

## The effects of Sun attraction and solar radiation pressure on Medium Earth orbit Satellites

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### Abstract

The effects of the sun perturbations ( solar radiation pressure and the attraction of the sun ) at medium Earth orbit satellite and relation of solar radiation pressure have been investigated . Computer simulation of the equation of motion and velocity with perturbations is designed by matlab a 7.10 where using Jacobian matrix method to increase the accuracy .

### حساب تأثير الاضطرابات الشمسية على الأقمار الاصطناعية متوسطة المدار

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### الخلاصة

تم في هذا البحث دراسة تأثير الاضطرابات الشمسية (ضغط الإشعاع الشمسي وجاذبية الشمس) على الأقمار الاصطناعية متوسطة المدار، حيث تم محاكاة سرعة وموقع الأقمار الاصطناعية واضطرابات باستخدام لغة الماتلاب وباستخدام طريقة مصفوفة جاكوبيان لغرض زيادة الدقة.

### 1-Introduction

There are two kinds of perturbations of satellite which effect position and velocity of satellite then at lifetime of satellite , gravitational and non-gravitational, gravitational perturbations include the spherical harmonics , Earth tide ,ocean tide effect and effect of sun and moon attraction, non-gravitational perturbations include atmospheric drag force , solar radiation pressure , magnetic forces etc. Rodolpho V. Moraes studied the joint effects of direct solar radiation pressure and atmospheric drag on the orbit of an artificial Earth satellite [1], D. Vokrouhlicky and his group studied the perturbation effect of the force due to direct solar radiation pressure on the dynamics of artificial satellites during penumbra [2], Some practical issues of interest such as the practical solar radiation pressure model formation flight of more than two satellites attitude control considerations sailing near earth space was performed by Zhong S. Wang [3]. The rotational dynamics of a small solar system body subject to solar radiation torques was investigated by Daniel J. and Sepidehsadat [4]. The relation between area to mass ratio of satellite was presented by R. Musci, and his group they assumed variation from 1 to 20 m<sup>2</sup>/kg for geosynchronous Earth orbits GEO [5].

In this paper , numerical simulation of the equation of motion of two body problem under the effects of solar radiation pressure and attraction of sun and moon at the high Earth orbit satellite by using Matlab a7.4 of the Jacobian matrix was supplemented to improve the accuracy of the numerical solution .

## 2-Equation of Motion

The second law of Newton's which expressed in mathematical form is [6]

$$\mathbf{k} = m\ddot{\mathbf{r}}$$

(1)

where  $\mathbf{k}$  is the vector sum of all forces acting on the mass  $m$  and  $\ddot{\mathbf{r}}$  is the vector of acceleration, and from Newton's law of universal gravitation

$$\mathbf{k} = G \frac{Mm}{r^2}$$

(2)

where  $M$  and  $m$  are two particles of matter and  $G$  is the universal constant of gravitation with  $G = (6.67259 \pm 0.00085) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ .

Within a Cartesian coordinate system with the  $x, y, z$  and with  $\alpha$  being the angles between the direction of force and the respective axis of the system we find from fig.(1).

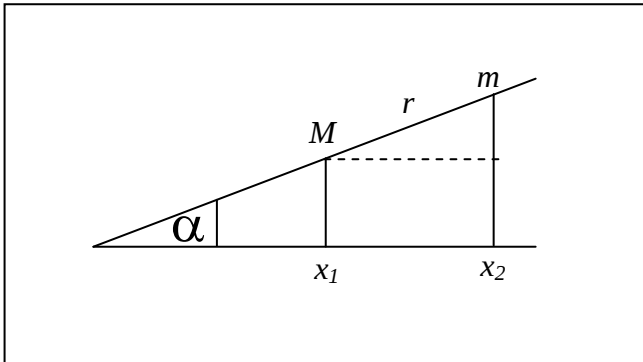


Fig (1) Components within Newton's equation of motion

$$M\ddot{x}_1 = K \cos \alpha$$

(3)

and after substituting eq.(1)

$$M\ddot{x}_1 = -G \frac{Mm}{r^2} \cos \alpha = -G \frac{Mm}{r^3} (x_1 - x_2)$$

(4)

After rearrangement we obtain for all three components

$$M\ddot{y}_1 = -G \frac{Mm}{r^3} (y_1 - y_2),$$

$$M\ddot{z}_1 = -G \frac{Mm}{r^3} (z_1 - z_2) \quad (5)$$

For the motion of  $m$  with respect to  $M$

$$m\ddot{x}_1 = -G \frac{Mm}{r^3} (x_1 - x_2),$$

$$m\ddot{y}_1 = -G \frac{Mm}{r^3} (y_1 - y_2),$$

$$m\ddot{z}_1 = -G \frac{Mm}{r^3} (z_1 - z_2)$$

Transferring the origin of the coordinate system to the center of mass  $M$ , using the substitutions  $x_2 - x_1 = x$ ,  $y_2 - y_1 = y$ ,  $z_2 - z_1 = z$

dividing eq.(5) by  $M$  and eq.(6) by  $m$  and using the result of position  $r^2 = x^2 + y^2 + z^2$  the equation becomes

$$\ddot{\mathbf{r}} = -G \frac{M+m}{r^3} \mathbf{r}$$

(7)

For artificial Earth satellites the mass  $m$  can be neglected so

$$\ddot{\mathbf{r}} = -\frac{GM}{r^3} \mathbf{r}$$

(8)

The general form of equation of motion including perturbations is

$$\ddot{\mathbf{r}} = -\frac{GM}{r^3} \mathbf{r} + \vec{a}_p$$

(9)

Where  $a_p$  is the sum of all the perturbing accelerations.

## 3-Solar Radiation Pressure

Solar radiation pressure is force acting on the satellite's surface caused by the sunlight. The force acting directly on the satellite is proportional to the effective satellite surface area, to the reflectivity of the surface and to the solar flux; it is inversely proportional to the velocity of light, the acceleration result from solar pressure radiation is [7,8,9]

$$a_{SPR} = -\mu p_s C_R \frac{A}{m} \quad (10)$$

where

$C_R$ =reflectivity coefficient

$A$ = cross sectional area of satellite

$m$ = mass of satellite

$\mu$ =shadow function

$\mu = 1$  for complete sun light

$\mu = 0$  for umbra phase

$0 < \mu < 1$  for penumbra phase

$P_s$  = which can be calculated from [10]

$$p_s = \frac{E}{c} \quad ( )$$

$E$ = Solar constant (nominal 1358 W/m<sup>2</sup>)

$c$ =vacuum speed of light.

#### 4-Attractions of Sun

The sun and moon causes periodic variations in all keplerian elements, but secular perturbations only to the right ascension of ascending node and argument of perigee[11]

$$\dot{\Omega}_{sun} = -0.00154 \frac{\cos i}{n} \quad (11)$$

$$\dot{\omega}_{sun} = 0.00077 \frac{5 \cos^2 i - 1}{n} \quad (12)$$

where  $n$  is the number of revolutions pr day,  $i$  inclination ,  $\dot{\Omega}_{sun}$  rate of right ascension of ascending node for sun and  $\dot{\omega}_{sun}$  rate of argument of perigee for sun are given in degrees/sec.

#### 5-Jacobian Matrix

The Jacobian matrix is the matrix of all first-order partial derivatives of a vector-valued function .That is ,the Jacobian of a function describes the orientation of a tangent plan to the function at a given point[12] . Let  $x \in R^n$  and

$y = f(x) \in R^n$  be a differentiable vector-valued function of  $x$ . In this case ,it is well known that the Jacobian matrix [13]

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} = \left( \frac{\partial f_i}{\partial x_j} \right)_{i,j=1,2,\dots,n} \quad (15)$$

#### 6. Discussion of Simulation Results

A computer simulation has been developed to the equation of orbital motion of two body problems with perturbations due to the effects of the solar radiation pressure and attractions of the sun using Matlab a 7.10 .The solar radiation pressure perturbation was calculated for 75 orbital cycles .

Figure (2) appear that the effect of sunny radiation pressure on position vector when we could be see that the position vector began to turn normally moving , but after many turn the position vector began to decreased , and can notes that . when we compared figure (2) with figure (3) which represent to the position vector with time without any effects of the sunny radiation pressure .

While the change of the orbit velocity as founded in figure (4) which appears that the relation between the orbit velocity with time under the effects of solar radiation pressure , where we cannot notes that any change in the beginning , but after many turns , we note that increasing in the orbit velocity with respect to figure (5) , and that appear the orbit velocity with time without the effects of solar radiation pressure .

The effect of attraction of the sun is shown in figures (6) and (7) , where this kind of perturbations affect the right ascension of ascending node and the argument of perigee only, it can be seen from these figures that there is a small differenc between the attractions of the sun on the right ascension of ascending

node and the argument of perigee actions .

It can be concluded from these figures that ,the solar radiation pressure impacts on the position and velocity of the orbit of the satellite and subsequently causes a reduction of the position of the

orbit and steps-up of the velocity of the orbit , then reduces the lifetime of the satellite .It is possible to decrease this kind of perturbation by controlling the area-to-mass ratio of the satellite depending on the equation (10) .

Fig.(2) Relationship between position vector with time under solar radiation pressure effect

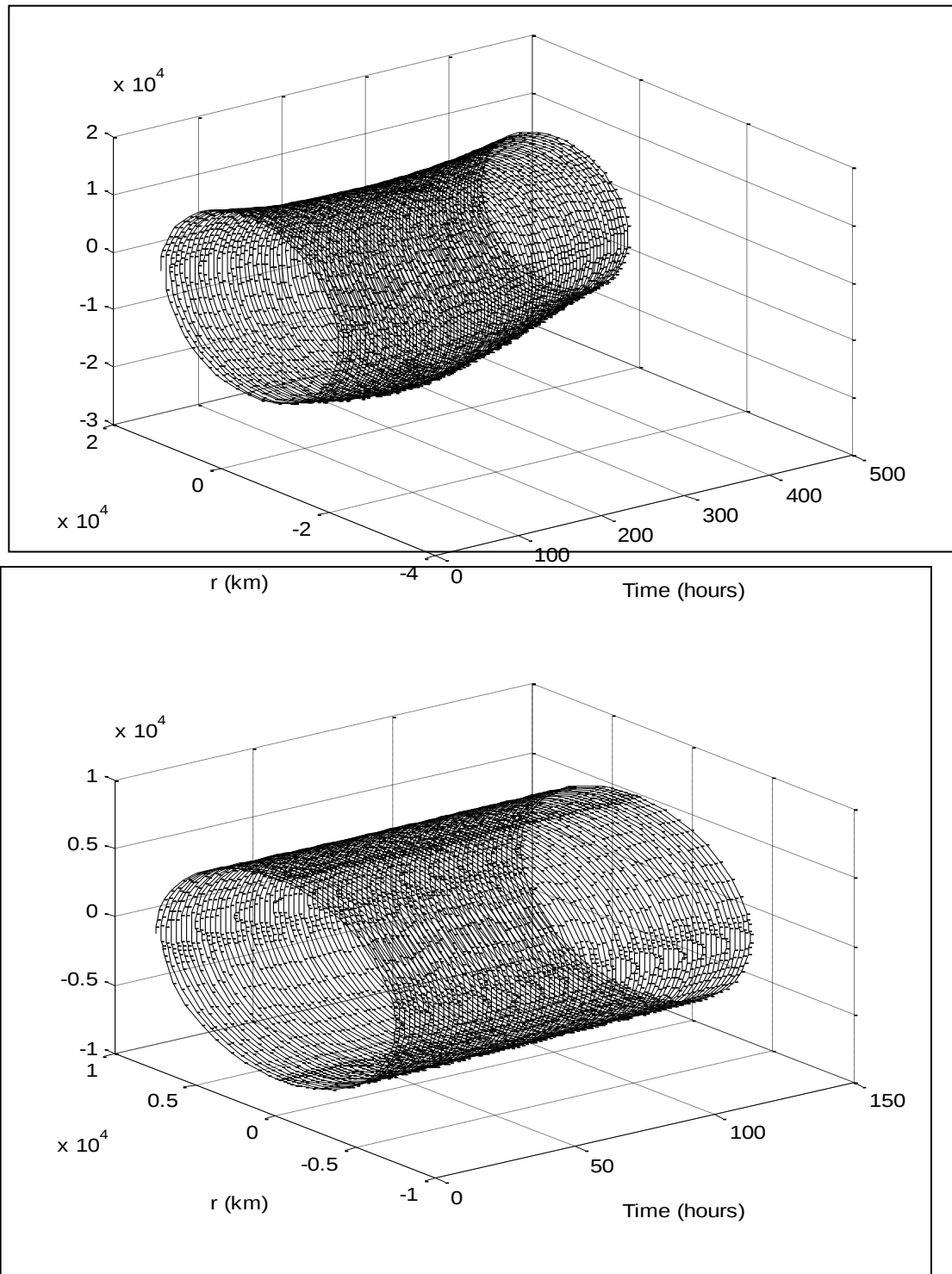


Fig.(3) Relationship between position vector with time without solar radiation pressure effect

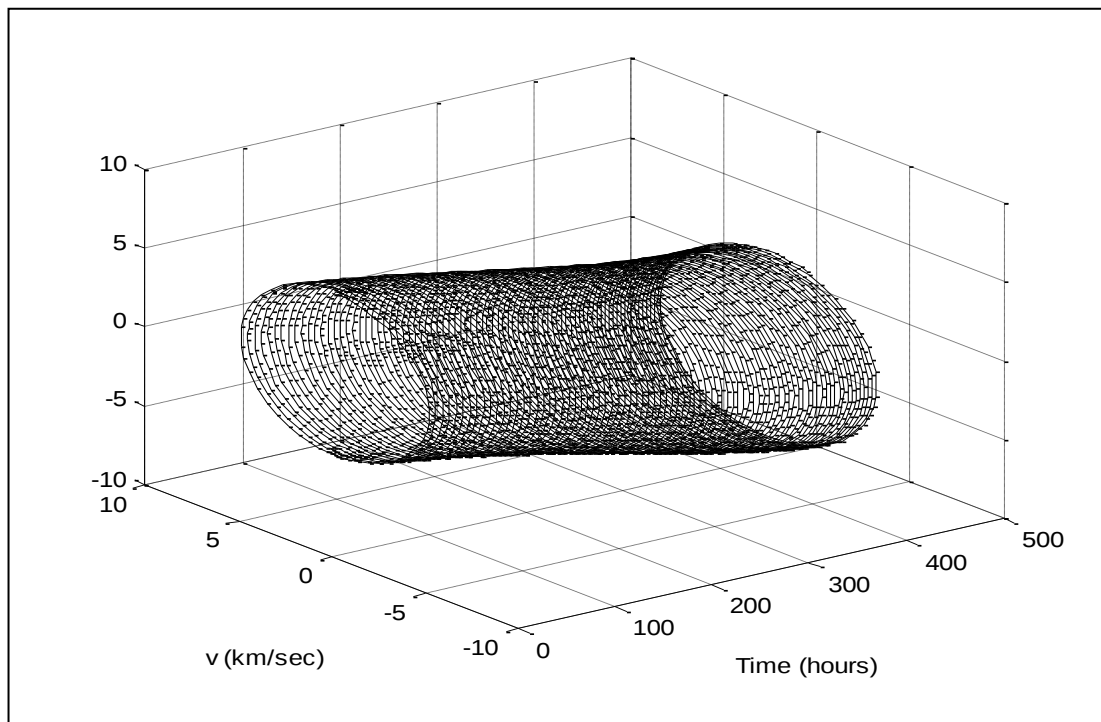


Fig.(4)The variation of orbital velocity with time under the effect solar radiation pressure

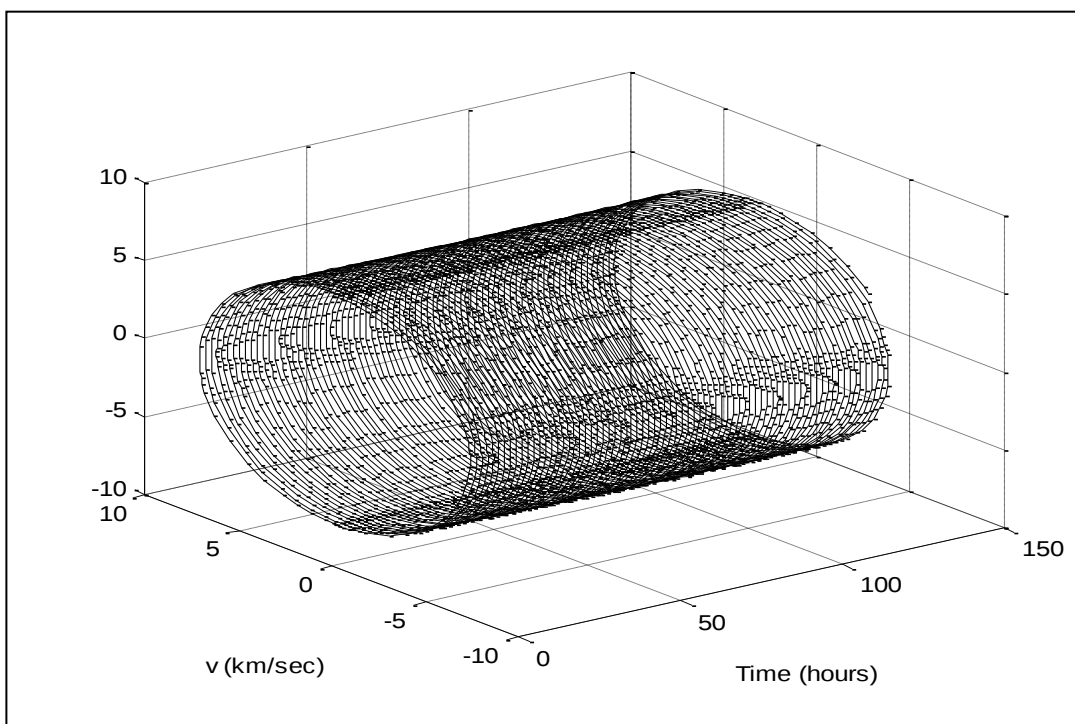


Fig.(5) velocity vector with time in the absence solar radiation pressure effect

Fig.(6) The effect of sun attraction on right ascension of ascending node

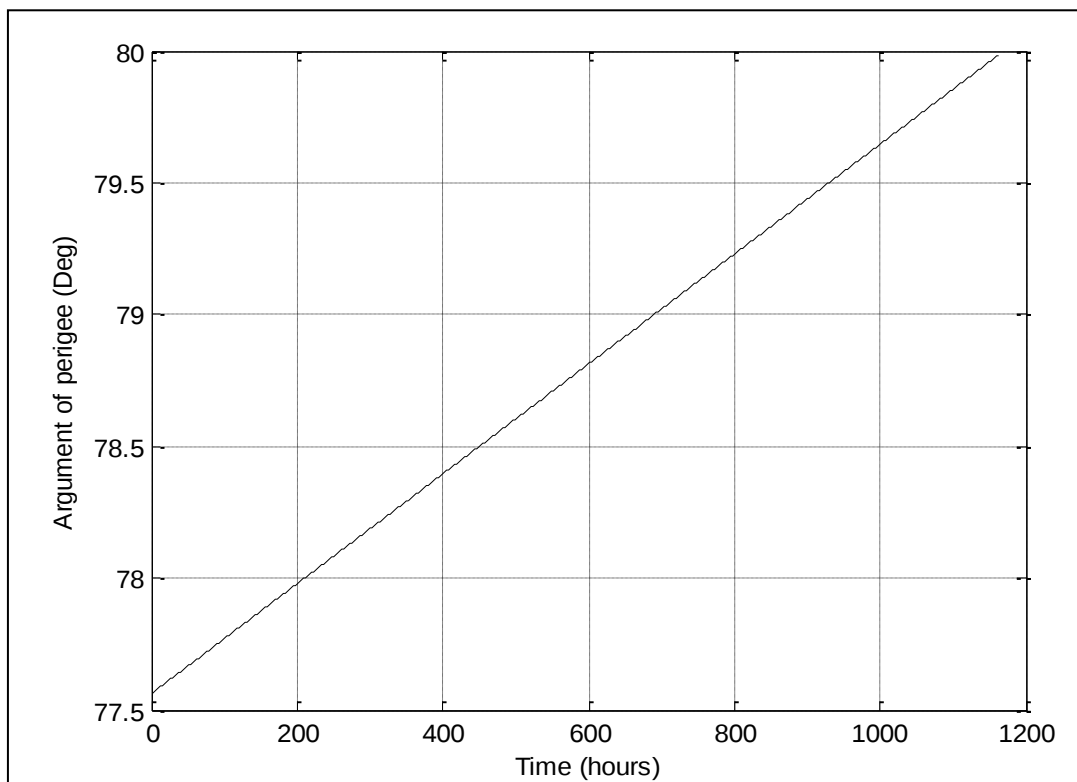
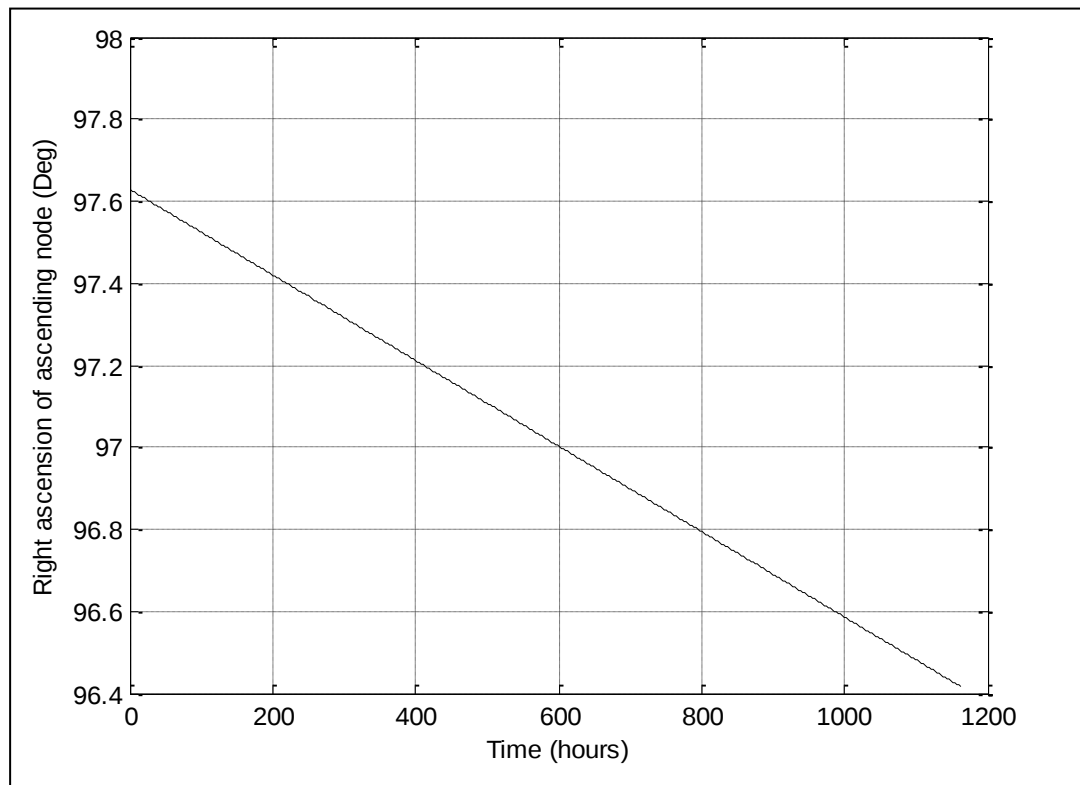


Fig.(7) The effect of sun attraction on argument of perigee

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