Calculation of the energy levels for $\left({}^{54}_{26}Fe \ \& \ {}^{54}_{28}Ni\right)$ nucleusby applying the nuclear shell model using modified surface delta interaction

D.N.Hamed **Q. B.M.Rashed**

Department of Physics, Faculty of Science, University of Kufa

Abstract:

In this work, we applied the nuclear shell model to study the energy levels for $\left({}^{54}_{26}Fe \ \& \ {}^{54}_{28}Ni\right)$ nucleus which contain two holes outside ${}^{56}_{28}Ni$ core and occupy the $0f_{\frac{7}{2}}$ shell .In these calculations used computer program MatLab 2016 to calculate Modified Surface Delta Interaction (MSDI) is employed as a residual interaction. Comparison with experimental values and found to be rather good agreement.

Keywords: nuclear shell model, modified surface delta interaction, energylevels,hole.

حساب مستويات الطاقة لنواتي $\left({}^{54}_{26}Fe \ \& \ {}^{54}_{28}Ni\right)$ بتطبيق انموذج القشرة النووي واستخدام جهد دلتا السطحي المعدل

دلال ناجي حميد **قصي باسم محمد رشيد**

قسم الفيزياء ,كلية العلوم , جامعة الكوفة

الخلاصة:

في هذا البحث تم اختيار انموذج القشرة النووي لدراسة مستويات الطاقة لنواتي $\left({}^{54}_{26}Fe \ \& \ {}^{54}_{28}Ni\right)$ والتي توصف بوجود اثنين من الفجوات خارج القلب المغلق ${}^{56}_{28}Ni$ يحتلان القشرة $0f_{\frac{7}{2}}$ في هذه الحسابات استخدم برنامج الماتلاب 2016 في حساب تفاعل دلتا السطحي المعدل كتفاعل متبقي بين جسيمتين . بعد مقارنة نتائج البحث بالنتائج العملية وجدنا تقارب جيد بينهما.

الكلمات المفتاحية: انموذج القشرة النووي , جهد دلتا السطحي المعدل, مستويات الطاقة , الفجوة.

1. Introduction

The nuclear shell model is the first quantum mechanical model of the nucleus. Its name has came from the results of empirical correlation of certain nuclear data. A shell model in which the system is thought to consist of individual particles moving in bound orbits in response to the remainder of the system. Each orbital has a well designated energy angular momentum and parity associated with it[1].

The nuclear shell model has played an indispensable role in the study of the nuclear structure, since it was conceived by Mayer and Jensen. The nuclear shell model has several important and basic features such as the independence of model assumption, the usage of realistic nucleon - nucleon interaction, and the common Hamiltonian for various types' Eigen states and different nuclei[2].

For many applications these complex configuration can be taken into account exactly by the digitalization of Hamilton matrix[3].
The shell model is suggested in analogy with the electronic structure of the atom. This model is sensitive to nucleon angular momentum. It can predict total angular momentum and parity if valid. The nucleon shells are filled up according to the Pauli principle. This principle results in a finite number of such particles occupying a given energy level and thus leads the concept of closed (filled levels) shell[1].

B. A.Brown& A. C. Larsen (2014) researchers studied large low-energy strength for(^{56,57}Fe) within the NSM[4]. P. Möller& other (2016) the researcher studiedground-stateand binding energiesof 9318 nuclei ranging from A= 16 to A= 339,including some nuclei containing proton holes and neutron holes[5].N.Shimizu& other (2016)researchers a successful description of both low-lying spectroscopy and the experimentally observed equilibration of ($J^{\pi}=2^{+}$& 2^{-}) states in (^{58}Ni) in a unified manner[6].

The $\left({}^{54}_{26}Fe\right)$ nucleus and $\left({}^{54}_{28}Ni\right)$ nucleuswere selected in this study because there was no study using the nuclear shell model and the modified delta surface using program MatLab.

The aim of research is studied the energy levels for all allowable total angular momentum and the parity for iron nucleus $\left({}^{54}_{26}Fe\right)$ and nickel nucleus $\left({}^{54}_{28}Ni\right)$ withinmodel space $\left(of_{\frac{7}{2}}\right)$ using the MSDI with respect to inert core $\left({}^{56}_{28}Ni\right)$.

2. The theory

Pandya succeeded in explaining the particle -hole theorem which relates the spectrum of a nucleus with one nucleon in each of the single-particle level j' and j to the spectrum of the nucleus with one particle in j' and another hole in j[2, 7]. This relationship will be used to correlate the spectrum observed in various nuclei.

The behavior of two-hole nuclei is very much the same of two particle nuclei except that the single-particle energies change sign[8, 9]. The hole-hole interaction energy in terms of the particle-particle matrix element which represented energy level is given by[9, 10]:

$$E_{JT}(j_1 j_2^{-1}; j_1 j_2^{-1}) = < j_a j_b \left| V^{MSDI}(1,2) \right| j_a j_b >_{JT} \quad (1)$$

Where $E_{J,T}(j_1 j_2^{-1}; j_1 j_2^{-1})$ matrix element, j_1, j_2, j_a & j_b the orbitals,(T)isospin , (J) total angular momentum.

A typical two body matrix element by using the MSDI is[7, 8]:

$$\langle j_a j_b | V^{MSDI}(1,2) | j_a j_b \rangle_{JT} =$$

$$\left\{ \begin{array}{l} -A_T \left[\dfrac{(2j_a+1)(2j_b+1)}{2(2J+1)(1+\delta_{ab})} \right] \\ \left[\left\langle j_b \dfrac{-1}{2} j_a \dfrac{1}{2} \middle| J\,0 \right\rangle^2 \left[1-(-1)^{l_a+l_b+J+T}\right] \right. \\ \left. + \left\langle j_b \dfrac{1}{2} j_a \dfrac{1}{2} \middle| J\,1 \right\rangle^2 \left[1+(-1)^T\right] \right] \\ +\left[2T(T+1)-3\right](B+C) \end{array} \right\} \quad (2)$$

where

$$\left\langle j_b \frac{-1}{2} j_a \frac{1}{2} \middle| J\,0 \right\rangle \& \left\langle j_b \frac{1}{2} j_a \frac{1}{2} \middle| J\,1 \right\rangle$$

the Clebsch-Gordan coefficients (CGC)[11],(T)isospin ,(A_T , B, C) are parameters obtained by fitting to experimental data in the various mass region.

The matrix element of Hamiltonian are given by[2, 10]:

$$\langle H \rangle_{11} = e_{\rho 1} + e_{\rho 2} + E(j_1 j_2^{-1}; j_1 j_2^{-1}) \quad (3)$$

where $\left(e_{\rho 1} \& e_{\rho 2}\right)$ are single particle energies.

Many theories for calculating the value of allowable total angular momentum of which, if found nucleons similar in the same orbit ($j_1 = j_2$) for single particle total angular momentum allowed are calculated from[10, 12]:

$$J = 0, 2, 4, \ldots\ldots, (2j-1) \quad (4)$$

3. Calculations and Results:

The current study was based on the use of program Matlab 2016 to calculate the CGC and matrix elements to find the energy levels of configuration (hole-hole).

3.1.Iron nucleus $\left({}^{54}_{26}Fe\right)$.

The $\left({}^{54}_{26}Fe\right)$ nucleus has 28 neutrons and 26protons, its neutrons fill the single particle state up to the magic shell closure at N=28 while 2 proton are less than the magic shell closure at Z=28. These protons (holes) occupy the model space $\left(of_{\frac{7}{2}}\right)$ taking $\left({}^{56}_{28}Ni\right)$ as an inert core. Todescribe the energy levels for each of the cases mentioned in the above for nucleons in the model space $\left(of_{\frac{7}{2}}\right)$, we will be basedon values the single particle energy $\left(e_{\rho}\right)$ for proton which is[1]:

$$\left(e_{\rho}\right)_{\text{Proton}} = -\,7.1667 \text{ MeV}$$

To determine of all allowable values of the total angular momentum and parity, applied the equation (4) where:

$$J^{\pi} = 0^+, 2^+, 4^+, 6^+$$

By applying the equation (2) using MatLab 2016 we get two body matrix element by using MSDI .The parameters values are (A_T =0.92 , B = 0.9 , C = 0) as shown in table (1)and applied equation (3) using the MatLab 2016, The energy levels values for configuration (hole-hole) were calculated listed in the table (2).

When taking the values of the energy levels for all allowable angular momentum for relative to the ground state, we were obtained the values of the final theoretical energy levels. In the table (3) comparison between the theoretical values and the experimental values of the energy levels with respect to ground state and shown in figure (1).

Table (1): The values of two body matrix element using the MSDIfor $\left({}^{54}_{26}Fe\right)$ nucleus.

J^{π}	$\left\langle of_{\frac{7}{2}}^{-2} \middle| V^{MSDI} \middle| of_{\frac{7}{2}}^{-2} \right\rangle$
0^+	-2.78
2^+	0.0238
4^+	0.4699
6^+	0.6856

Table (2): Values of energy levels for configuration (hole-hole) using the MSDI for $\left({}^{54}_{26}Fe\right)$ nucleus.

J^{π}	Energy (MeV)
0^+	-17.1134
2^+	-14.3096
4^+	-13.8636
6^+	-13.6479

Table (3): Comparison between theoretical values and experimentalvalues of the energy levels for the $\left({}^{54}_{26}Fe\right)$ nucleus using theMSDI relative togroundstate (-17.1134 MeV)[13].

J^{π}	Energy (MeV)	
	The.Res	Exp.Res[14]
0^+	0	0
2^+	2.804	2.8
4^+	3.25	3.2
6^+	3.465	3.7

.

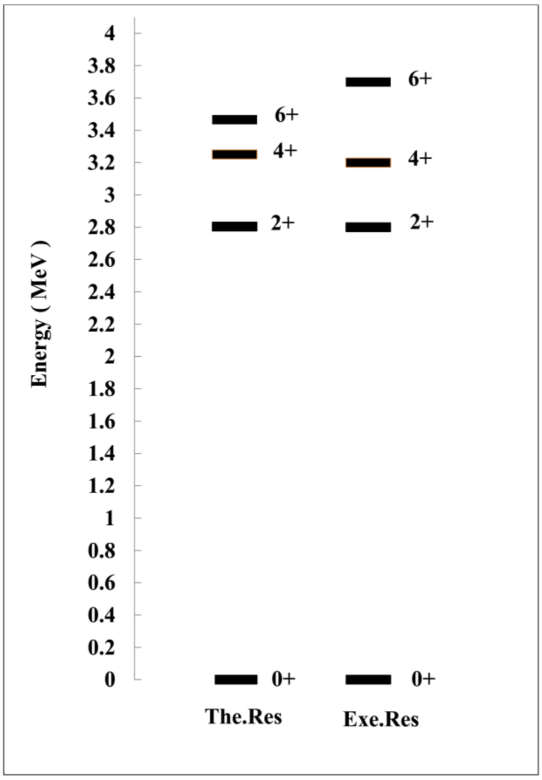

Figure (1): Comparison between theoretical values and experiment values for $\left({}^{54}_{26}Fe\right)$ nucleus.

3.2. Nickel nucleus $\left({}^{54}_{28}Ni\right)$.

The nuclei ${}^{54}_{28}Ni$ is consist of Z=28 and N=26 which has two holes in shell $of_{\frac{7}{2}}$ outside close shell ${}^{56}_{28}Ni$ for description the energy levels. We have the single particle energy $\left(e_{\rho}\right)$ which are[1]:

$\left(e_{\rho}\right)_{\text{Neutron}}$ = -16.6431 MeV

The matrix element $\left\langle of_{\frac{7}{2}}^{-2} \middle| V^{MSDI} \middle| of_{\frac{7}{2}}^{-2} \right\rangle$ is calculate from equation (2) by using the parameters (A_T =0.81, B = 0.7, C = 0.0) table (4). In order todetermine the value of all angular momentum allowableby using equation (4) as:

$J^{\pi} = 0^{+}, 2^{+}, 4^{+}, 6^{+}$

The ${}^{54}_{28}Ni$ levels are get by equation (2) and (3) as shown with respect to ground state in table (5) with the experimentalvalues[14].

In table (6) and figure (2) comparison between the theoretical and experimental values.

Table (4): The values of the matrix element using the MSDI for ${}^{54}_{28}Ni$ nucleus.

J^{π}	$\left\langle of_{\frac{7}{2}}^{-2} \middle| V^{MSDI} \middle| of_{\frac{7}{2}}^{-2} \right\rangle$
0^{+}	-2.54
2^{+}	-0.0714
4^{+}	0.3213
6^{+}	0.5112

Table (5): Values of energy levels for configuration (hole-hole) using the MSDI for ${}^{54}_{28}Ni$ nucleus.

J^{π}	Energy (MeV)
0^{+}	-35.8263
2^{+}	-33.3577
4^{+}	-32.9650
6^{+}	-32.7751

Table (6): Comparison between theoretical values and experimentalvalues of the energy levels using the MSDI for the ${}^{54}_{28}Ni$ nucleus withrespect to ground state (-35.8263MeV)[13].

J^{π}	Energy (MeV)	
	The.Res	Exp.Res[14]
0^{+}	0	0
2^{+}	2.468	1.39
4^{+}	2.861	2.62
6^{+}	3.051	3.07

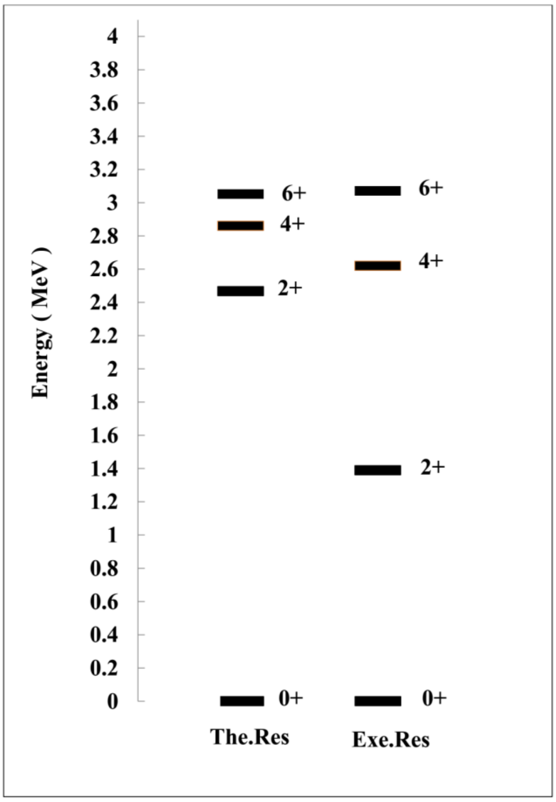

Figure (2): Comparison between theoretical values and experiment values for the $\left({}^{54}_{28}Ni\right)$ nucleus…

4. Discussion

4.1. Iron nucleus $\left({}^{54}_{26}Fe\right)$.

Through table (4) and figure (1)

we note:

1. Theoretical value for energy level (2.804 MeV) is identical to the experimental value of energy level (2.8 MeV) and same value of the total angular momentum and parity (2^+).
2. The level value (3.25 MeV) with total angular momentum and parity (4^+) corresponds to the experimental value (3.2 MeV) by the same total angular momentum and parity.
3. The energy level was compared (3.465 MeV) by total angular momentum and parity (6^+) with the experimental value (3.7 MeV)with the same total angular momentum and parity.

4.2. Nickel nucleus $\left({}^{54}_{28}Ni\right)$.

From table (7) and figure (2) we can shown the following conclusions:

1. Many calculated levels with their spin in good agreement with the experimental data where (2.62MeV and 3.07MeV) with theoretical values (2.861MeV and 3.051MeV).
2. We predict the experimental energy level (1.39MeV) with angular momentum and parity (2^+) is theoretical value (2.468MeV) in same angular momentum and parity.

5. Conclusions

By application the nuclear shell model using modified surface delta interaction to calculate energy levels for $\left({}^{54}_{26}Fe\ \&\ {}^{54}_{28}Ni\right)$ found all calculate levels with allowable angular momentum in good agreement with experimental values. From results obtained in this work and experimental data show that the shell model by using (MSDI) is a successful model to describe the spectrum $\left({}^{54}_{26}Fe\ \&\ {}^{54}_{28}Ni\right)$ and the agreement with experimental values is fairly good in general. The shell model calculation using (MSDI) is successful in an introducing the energy spectrum of nuclei.

6. Reference

[1] K. S. Krane and D. Halliday, Introductory nuclear physics . Wiley New York, 1988.

[2] A. Hasan and D. Hamed, Journal of Kufa-Physics, Vol. 5, No.2, 2013.

[3] B. A. Brown, "Lecture notes in nuclear structure physics," National Super Conducting Cyclotron Laboratory, 2005.

[4] B. A. Brown and A. Larsen, Physical review letters, Vol. 113, No.1, 2014.

[5] P. Möller, A. J. Sierk, T. Ichikawa, and H. Sagawa, Atomic Data and Nuclear Data Tables, Vol. 109, No.1, 2016.

[6] N. Shimizu, Y. Utsuno, Y. Futamura, T. Sakurai, T. Mizusaki, and T. Otsuka, Physics Letters B, Vol. 753, No.1, 2016.

[7] P. J. Brussaard and P. W. M. Glaudemans, Shell-model applications in nuclear spectroscopy: North-Holland Pub. Co., 1977.

[8] A. H. Taqi, SoftwareX,Vol. 5,. No.1, 2016.

[9] J. Suhonen, From Nucleons to Nucleus: Concepts of Microscopic Nuclear Theory: Springer Science & Business Media, 2007.

[10] R. Lawson, Theory of the nuclear shell model: Clarendon Press Oxford, 1980.

[11] S. S. Wong, Introductory nuclear physics: John Wiley & Sons, 2008.

[12] A.H.Uraybie, "Thesis," University of Kufa, 2007.

[13] M. Wang, G. Audi, F. Kondev, W. Huang, S. Naimi, and X. Xu, Chinese physics C, Vol. 41, No.3, 2017.

[14] Y. Dong and H. Junde, Nuclear Data Sheets, Vol. 121, No.1, 2014.