Estimation the 3-D electrical characteristics of an electrostatic precipitator using FDM and MATLAB

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Abstract:

This paper used a new program implemented in MATLAB to calculate the electric potential, electric field and charge density distributions in three dimensions (3-D) of an electrostatic precipitator for wire-plate type, the principle of the program is Finite Difference Method (FDM) which is applied to solve Poisson's equation and the relation between electric potential and electric field with suitable boundary conditions and programmed in MATLAB. In the present study the software's appears both accuracy results and the computation time required for solving the problem is less. Also the results obtained through their implementation in computer programs are better than FDM or FEM techniques when it use without combined of other technique.

Keywords: Electrostatic precipitator, electric field distribution, numerical simulation, MATLAB program.

تخمين الخصائص الكهربائية بثلاث ابعاد للمرسب الاليكتروستاتيكي باستخدام FDM and MATLAB

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قسم الفيزياء- كلية العلوم-جامعة كربلاء

الخلاصة:

استخدم في هذا البحث برنامج جديد مطبق بالمتلاب لحساب توزيعات الجهد الكهربائي ، المجال الكهربائي وكثافة الشحنة للمرسب الاليكتروستاتيكي نوع سلك لوح.ان مبدا البرنامج اعتمد على تقنية Finite Difference في حل معادلة بوازون والعلاقة بين الجهد الكهربائي والمجال الكهربائي تحت شروط حدودية ملائمة وتم برمجة المعادلات بالماتلاب .اظهرت البرامج التي استخدمت في هذة الدراسة نتائج دقيقة وزمن الحساب المطلوب اقل في حل المشكلة. ايضا النتائج التي تم الحصول عليها بتطبيق هذه البرامج الحاسوبية افضل من تقنيات Finite Element or Finite Difference عندما تستخدم بدون الاشتراك مع اي تقنية اخرى.

الكلمات المفتاحية: ثالوسيانين، درجة حرارة القاعدة (الأرضية)، الحجم الحبيبي، التبلور، قضبان نانوية.

1. Introduction

The electrostatic precipitator ESP is an industrial technology serving an industrial plant such as cement production, chemical processing, electric power plants and domestic cleaning air [1].which use corona discharge for charging electrically the dust particles (resulted from combustion) that cross it. The collecting chamber of the electrostatic precipitator includes a series of equidistant discharge wires placed in the middle between two parallel collecting plate's The development of the computing performance of ESP play an important role in predicting the design and optimization stages of the equipment or of the installation in the laboratory and test bench in the virtual environment. In order to get high efficiency of ESP it is appropriate to develop some software packages, a different numerical method can be used to estimate the electric field distribution of an electrostatic precipitator [2] such as Finite Difference Method FDM, Finite Element Method FEM, Finite Volume Method FVM, Boundary Element Method BEM. The Electric field and charge density distribution in the duct ESP are computed numerically [3] using the commercially available solver FEM Comsol Multi physics ,the numerical solution provides details insight into a number of basic phenomena in ESP. Both FDM and FEM methods [4] are used to find two dimensional electric field distributions with suitable boundary conditions using program MATLAB.The basic phenomena in ESP, the variations of the velocity and Electric fields in electro hydrodynamic (EHD) flow in a positive single-wire electrostatic precipitator are studied under both laminar and turbulent flow conditions [5], the different values of the voltage, charge density and flow streamlines are numerically simulated. Numerical solution used to obtain optimizing device characteristics through control of the inner pump electric field profile of an electrostatic air pump. The electrostatic that contains a sharp-edge-to-parallel-plane electrode geometry with unipolar positive corona is chosen to generate linear electric field distribution [6]. This paper aims to Find electric field Distributions, which are inevitable tool in various electricity concerned technologies, in particular, for analyzing discharge phenomenon and designing high voltage equipment**.** This study is applied for the determination of the potential and electric field distributions inside an electrostatic precipitator. The calculated results, obtained using FDM and our own programs created in MATLAB. The data obtained from the technique were accurate and the time required for solving the problem is less and get a good computation, in comparison with the results obtained using each method separately.

2. Numerical Solution

In order to solve the Poisson's and current continuity equations for calculating the electric field and charge density distribution .boundary conditions are needed for the electric potential u Dirichlet conditions are used ,the ground collecting plates gives u=0 and the potential at discharge wire is taken as the ESP operating voltage with negative polarity . The electric potential distribution in problem domain is estimated at all points in the grid using Cooperman's equation [7] which is depending as first boundary conditions.

$$u(x,y) = u_\circ \frac{\sum_{m=-w}^{w} ln\left\{\frac{cosh\left[\frac{\pi(y-2mS_y)}{2S_x}\right] - cos\left(\frac{\pi x}{2S_x}\right)}{cosh\left[\frac{\pi(y-2mS_y)}{2S_x}\right] + cos\left(\frac{\pi x}{2S_x|}\right)}\right\}}{\sum_{m=-w}^{w} ln\left\{\frac{cosh\left(\frac{\pi m S_x}{S_x}\right) - cos\left(\frac{\pi r}{2S_x}\right)}{cosh\left(\frac{\pi m S_x}{S_x}\right) + cos\left(\frac{\pi r}{2S_x}\right)}\right\}} \qquad (1)$$

Where:

u_0 = the applied voltage on the wire

x, y = the coordinate position in meters measured from the discharge wire as origin

S_x = wire - plate spacing

S_y = half wire-wire spacing

m = the number of discharge wires.

The boundary condition is also needed for charge density on the discharge wireρ_0. Calculation of space charge density at the discharge wire is used as another boundary conditions as [8] approximating the plasma region surrounding the electrical discharge as cylindrical can be evaluated in terms of estimation average current density Jp.

$$\nabla.(\rho E\, b) = 0$$

$$(\rho b)\nabla.E + b(E.\nabla\rho) + \rho(E.\nabla b) = 0$$

Since the mobility of charge carriers is assumed to be constant [11], the last term will be zero, rewriting the equation

$$(\rho b)\nabla.E + b(E.\nabla\rho) = 0$$

The above equation can be further expand [12] as rewriting in the form.

$$(\rho b)\, \rho/\varepsilon_\circ + b\left(E_x\frac{\partial\rho}{\partial x} + E_y\frac{\partial\rho}{\partial y}\right) = 0$$

$$\left(\frac{\rho^2}{\varepsilon_\circ} + E_x\frac{\partial\rho}{\partial x} + E_y\frac{\partial\rho}{\partial y}\right)_{i,j} = 0 \qquad (7)$$

Using finite difference method

$$\frac{\partial\rho_{(i,j)}}{\partial x} = \frac{\rho_{(i,j)}-\rho_{(i-1,j)}}{\Delta x} \quad and \quad \frac{\partial\rho_{(i,j)}}{\partial y} = \frac{\rho_{(i,j)}-\rho_{(i,j-1)}}{\Delta y} \qquad (8)$$

Substituting Eq.(8) in Eq.(7)

$$\frac{{\rho_{(i,j)}}^2}{\varepsilon_\circ} + E_x\frac{\rho_{(i,j)}-\rho_{(i-1,j)}}{h} - E_y\frac{\rho_{(i,j)}-\rho_{(i,j-1)}}{h} = 0$$

$$\frac{\rho_{(i,j)}^2}{\varepsilon_\circ} + \frac{E_x\rho_{(i,j)}-E_x\rho_{(i-1,j)}+E_y\rho_{(i,j)}-E_y\rho_{(i,j-1)}}{h} = 0$$

$$\frac{h}{\varepsilon_\circ}\rho_{(i,j)}^2 + (E_x+E_y)\rho_{(i,j)} - (E_x\rho_{(i-1,j)} + E_y\rho_{(i,j-1)}) = 0 \qquad (9)$$

From Eqs. (2) And (9), $\rho(i,j)$ at any point in the grid can be obtained

Poisson's equation relates the electric field intensity E to the space charge density ρ.

The continuity (Eq. (5)), expanded using vector algebra, is given by [10],

A new MATLAB program employed for determined the electric potential, electric field and charge density distributions in any point in the grid for ESPs domain.

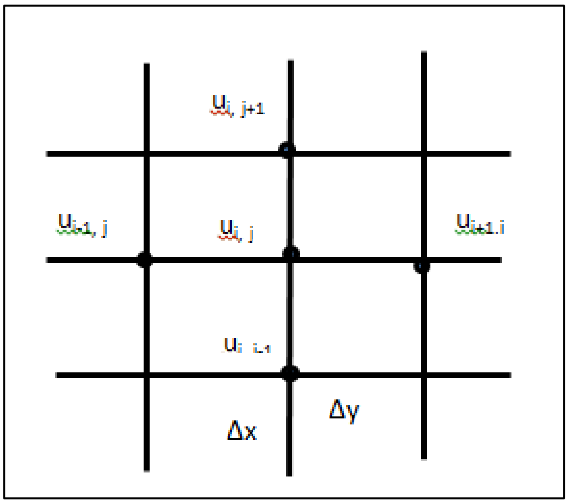

Fig.(2) Finite difference solution region

Due to the symmetry, see Fig. (1) Only a quarter section of the Electrostatic precipitator channel needs to be modeled. The distributions of electric potential, electric field and charge density are evaluated at area (15×15) mm^2 for uniform mesh which contains (60×60) points. This number of points is sufficient to get high efficiency of ESP.

3. Results and Discussion:

The calculations were done using a computer program. The problem domain was divided into 80x80 and the solution of the Eq's.(1,2,4,9,15) was obtained by a new program in MATLAB. Assumed initial voltage at the wire, wire radius, half wire-to-wire spacing, wire to plate spacing, measured (or assumed) current density at the plate, effective mobility of charge carriers, air density factor at NTP and roughness factor of the wire are the required input data for dust free simulation.

as

$$\nabla^2 u = -\frac{\rho}{\varepsilon_\circ} \qquad (10)$$

Where, ρ is the charge density and $\varepsilon_\circ$is Permeability of free space

In two dimensions equation (10) becomes:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\frac{\rho}{\varepsilon} \qquad (11)$$

In applying the methods of finite difference (central difference method) equation (5) become

$$\frac{\Delta^2 u}{\Delta^2 x} = \frac{u_4 + u_2 - 2u_0}{(\Delta x)^2} \quad \text{and} \quad \frac{\Delta^2 u}{\Delta^2 y} = \frac{u_1 + u_3 - 2u_0}{(\Delta y)^2}$$

(12)

Substituting Eq. (12) in Eq.(11) and letting $(\Delta x = \Delta y = h)$ obtained

$$\frac{u_4 + u_2 - 2u_0}{h^2} + \frac{u_1 + u_3 - 2u_0}{h^2} = -\frac{\rho}{\varepsilon_\circ} \qquad (13)$$

$$u_\circ = \frac{1}{4}\left[(u_1 + u_2 + u_3 + u_4) + \frac{h^2}{\varepsilon_\circ}\rho\right]$$

(14)

By discretizing equation (14) using Finite Differences around an interior node, (i, j), for a uniform mesh both on the x-axis direction (of step h) and on the y-axis direction (of step h) [9],

$$u(i,j) = \frac{1}{4}\left[u_{i,j+1} + u_{i-1,j} + u_{i,j-1} + u_{i+1,j} + \frac{h^2\rho}{\varepsilon_\circ}\right] \qquad (15)$$

From equation (15) and see fig.2 it is clear that voltage at the central node u(i, j) is the mean of the voltages at the other four nodes, the values of potential at all point in the grid can be calculated by repeating the central node at another point by iteration method. The difference between the values of electric potential calculated from Eq.(1) and Eq.(15) must equal one volt or less ,this condition is satisfied the numerical solution.

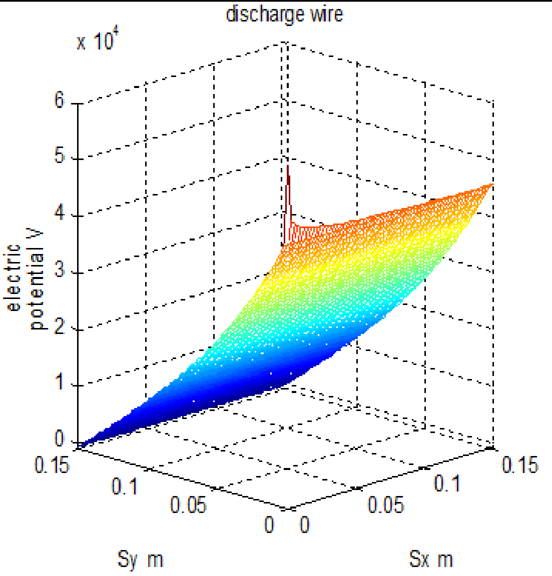

Fig. 3a electric potential distribution for quarter section of ESP, V=60kV,J=5×10^{-4} A/m^2 with 60×60 cells

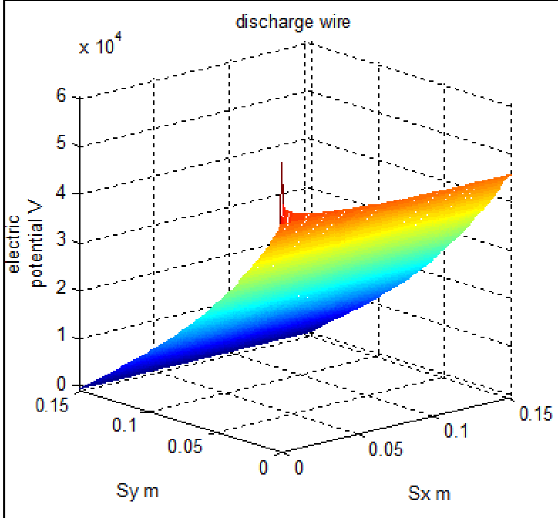

Fig. 3b electric potential distribution for quarter section of ESP,V=60kV,J=5×10^{-4}A/m^2 with150×150 cells

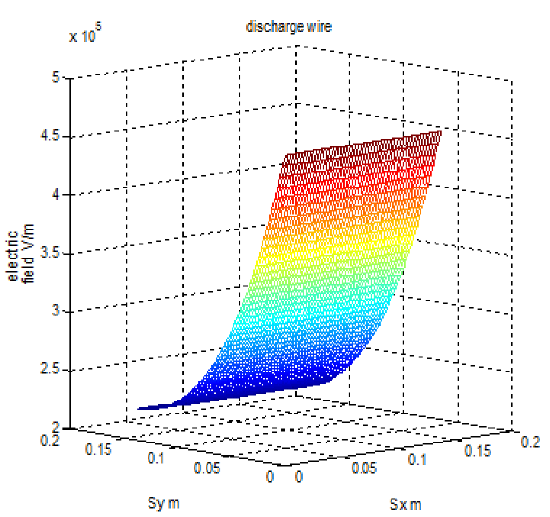

Fig. 4a electric field distribution for quarter section of ESP,V=60kV,J=5×10^{-4}A/m^2 with60×60 cells

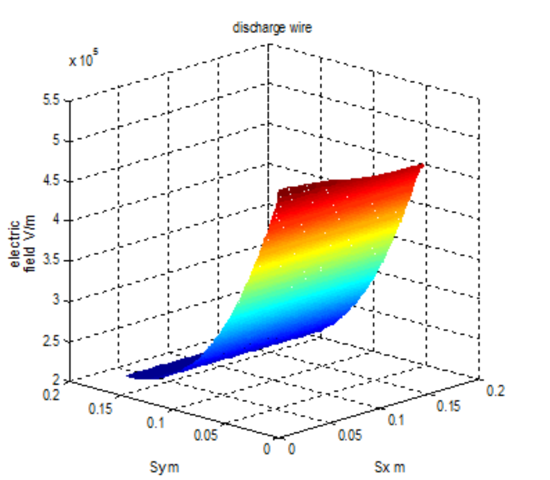

Fig.4b electric field distribution for quarter section of ESP ,V=60 kV ,J=5×10^{-4} A/m^2 150×150 cells

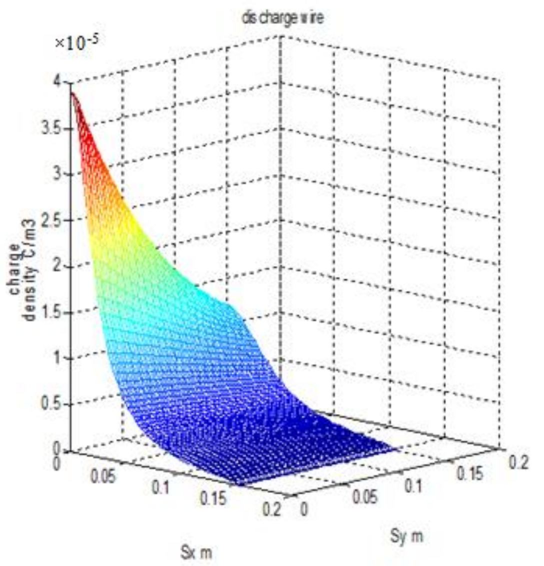

Fig. 5a charge density distribution for quarter section of ESP,V=60kV,J=5×10⁻⁴A/m² with60×60 cells

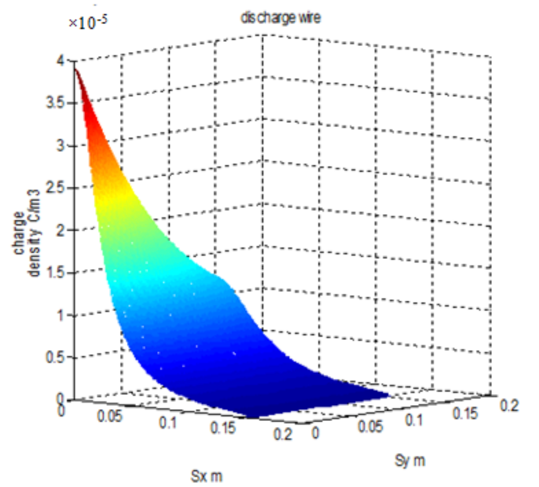

Fig. 5 b charge density distribution for quarter section of ESP,V=60kV ,J=5×10⁻⁴ A/m² 150×150 cells

The point (0,0)represent the position of discharge wire. The high values of potential, electric field and charge density at the discharge wire decrease slowly toward the collecting plates. This is clear in figures 3,4,5. Fig.(6,7,8) show the distribution of electric potential ,electric field and charge density in case of applied voltage 90kVand current density

5×10^{-3} A/m^2, it is clear that the high voltage lead to increasing in electric field and charge density. The electric field distribution in fig.7 show the same behavior of fig.4 but the charge density distribution in fig.8 show different behavior as in fig.5,it is evident that the charge distribution round the discharge wire was uniform and equal to zero in the bottom area or near the collecting plates, this result gives a certain guarantee to charging all the ESP.

The distribution of electric potential, electric field and charge density which have been shown in figures (3,4,5) for wire–plate electrostatic precipitator that has geometrical parameters such as distance between wire-plate Sx=0.15m,half distance between wire-wire Sy=0.15m,the radius of discharge wire a=0.001m, mobility of charge carrier=2×10^{-4} (m^2/V.s) in case of dust free conditions and the applied voltage on the discharge wire is 60kV. Fig.3 shows electric potential distribution of the problem domain which has area (15×15 cm^2), fig.3a refers to potential distribution in problem domain with 60×60 points in the grid while fig.3b show the potential at 150×150 points in the grid, For a suggestive program used in this paper it was aimed to increase the number of points in the grid when it compares with another technique, the results at high precision was 13343 nodes in the entire domain in finite element method (FEM) [2] this number of points was less than the number obtained using program created in MATLAB. The same results were obtained in fig.4b for electric field distribution when it compared with fig.4a, also the comparison in figures 5b and 5a for charge density distribution gives the similar behavior .The charging of toxic particle in ESP required potential distribution at large number of points in the grid of the problem domain(fine grid). Finally dividing the domain of ESP at large number lead to high efficiency.

There are many parameters that are affecting on efficiency of ESP such as radius of discharge wire, applied voltage and the radius of particle which must be collecting. If the potential distributed at large number of points in grid of problem domain all the particle with small radius about 1 micron or less can be charging and collecting.

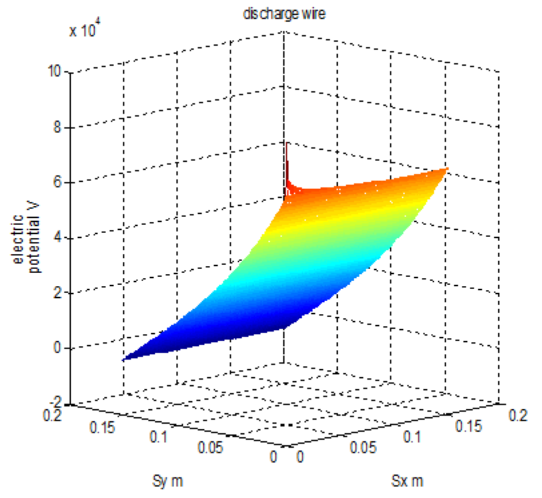

Fig.6 electric potential distribution for quarter section of ESP ,V=90 kV ,J=5×10^{-3} A/m^2 150×150 cells

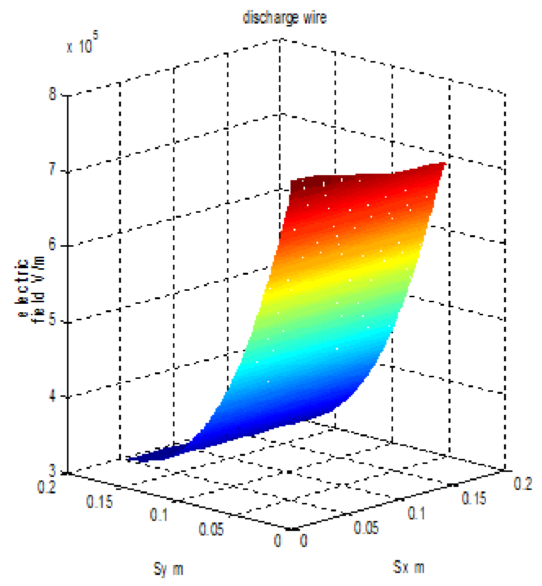

Fig.7 electric field distribution for quarter section of ESP ,V=90 kV ,J=5×10^{-3} A/m^2 150×150 cells

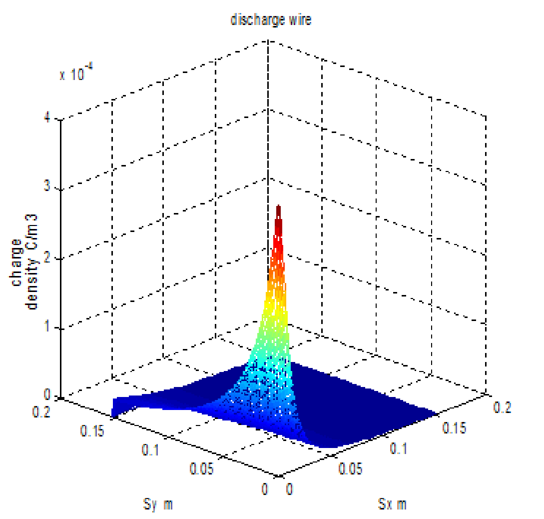

Fig.8 charge density distribution for quarter section of ESP ,V=90 kV ,J=5×10^{-3} A/m^2 150×150 cells

4. Conclusions

A numerical model can be applied for geometrical configurations which is different in geometrical and operation parameters such as wire –plate distance, half distance between wire to wire and another .The model describes the distribution of electric potential, electric field and charge density at high number of points in the grid of problem domain when it compare with other technique, because of high number of point the collection efficiency of ESP become more accuracy than the collection efficiency calculated by prior technique such as finite difference method (FDM), finite element method (FEM).

5. Reference

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