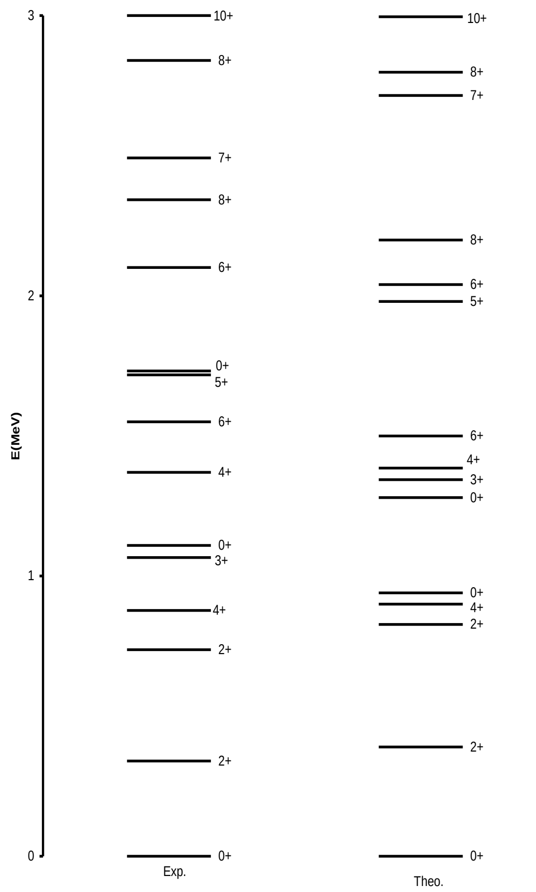

Fig. (5): A comparison between theoretical values of energy levels and the corresponding experimental one for ^{116}Pd[17]

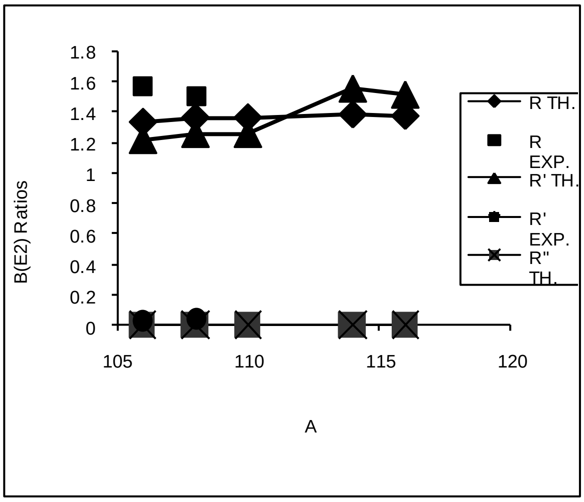

Fig.(6):Exeperemental [13-17] and calculated B(E2) Ratios R,R'and R "of $^{116-106}$Pd isotopes .

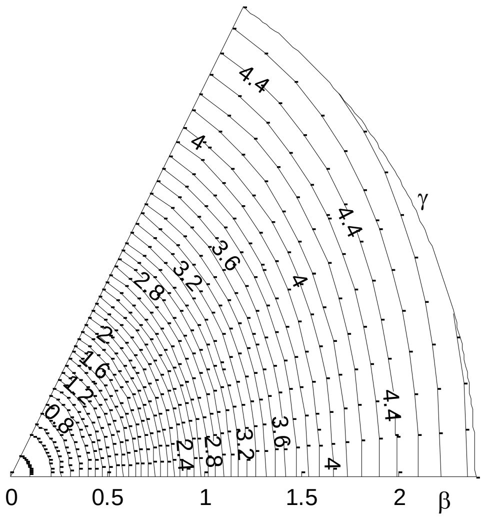

Fig.(7): Equipotential energy surfaces for Pd^{106}.

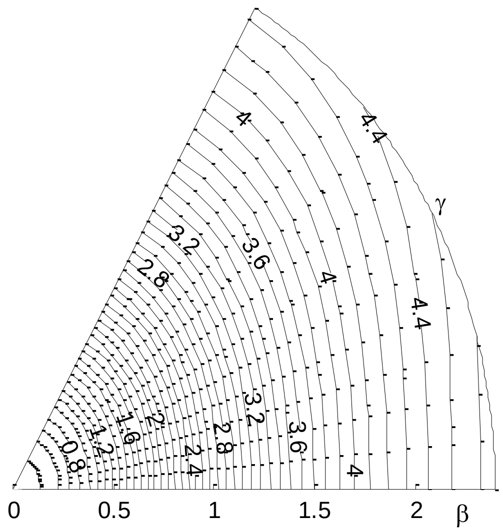

Fig.(8): Equipotential energy surfaces for Pd^{108}

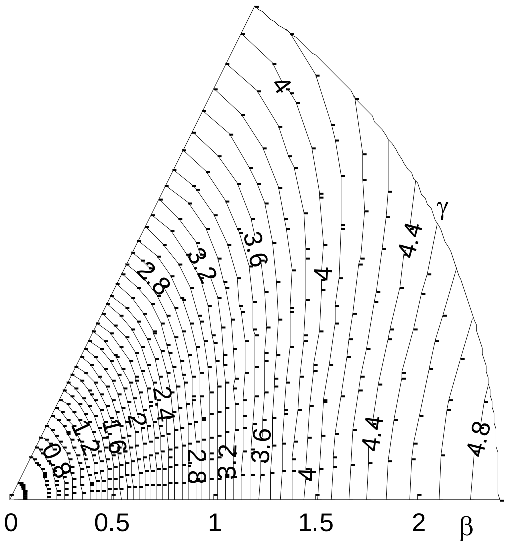

Fig. (9): Equipotential energy surfaces for Pd^{110}

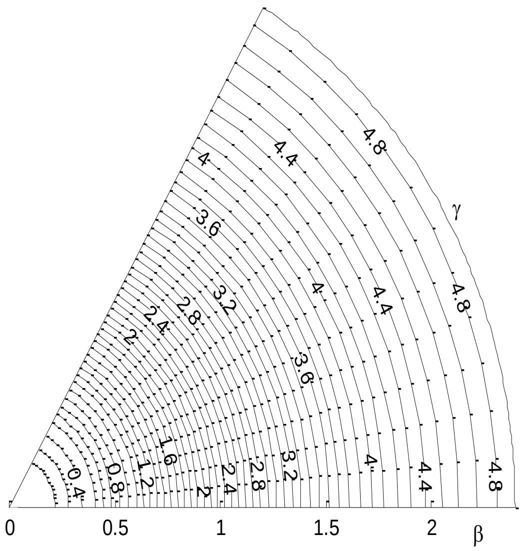

Fig. (10):Equipotential energy surfaces for Pd^{114}.

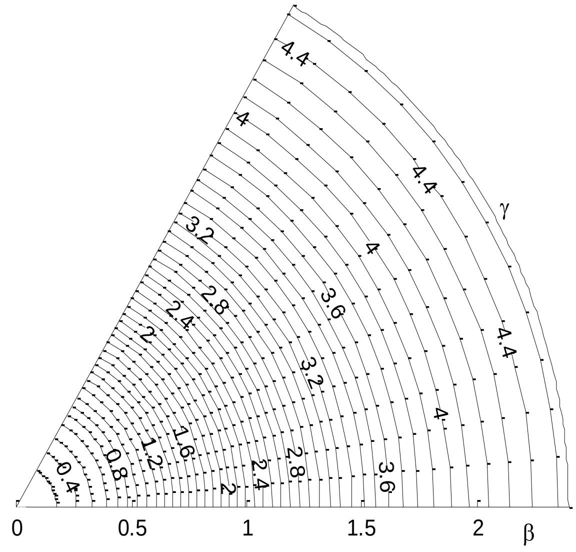

Fig. (11):Equipotential energy surfaces for Pd^{116}.

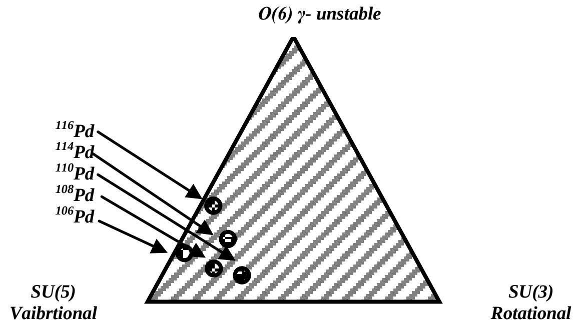

Fig.(13) the location of the $^{106-110,114,116}$Pd nuclei on the Casten's triangle

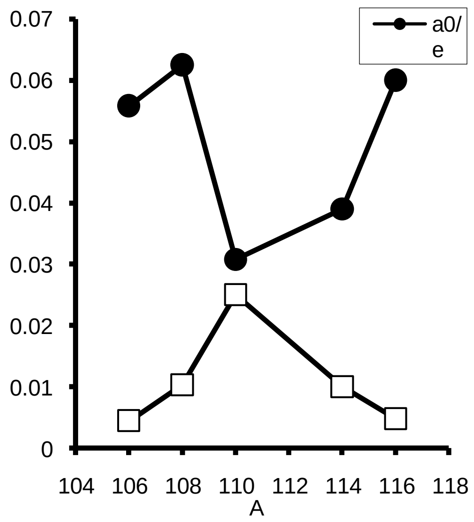

Fig. (12): The ratios of a_0/ε and a_2/ε for $^{106-110,114,116}$Pd nuclei

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Appendix I

```
% program to plot equipotential energy surfaces
clc
[th,r] = meshgrid((0:5:60)*pi/180,0:.2:2.4);
[x,y] = pol2cart(th,r);
es=.032;ed=.375;f1=.026;f2=.018;f3=.022;f4=.002;n=9;
%es=.016;ed=.492;f1=.027;f2=.009;f3=.003;f4=.005;n=8;
%es=.01;ed=.638;f1=.028;f2=.006;f3=-.004;f4=.006;n=7;
z=n./(1+r.^2).*(es+ed.*r.^2)+n.*(n-1)./(1+r.^2).^2
.*(f1.*r.^4+f2.*r.^3.*cos(3.*th)+f3.*r.^2+f4);
disp(z)
figure('Color',[1 1 1]);
axes('YColor',[1 1 1],'xlim',[0 2.4]);
axis square
axis equal
hold on
v=.2:.2:6;
v1=.4:.4:6;
[c,h] = contour(x,y,z,v,'k');
clabel(c,h,v1);
text(2.1,1.25,texlabel('gamma'))
text(2.2,-.1,texlabel('beta'))
plot ([0 2.4*cos(pi/3)] ,[0 2.4*sin(pi/3)],'k')
arc = rsmak('arc',2.4,[0;0],[0 pi/3]);
fnplt(arc,'k',1), axis square
box('off');
hold off
```