Nuclear Shell Model Application To Calculate Energy Levels For ($^{14}{}_{8}O_{6}$,$^{14}{}_{6}C_{8}$) Nuclei

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ABSTRACT

In this paper, energy levels were calculated for two nuclei ($^{14}{}_{8}O_{6}$,$^{14}{}_{6}C_{8}$) by applying the nuclear shell model by using modified surface delta interaction (MSDI).The (MSDI) succeeded in energy levels calculation for the configuration (hole-hole) ,this case facilitated the calculation of many holes outside closed shell by using the configuration (hole-hole). Computer programs were used to calculate important coefficients involved mainly in calculating energy levels include Clebch - Gordan , Racha coefficient and matrix element for two bodies by using modified surface delta interaction. Comparison the current values with the experimental values , it was found current values were in perfect agreement with the experimental values .

تطبيق أنموذج القشرة النووي لحساب مستويات الطاقة للنواتين ($^{14}{}_{8}O_{6}$,$^{14}{}_{6}C_{8}$)

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الخـلاصـــة

تم حساب مستويات الطاقة لنواتي ($^{14}{}_{8}O_{6}$,$^{14}{}_{6}C_{8}$) بتطبيق أنموذج القشرة النووي وباستخدام جهد دلتا السطحي المحور (Modified Surface Delta Interaction) الذي نجح في حساب مستويات الطاقة للترتيب (فجوة - فجوة) . ،لقد استخدمت برامج حاسوب في حساب المعاملات المهمة التي تدخل بشكل أساسي في حساب مستويات الطاقة ومنها معاملات كلبش- كوردن ومعاملات راكاه وعنصر المصفوفة لإثنين من الجسيمات بإستخدام جهد دلتا السطحي المحور ، تمت مقارنة النتائج الحالية مع النتائج العملية وكان هناك تطابقاً مقبولاً جداً فيما بينهما.

Introduction

Nuclear shell model has played an indispensable role in the study of the nuclear structure, since it was conceived by Mayer and Jensen . The shell model has several important and basic feature such as the independence of model assumption ,the usage of realistic nucleon- nucleon interaction, and the common Hamiltonian for various types eigenstates and different nuclei [1] . For many applications these complex configuration can be taken into account exactly by the digitalization of Hamilton matrix [2]. In this work we study the energy levels for all allowable total angular momentum and the parity for mirror nuclei ($^{14}_{8}O_6$,$^{14}_{6}C_8$) using modified surface delta interaction MSDI .

Theory

Pandya succeeded in explaining the particle - hole theorem which relates the spectrum of a nucleus with one nucleon in each of the single-particle level j' and j to the spectrum of the nucleus with one particle in j' and another hole in j [3] . This relationship will the be used to correlate the spectrum observed in various nuclei .

The hole-hole interaction energy in terms of the particle-particle matrix element which represented energy level is given by[4]:

$$E(j_1 j_4{}^{-1}, j_3 j_2{}^{-1}) = -\frac{1}{2}(-1)^{j_1+j_2+j_3+j_4}$$
$$\sum_J (2J+1) W(j_1 j_2 j_4 j_3; JK) * \{ E_{J0}(j_1 j_2; j_3 j_4)$$
$$+ E_{J_1}(j_1 j_2; j_3 j_4) \} \text{.............(1)}$$

Where $W(j_1 j_2 j_4 j_3; JK)$ is Racha coefficient [5]

J angular momentum ,

$E_{J0}(j_1 j_2; j_3 j_4)$ and $E_{J1}(j_1 j_2; j_3 j_4)$ are two body matrix element which given by :

$$E_{J0}(j_1 j_2; j_3 j_4) = < j_a j_b \left| V^{MSDI}(1,2) \right| j_a j_b >_{J0}$$
$$\text{................(2)}$$

$$E_{J1}(j_1 j_2; j_3 j_4) = < j_a j_b \left| V^{MSDI}(1,2) \right| j_a j_b >_{J1}$$
$$\text{...............(3)}$$

A typical two body matrix element by using modified surface delta interaction is [3, 6,7]:

$$< j_a j_b \left| V^{MSDI}(1,2) \right| j_a j_b >_{JT} = -A_T \frac{(2j_a+1)(2j_b+1)}{2(2J+1)(1+\delta_{ab})} *$$
$$\{ (j_b -\frac{1}{2} j_a \frac{1}{2} | J0)^2 \left[1-(-1)^{l_a+l_b+J+T} \right] +$$
$$(j_b \frac{1}{2} \;\; j_a \frac{1}{2} \;\; | J1)^2 \left[1+(-1)^T \right] \}$$
$$+ \left[2T(T+1)-3 \right] + B + C \text{............(4)}$$

$$\left| < j_a j_b \left| V^{MSDI}(1,2) \right| j_c j_d >_{JT} \right| = R$$
$$\sqrt{< j_a j_b \left| V^{MSDI} \right| j_a j_b >_{JT} < j_c j_d \left| V^{MSDI} \right| j_c j_d >_{JT}}$$
$$\text{.................(5)}$$

Where $(j_b \frac{1}{2} \;\; j_a \frac{1}{2} \;\; | J1)$ is Clebch - Gordan Coefficient [7,8,9], T isospin, A_T , B and C are parameters obtained from fitting to experimental data in various mass region, eq(4) the matrix element

Where R=1

The matrix element of Hamiltonian are[4,10,11] given by:

$$\langle H \rangle_{11} = e_{\rho_1} + e_{\rho_2} + E(j_1 j_4{}^{-1}; j_1 j_4{}^{-1})$$
$$\text{.......................(6)}$$

$$\langle H \rangle_{22} = e_{\rho_1} + e_{\rho_2} + E(j_3 j_2{}^{-1}; j_3 j^{-1}{}_2)$$
$$\text{..........................(7)}$$

$$\langle H\rangle_{12}=\langle H\rangle_{21}=E(j_1 j_4^{-1};j_3 j^{-1}{}_2)$$
........................(8)

Where e_{ρ_1} *and* e_{ρ_2} are single particle energies.
The energy levels are obtained as the eigenvalues of the Hamiltonian matrices .

Calculations and Results

1. ^{14}C Nucleus

The nucleus $^{14}{}_6C_8$ has 8 neutrons and 6 protons ,its neutrons fill the single particle state up to the magic shell closure at N=8 while 2 proton are less than the magic shell closure at Z=8. These protons (holes) occupy the model space ($0p_{3/2}$ $0p_{1/2}$) taking $^{16}{}_8O_8$ as an inert core. To describe the energy levels we have the single particle energy which is[12] :

$e_{0P_{1/2}}$ =21.3887 Mev

$e_{0P_{3/2}}$ =16.731 Mev

The values of all allowable angular momentum and parity are:

$$j^{\pi}=0^{+},1^{+},2^{+}$$

By eq(4) we get two body matrix element by using (MSDI)

$< j_a j_b|V^{MSDI}(1,2)|j_a j_b >_{JT}$

in which the parameters are :

A=1.82, B=1.95 and C=0.0

Were used as shown in table (2). By applying the matrix element resulting above and the Racha coefficient value in eq(1)
the matrix element for (hole-hole) configuration was obtained as shown in table (3).Using matlab 2011, we can find the the energy levels by eqs (6,7,8) in which we found eigenvalues . The eigenvalues explicated , the energy levels of all allowable angular momentum were obtained in regard to the ground state as shown in table (4) and figure (1).

Table (1): Values of Racha coefficients

$W(j_1 j_2 j_4 j_3;JK)$	**Value**
$W(\frac{3}{2}\frac{3}{2}\frac{3}{2}\frac{3}{2};00)$	**-0.25**
$W(\frac{3}{2}\frac{3}{2}\frac{3}{2}\frac{3}{2};01)$	**0.24999997**
$W(\frac{3}{2}\frac{3}{2}\frac{3}{2}\frac{3}{2};02)$	**-0.25**
$W(\frac{3}{2}\frac{3}{2}\frac{3}{2}\frac{3}{2};03)$	**0.24999997**
$W(\frac{3}{2}\frac{3}{2}\frac{3}{2}\frac{3}{2};20)$	**-0.25**
$W(\frac{3}{2}\frac{3}{2}\frac{3}{2}\frac{3}{2};21)$	**0.05**
$W(\frac{3}{2}\frac{3}{2}\frac{3}{2}\frac{3}{2};22)$	**0.1500002**
$W(\frac{3}{2}\frac{3}{2}\frac{3}{2}\frac{3}{2};23)$	**0.04999994**
$W(\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2};00)$	**-0.4999994**
$W(\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2};01)$	**0.5**
$W(\frac{1}{2}\frac{1}{2}\frac{3}{2}\frac{3}{2};01)$	**-0.35355338**
$W(\frac{1}{2}\frac{1}{2}\frac{3}{2}\frac{3}{2};02)$	**-0.35355338**
$W(\frac{3}{2}\frac{1}{2}\frac{1}{2}\frac{3}{2};20)$	**0**
$W(\frac{3}{2}\frac{1}{2}\frac{1}{2}\frac{3}{2};21)$	**0.24999997**
$W(\frac{3}{2}\frac{1}{2}\frac{1}{2}\frac{3}{2};22)$	**0.1**
$W(\frac{3}{2}\frac{3}{2}\frac{1}{2}\frac{3}{2};21)$	**0.22360679**
$W(\frac{3}{2}\frac{1}{2}\frac{1}{2}\frac{3}{2};10)$	**0**
$W(\frac{3}{2}\frac{1}{2}\frac{1}{2}\frac{3}{2};11)$	**-0.08333331**

Table (2): Values of two body matrix elements for ${}^{14}_{6}C_8$ using (MSDI)

J	T	$<j_a j_b \vert V^{MSDI}(1,2) \vert j_a j_b>_{J,T}$	VALUE
0	1	$<\frac{1}{2}\frac{1}{2}\vert V^{MSDI}(1,2)\ \vert\frac{1}{2}\frac{1}{2}>$	96.79632724
0	0	$<\frac{1}{2}\frac{1}{2}\vert V^{MSDI}(1,2)\ \vert\frac{1}{2}\frac{1}{2}>$	-50.1204488
0	1	$<\frac{3}{2}\frac{3}{2}\vert V^{MSDI}(1,2)\ \vert\frac{3}{2}\frac{3}{2}>$	-87.2483
0	0	$<\frac{3}{2}\frac{3}{2}\vert V^{MSDI}(1,2)\ \vert\frac{3}{2}\frac{3}{2}>$	32.4807484
1	1	$<\frac{3}{2}\frac{1}{2}\vert V^{MSDI}(1,2)\ \vert\frac{3}{2}\frac{1}{2}>$	-89.77626
1	0	$<\frac{3}{2}\frac{1}{2}\vert V^{MSDI}(1,2)\ \vert\frac{3}{2}\frac{1}{2}>$	43.59287748
2	1	$<\frac{3}{2}\frac{3}{2}\vert V^{MSDI}(1,2)\ \vert\frac{3}{2}\frac{3}{2}>$	-61.33215
2	0	$<\frac{3}{2}\frac{3}{2}\vert V^{MSDI}(1,2)\ \vert\frac{3}{2}\frac{3}{2}>$	33.29509345
2	1	$<\frac{3}{2}\frac{1}{2}\vert V^{MSDI}(1,2)\ \vert\frac{3}{2}\frac{1}{2}>$	90.388766
2	0	$<\frac{3}{2}\frac{1}{2}\vert V^{MSDI}(1,2)\ \vert\frac{3}{2}\frac{1}{2}>$	-30.84607621
0	1	$\vert <\frac{1}{2}\frac{1}{2}\vert V^{MSDI}(1,2)\ \vert\frac{3}{2}\frac{3}{2}> \vert$	1.5144821
0	0	$\vert <\frac{1}{2}\frac{1}{2}\vert V^{MSDI}(1,2)\ \vert\frac{3}{2}\frac{3}{2}> \vert$	1.961846361
2	1	$\vert <\frac{3}{2}\frac{3}{2}\vert V^{MSDI}(1,2)\ \vert\frac{3}{2}\frac{1}{2}> \vert$	0.41100823

Table (3) Values of matrix element for (hole-hole) configuration for ${}^{14}_{6}C_8$ using (MSDI)

J^{+}	$E(j_1 j_4^{-1}; j_3 j_2^{-1})$	Value (MeV)
0	$E(\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2})$	-14.02763534
0	$E(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{3}{2})$	-10.953563632
0	$E(\frac{1}{2}\frac{1}{2};\frac{3}{2}\frac{3}{2})$	1.0501221
2	$E(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{3}{2})$	-4.06353732
2	$E(\frac{3}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2})$	-7.44283533
2	$E(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{1}{2})$	0.2187112
1	$E(\frac{3}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2})$	-9.67153533

Table (4): Comparison between theoretical and experimental values for ${}^{14}_{6}C_8$ with respect to ground state (-22.3381 MeV) using (MSDI)

J^{π}	Energy(MeV)	
	Pre.Res	Exp.Res[13]
0^{+}	0	0
1^{+}	6.1109	6.0938
0^{+}	6.5837	6.5894
2^{+}	7.0313	7.012
2^{+}	8.3706	8.3179

2. ^{14}O Nucleus

The nucleus $^{14}{}_{8}O_{6}$ has N= 6 neutron (2 neutron less of the magic closed shell in N= 8) and 2 protons working as magic closed shell .These neutrons (holes) occupy the model space($0p_{3/2}$ $0p_{1/2}$) taking $^{16}{}_{8}O_{8}$ as an inert core . To describe the energy levels we have the single particle energy which are [12] .

$e_{0P_{1/2}}$ =15.6638 Mev

$e_{0P_{3/2}}$ =9.4875 Mev

The values of all allowable angular momentum and parity are:

$$j^{\pi} = 0^{+}, 1^{+}, 2^{+}$$

By applying the two body matrix element by using eq (4)as shown in table (5),where the parameters used are:

A= 1.721 , B= 1.87 and C= 0.0

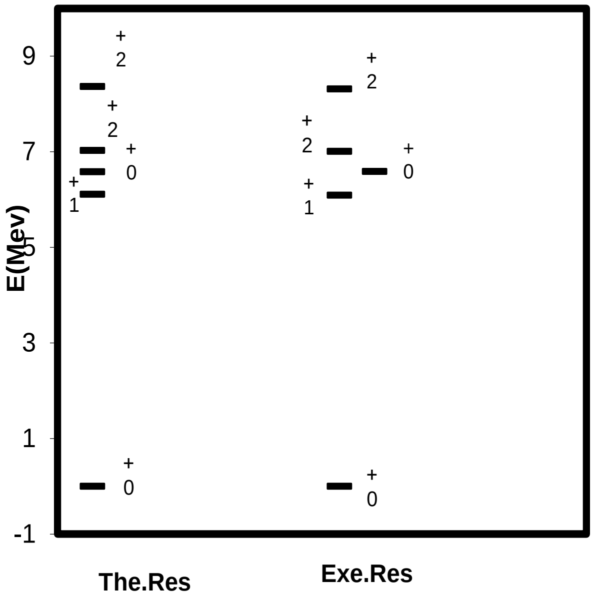
Apply the resulting above result and the Racha coefficients values in eq (1) the matrix element for (hole – hole) configuration was obtained as shown in table (6) . using matlab 2011, we can find the energy levels by eqs (6,7,8) in which we found eigenvalues . Though the eigenvalues explicated , the energy levels of all allowable were obtained in regard to the ground state as shown in table (7) and figures (2).

Table (5): Values of two body matrix elements for $^{14}{}_{8}O_{6}$ using (MSDI)

J	T	$< j_a j_b \vert V^{MSDI}(1,2) \vert j_a j_b >_{J,T}$	Value(MeV)
0	1	$< \frac{1}{2}\frac{1}{2} \vert V^{MSDI}(1,2) \vert \frac{1}{2}\frac{1}{2} >$	-40.524789
0	0	$< \frac{1}{2}\frac{1}{2} \vert V^{MSDI}(1,2) \vert \frac{1}{2}\frac{1}{2} >$	-16.03307767
0	1	$< \frac{3}{2}\frac{3}{2} \vert V^{MSDI}(1,2) \vert \frac{3}{2}\frac{3}{2} >$	33.859671
0	0	$< \frac{3}{2}\frac{3}{2} \vert V^{MSDI}(1,2) \vert \frac{3}{2}\frac{3}{2} >$	24.613774
1	1	$< \frac{3}{2}\frac{1}{2} \vert V^{MSDI}(1,2) \vert \frac{3}{2}\frac{1}{2} >$	31.547896
1	0	$< \frac{3}{2}\frac{1}{2} \vert V^{MSDI}(1,2) \vert \frac{3}{2}\frac{1}{2} >$	22.1781389
2	1	$< \frac{3}{2}\frac{3}{2} \vert V^{MSDI}(1,2) \vert \frac{3}{2}\frac{3}{2} >$	88.235689
2	0	$< \frac{3}{2}\frac{3}{2} \vert V^{MSDI}(1,2) \vert \frac{3}{2}\frac{3}{2} >$	27.1852009
2	1	$< \frac{3}{2}\frac{1}{2} \vert V^{MSDI}(1,2) \vert \frac{3}{2}\frac{1}{2} >$	-75.258417
2	0	$< \frac{3}{2}\frac{1}{2} \vert V^{MSDI}(1,2) \vert \frac{3}{2}\frac{1}{2} >$	-15.07466904
0	1	$\vert < \frac{1}{2}\frac{1}{2} \vert V^{MSDI}(1,2) \vert \frac{3}{2}\frac{3}{2} > \vert$	1.334898
0	0	$\vert < \frac{1}{2}\frac{1}{2} \vert V^{MSDI}(1,2) \vert \frac{3}{2}\frac{3}{2} > \vert$	2.051010639
2	1	$\vert < \frac{3}{2}\frac{3}{2} \vert V^{MSDI}(1,2) \vert \frac{3}{2}\frac{1}{2} > \vert$	0.24823
2	0	$\vert < \frac{3}{2}\frac{3}{2} \vert V^{MSDI}(1,2) \vert \frac{3}{2}\frac{1}{2} > \vert$	0.18656931

Table (6) Values of matrix element for (hole-hole) configuration for $^{14}_{8}O_6$ using (MSDI)

J^+	$E(j_1 j_4^{-1}; j_3 j_2^{-1})$	Value(MeV)
0	$E(\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2})$	1.696736
0	$E(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{3}{2})$	11.694689
0	$E(\frac{1}{2}\frac{1}{2};\frac{3}{2}\frac{3}{2})$	2.713789
2	$E(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{3}{2})$	16.7360304
2	$E(\frac{3}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2})$	11.2916344
2	$E(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{1}{2})$	0463121
1	$E(\frac{3}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2})$	8.9095204



Figure(1):Comparisom Between Theortical and Experimental Values For Nucleus

Table (7): Comparison between theoretical and experimental values for $^{14}_{8}O_6$ with respect to ground state (-28.889 MeV) using(MSDI)

J^{π}	Energy(MeV)	
	Pre.Res	Exp.Res[13]
0^+	0	0
1^+	5.172	5.173
0^+	5.9164	5.920
2^+	6.5912	6.590
2^+	7.7554	7.768

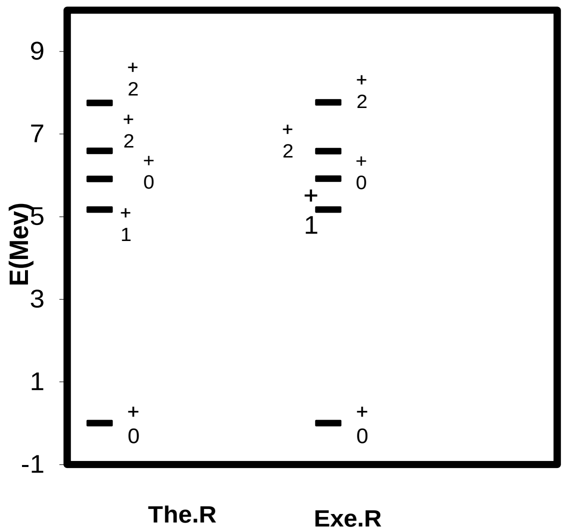


Figure (2):Comparisom Between Theortical and Experimental Values For Nucleus

Discussion and Conclusions

Through the tables (4,7) and figures(1,2) we noticed that all the calculated of energy levels with allowable angular momentum and parity is in good agreement with experimental values with same angular momentum and parity, the following is to be concluded the shell model by using modified surface delta interaction was successful in calculation energy levels for these nuclei .

Reverences

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