



EVALUATION OF MODULUS OF ELASTICITY FOR HIGH STRENGTH CONCRETE USING AGGREGATES IN IRAQ

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ABSTRACT

There are numbers of equations to evaluate the modulus of elasticity by current codes and researchers. This paper aims in developing an empirical equation to evaluate the modulus of elasticity for High Strength Concrete (HSC) considering the Iraqi aggregates used in producing the HSC. The analysis consider 78 tests from the available literature using Iraqi aggregates with a wide range of compressive strength of concrete from 41 to 83.3 MPa. The comparisons between the proposed equation here and the current codes ACI 318-14, EC2-02 and the equation by Noguchi show that for the ACI 318-14, there is an overestimation of 80% of the tests that are below it, while EC2 values are too conservative as they are below 78% of test results. The equation by Noguchi et al. showed better evaluations as it is very close to the proposed equation. More tests are required to finalize the developed equation, which cover all the parameters influencing modulus of elasticity in the produced HSC.

KEYWORDS: High-strength concrete; Modulus of elasticity; Coarse aggregates;

Admixture; Correction factors

تقييم معامل المرونة للخرسانة عالية المقاومة باستعمال الركام العراقي

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الخلاصة

توجد عدد من المعادلات لاجاد معامل المرونة في المدونات الحالية و الباحثين. هذا البحث يهدف الى تطوير معادلة تجريبية لتقييم معامل المرونة للخرسانة ذات القوة العالية مع الاخذ بنظر الاعتبار دور الركام العراقي المستخدم في انتاج الخرسانة. التحليلات الضرورية لهذا البحث تمت باتخاذ نتائج 78 اختبارا من ضمن الاختبارات المذكورة في الادبيات و التي اجريت من قبل الباحثين العراقيين الذين استعملوا الركام العراقي، و قوة الخرسانة في هذه الاختبارات تتراوح بين 41 الى 83.3 ميغا پاسكال و تمت مقارنة نتائج المعادلة التجريبية المقترحة مع كل من المعادلات في ACI 318-2014 و EC2-02 و معادلة NOGUCHI الاكثر شيوعا. نتائج ACI كانت فوق نتائج الموجودة بنسبة 80%، بينما نتائج EC2-02 اكثر تحفظا بنسبة 78% تحت النتائج الاختبارية و اخيرا كانت النتائج الحاصلة قريبة جدا من نتائج معادلة NOGUCHI. ان المعادلة التجريبية المقترحة ليست في صيغتها النهائية بل تحتاج الى اختبارات اكثر لكي تاخذ بنظر الاعتبار كل العوامل المؤثرة على قيمة معامل المرونة للخرسانة عالية القوة.

1. INTRODUCTION

It is agreed that one of the most important elastic properties of concrete for design of plain, reinforced, and prestressed concrete required to be defined is a modulus of elasticity. This is to assess for the performance of the structure, serviceability concrete members, and then to calculate the deflection and stresses under short-term and long-term loading.

The modulus of elasticity is considered as a function of compressive strength of concrete, therefore all the parameters that have influence on the properties of concrete should necessarily have its effects on the value of the modulus of elasticity. These parameters should take accounts of water to cement ratio, properties and proportions of fine and coarse aggregates, age of concrete, rate of loading, and other factors.

There are different equations by current codes of practice and researchers for prediction of the modulus of elasticity. ACI 318-14 and EC2 express the modulus of elasticity in terms of the secant modulus, and they differ in their definitions. The definition of ACI 318-14 expression is based on the specified concrete strength while EC2 bases on the mean strength which is somewhat higher than the specified strength.

EC2-02 gives the secant modulus to be defined as the slope of a line drawn from a zero stress to a compressive stress equal to $0.40 f_{cm}$.

$$E_{cm} = 22,000 \left(\frac{f_{cm}}{10} \right)^{1/3} \text{ MPa} \quad (1)$$

(The definitions are given in the Notation)

ACI 318-02 gives an expression for the secant modulus of elasticity-defined as stress may be drawn by a slope of a line from zero to a compressive stress equal to $0.45 f'_c$, given as

$$E_c = 0.043 \gamma_c^{1.5} \sqrt{f'_c} \quad (2)$$

(The definitions are given in the Notation)

Where γ_c is the concrete unit weight varying from 1440 to 2560 kg/m^3 according to ACI 318-14.

For normal density concrete, the expression simplifies to $E_c = 4700 \sqrt{f'_c} \text{ MPa}$.

Carrasquillo et al., (1988) modified versions of ACI 318-14 and reported by ACI Committee 363 as follows:

$$E_c = \left(3320 \sqrt{f_{ck}} + 6900 \right) \left(\frac{\gamma_c}{2300} \right)^{1.5} \text{ MPa} \quad (3)$$

(The definitions are given in the Notation)

There are many available expressions for evaluating the modulus of elasticity for HSC, which are mostly based on the tests using local materials. Codes like ACI 318-14 and EC2 use expressions in term of compressive strength only without factors for types of aggregate and

admixtures. A specified concrete strength is not limited by ACI as $21 < f'_c < 83 \text{ MPa}$ while EC2 takes the limit up to $f'_c = 90 \text{ MPa}$. MC90 clause 2.1.4.2 gives value of E_{ci} , the tangent modulus for concretes using a type of quartzite aggregates, for cylinder strengths up to 80 MPa. It recommends factors (α_ϵ) to account for the influences from type of aggregates as in Table 2.1.5 in MC90 (It is reproduced in Table 1).

Concrete Society Report 49 suggested a direct measurement of E_{ci} to take full account of the differences in aggregate stiffness and for higher strengths. The E_{cm} should be determined by testing.

Table 1. Factor of α_ϵ to take account of different types of aggregate (by MC90)

Aggregate type	α_ϵ
Basalt, dense limestone aggregates	1.2
Quartzitic aggregates	1.0
Limestone aggregates	0.9
Sandstone aggregates	0.7

Rashid et al. (2002) evaluated the influence of coarse aggregate on the modulus of elasticity as shown in Fig. 1. The authors used 641 results from literature. They found that for a particular concrete strength the smallest elastic modulus was given by sandstone followed by gravel, whereas dolomite provides the largest value. All types are found to give comparable elastic modulus values.

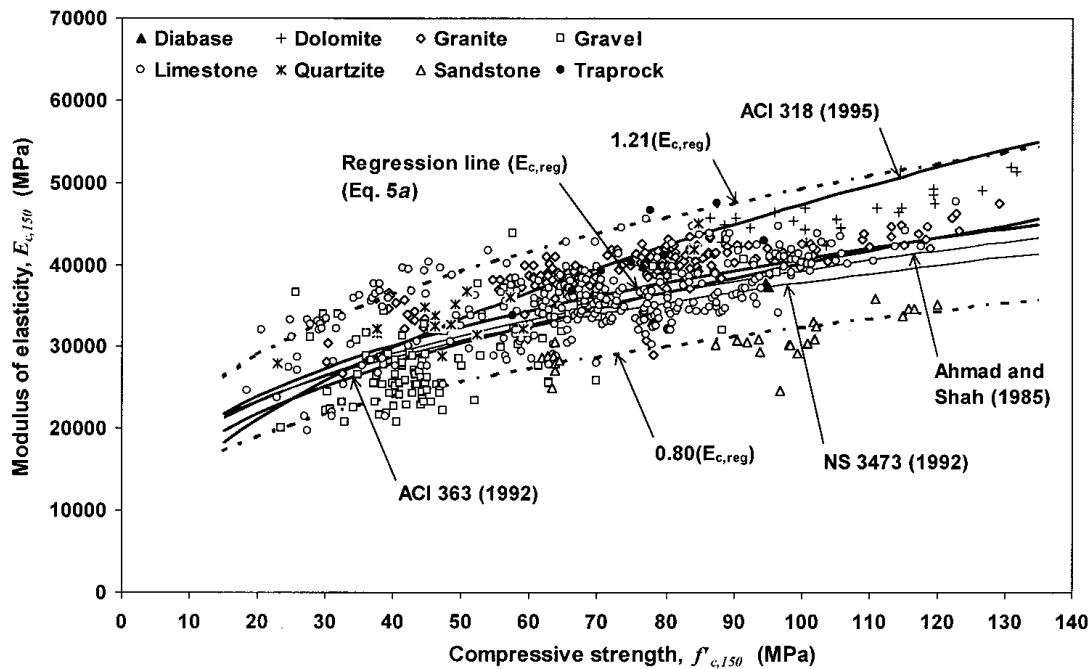


Fig. 1. Modulus of elasticity as function of compressive Strength (reproduced).

Paultre P. and Mitchell D in 2003 used a broad test results from different researchers from several countries to investigate the predictions by ACI 318-14, EC2-02, CSA A23.3-94, and NZS 3101 as shown in Fig. 2. They found a wider scatter of values of modulus of elasticity. They concluded that the North American data in the consideration provide a more accurate prediction of E_c than others in the comparison.

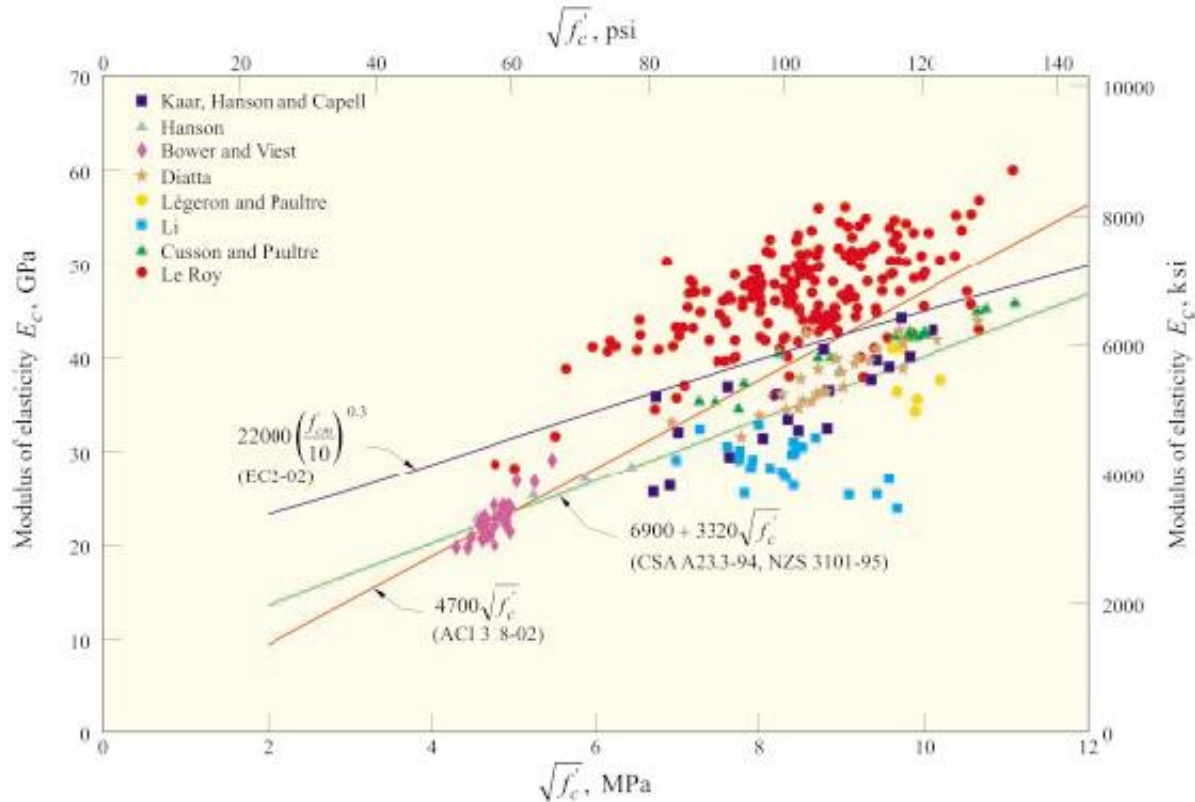


Fig. 2. is the Secant modulus of elasticity predicted using the simplified ACI 318-14, the EC2, CSA A23.3.94 and NZS 3101-95, as a function of $\sqrt{f'_c}$.

Noguchi et al. (2009) conducted a regression analysis for more than 3000 data tests to introduce a practical equation as in eqn. (4) that is applicable to a wide range of aggregate types and mineral admixture.

$$E_c = k_1 k_2 \cdot 3.35 \times 10^4 (\gamma / 2400)^2 (f'_c / 60)^{1/3} \text{ MPa} \quad (4)$$

(The definitions are given in the Notation)

They concluded that the modulus of elasticity should be expressed as a function of type of aggregate and the type and amount of admixtures. They introduced the values of k_1 and k_2 for the types of aggregates and mineral admixtures as in Table 2 and 3, respectively.

Table 2. Practical values of correction factor k_1

Type of aggregate	Correction factor
Lithological type of coarse aggregate	1.20
Crushed limestone, calcined bauxite	0.95
Crushed quartzitic aggregate, crushed andesite, crushed basalt, crushed claystone, crushed cobblestone	1.00

Table 3. Practical values of correction factor k_2

Type of addition	k_2
Silica fume, ground-granulated blast-furnace slag, fly ash fume	0.95
Fly ash	1.10
Addition other than above	1.00

Iraqi researchers carried out considerable experimental works to study the mechanical properties of HSC including the modulus of elasticity on very limited specimens. However, there is no effort towards introducing expressions for predicting the modulus of elasticity using Iraqi aggregates in producing HSC with and without industrial admixtures. These tests are not reported thoroughly in order to be used for further investigations like this particular study.

2. PROPOSING AN EQUATION FOR MODULUS OF ELASTICITY

In fact, a proposing an equation for the modulus of elasticity for concrete made from local aggregate needs massive data that represents the interactions of the properties of concrete. However, this study is far from proposing a reliable equation rather than an attempt towards a complete programme to achieve that. In this study, a total of 78 test results as shown in [Table 4](#) have been collected from the literature, and the concrete strength is ranging from 41 to 83.3 MPa. The data are from ([Khalil W. and Abdulrazaq A., 2011](#); [Al-Baghdadi H., Al-Ameeri A. 2010](#); [Salh S., Rejeb et. al. 2013](#); [Al -Azzawi A. et. al. 2011](#); [Al-Ameeri, 2013](#); [Hassan M., 2002](#); [Aziz Q., 2013](#); and [Al-Khafaji, 2008](#)). Regression analyses was performed to establish an empirical relationship between the HSC and modulus of elasticity of concrete.

The unit weight factor of concrete for all tests is ignored and assumed to be equal to 1.0. This is because of the limited available local test results and the reported information related to these tests. Also, because the data are for HSC, and the regression is conducted to predict the modulus of elasticity for HSC, it is preferred to consider the effect from type of aggregate only.

The approach that is used by [Noguchi et al. \(2009\)](#) is preferred to be followed therefore the influence from admixture is evaluated by using the correction factors by Noguchi et al. in [Table 3](#).

The collected data include tests with different sizes and types of specimens, (100x100, 50x50) mm cubes and (100x200 and 150 x300) mm cylinders.

[Rashid et al. \(2002\)](#) used 415 test results to examine the specimen size effect on the relationships between the strength and the modulus of elasticity for both cylinder sizes as in

Fig. 3. It is found that most of the results fall below the line of equality, and this is a clear indication to show that the used results of tests with a 100 mm cylinder are higher than those with a 150 mm cylinder. They proposed a relationship may be the simplest one in literature to convert the value of the 100 to 150 mm for compressive strength ranging from 10 to 120 MPa:

$$f'_{c,150} = 0.96f'_{c,100} \text{ MPa} \quad (5)$$

(The definitions are given in the Notation)

For converting the measured value of modulus of elasticity from 100 to 150 mm cylinder, a regression analysis for 644 test results from literature have been used to propose an equation as follow:

$$E_{c,150} = 0.76E_{c,100} + 6.35 \text{ MPa} \quad (6)$$

(The definitions are given in the Notation)

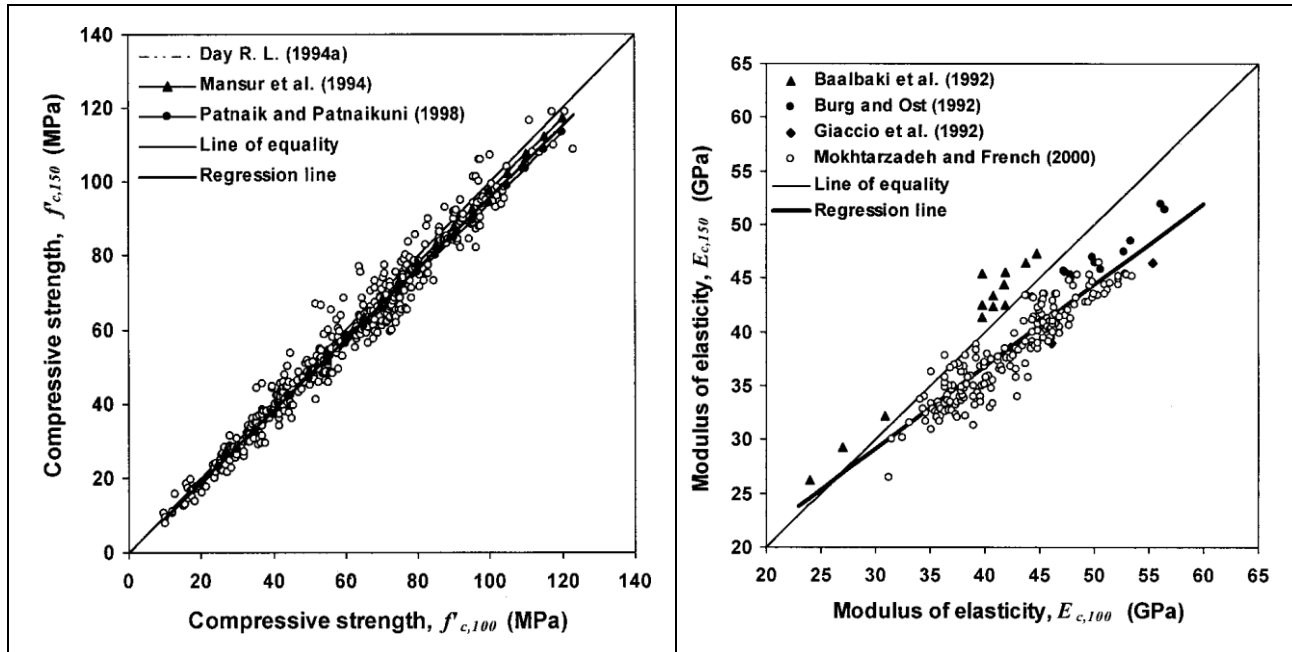


Fig. 3. Effect of size of specimen on compressive strength and modulus of elasticity (reproduced).

Therefore, it is observed that for various sizes of cubic and cylinder tests for compressive strength of concrete the correlation is needed to take account of the specific size of the specimens and to be equivalent to 150x300 mm cylinder.

Table 4. Data of the experimental works from literature

Author	No. of tests	Type of coarse aggregate	Size of agg. (mm)	Compressive strength $\left(\frac{N}{mm^2}\right)$	Specimen size (mm)	$E_{C,test}$ $\left(\frac{N}{mm^2}\right)$
Al-Baghdadi et al.2010	14	Crushed stone	20	53.0-68.4	100x200 Cylinder	32.7-39.2
Al-Azzawi et al.2010	13	Natural gravel	10	66.5-73.8	100x200 Cylinder 150x300 Cylinder 100x100 Cube 150x150 Cube	32.69-35.16
Al-Khafaji 2008	9	Crushed gravel	14	46.5-80.5	100x100 Cube	36.73-45.06
Hassan M. 2002	9	Natural gravel	9.5	46.1-50.0	150x300 Cylinder	33.8-35.2
Aziz O.Q. et al. 2013	9	Natural gravel	9.5	62.77-84.55	150x300 Cylinder	40.0-48.19
Al-Khalil et al.2013	7	Crushed gravel	10	55.0-83.3	100x100 Cube	32.54-45.27
Al-Ameeri 2013	7	Crushed gravel	10	45.1-48.3	150x150 Cube	27.0-29.0
Khalil et al.2011	5	Crushed gravel	12.5	78.8-83.5	100x100 Cube	36.2-39.0
Salih et al	5	Crushed gravel	10	54.0-76.0	100x100 Cube	38.87-44.72

The available equations for evaluating the modulus of elasticity by current codes and researchers are generally expressed in two forms. The first one is recommended in ACI 363, Architectural Institute of Japan and Shah and Ahmad ([Shah et al., 1994](#)). It is written in the general form as:

$$E_c = A(f_c)^B + C \quad (7)$$

The second form of this equation is expressed as a product of some variables commencing the effects of types of aggregate, concrete admixtures, and unit weight of concrete into consideration. This type is recommended in CEB-FIP, Model Code, EC2, by [Iravani \(1996\)](#), and [Noguchi et al. \(2009\)](#).

$$E_c = \alpha(f_c)^B \gamma^c \quad (8)$$

In this regression analysis, the first form has been chosen to represent the relationship between the concrete compressive strength and the modulus of elasticity; this is because the ACI 318-14 is a wider practical code of practice throughout the state. The authors compelled to work out this study depending on limited properties of aggregate without attempts to specify factors for the size of aggregate.

The regression analysis for all data has been conducted considering the compressive strength of concrete and the measured modulus of elasticity as shown in [Fig. 4](#), and the prediction of the evaluated modulus of elasticity could presented as in equation (9):

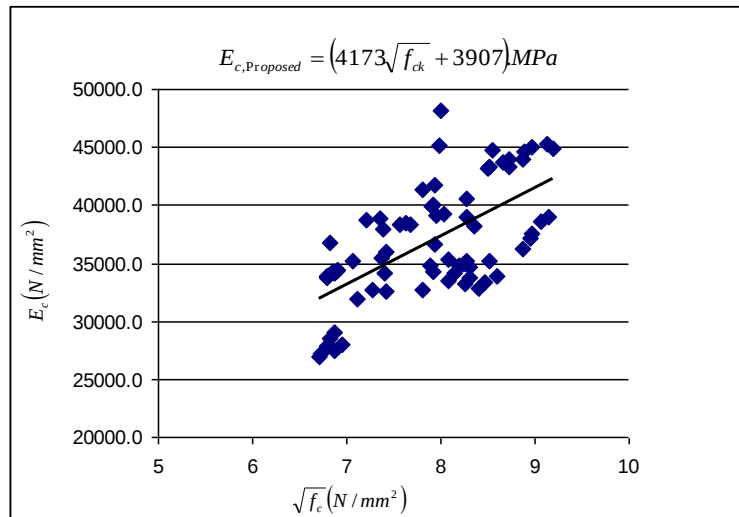


Fig. 4. Relationships between the measured modulus of elasticity and compressive strength of concrete.

$$E_{c,Proposed} = (4173\sqrt{f_{ck}} + 3907) \text{ MPa} \quad (9)$$

To account for the aggregate type in the proposed equation, further considerations have been made as shown in Fig. 5 to introduce a factor of k to the first part of the equation as shown in equation (10). It was found that the values of k for both crushed gravel aggregate 0.95 and crushed stone are similar and equal to 0.974, while for natural aggregate is different and equal to 1.00.

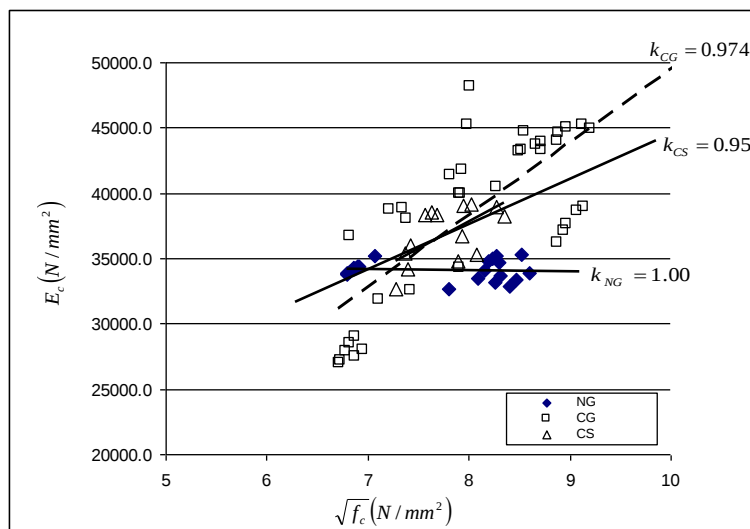


Fig. 5. Estimation of aggregate types in the proposed equation for modulus of elasticity.

Note: NG: Natural River aggregate, CG: Crushed River aggregate and CS: Crushed stone

$$E_{c,Proposed} = (k \cdot 4173\sqrt{f_{ck}} + 3907) \text{ MPa} \quad (10)$$

The assessment for this proposed equation is shown in comparisons between the prediction by it and the codes of ACI 318-14, EC2-02 and equation by Noguchi as shown in Fig.4. The E_{cm} in EC2-02 is calculated by assuming $f_{cm} = f_{ck} + 8 \approx f'_c + 8$ as shown in Fig. 6.

The figure shows that there are obvious differences in the predictions between them and the proposed equation. The ACI predictions are overestimating the modulus of elasticity, and as they are below 80% of the entire data fall below this line, while EC2 values are noticeably conservative as they are below 78% of the entire data fall above this line. The evaluations by the proposed equation and by Noguchi et al. (2009) showed fair results as they are very close to each other, and their lines lie between the two codes. However, the estimated values of the correction factor k seem to agree with those proposed by Noguchi as in Table 2.

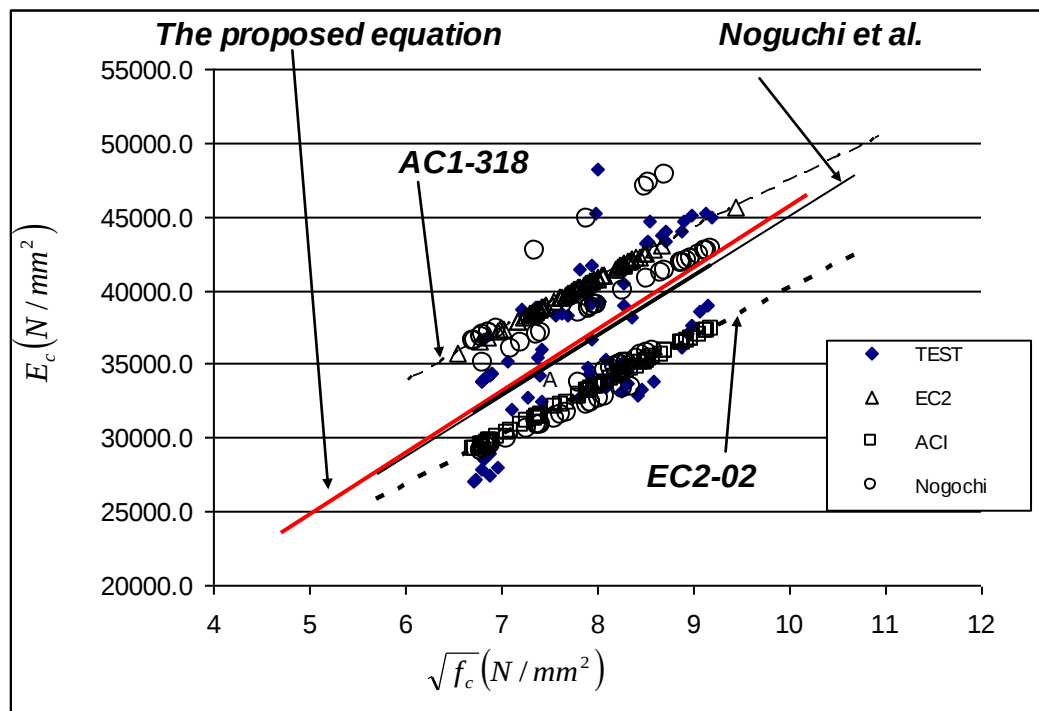


Fig. 6. Predictions of modulus of elasticity by the proposed equation, EC2, ACI and Noguchi et al.

3. CONCLUSIONS

There are various equations to evaluate the modulus of elasticity for concrete, which are by Codes of like ACI, EC2, and MC90. Also, there are serious studies conducted by various researchers for this purpose have shown that the type of aggregate has a moderate influence on the modulus of elasticity. However, most of these equations are based on the local materials of U.S. or Europe countries and often used by engineers all around the world. It was believed to make an attempt toward proposing an equation to take account of the influence of local materials accurately.

A correction factor is considered to define the influence of the type of local aggregate into the equation, but the effects from various types of admixtures are not included due to the difficulties in obtaining sufficient test results.

The evaluation by the proposed equation for modulus of elasticity for High Strength Concrete (HSC) is compared to the evaluation by others, and the following conclusions are obtained:

1. It is observed that the predictions by the proposed equation for the local types of coarse aggregates give better results than those by codes of practice.
2. The predictions by ACI 318-14 were found to overestimate the modulus of elasticity, and 80% of the results are below its regression line while EC2 showed a clear conservative values as 78% of test values are above its line, but they are not far from the prediction by Noguchi et al.'s equation.
3. This study is limited as the tests results are not comprehensive to involve all parameters.
4. It is recommended for an extensive experimental programme to consider a wide range of type and size of aggregates from different sources overall Iraq and include the available admixtures in producing HSC.

4. REFERENCES

- ACI Committee 363, State –of-the Art Report on High –Strength Concrete, ACI Journal, Proceedings VOL.81, No.4, July-Aug.1984, pp. 364-411.
- Committee Euro-International du Beton, High Performance Concrete, Recommended Extensions to the Model Code 90-Research Needs, CEB Bulletin' Information, No.228, 1995, 46pp.
- ENV 1992-1-1, Eurocode 2. Design of Concrete Structures-Part 1: General Rules and Rules for Buildings, 2004, 225pp.
- Concrete Society, Design Guide for High Strength Concrete, Technical Report 49, Slough, The Concrete Society, 1998(amended 2004)
- Architectural Institute of Japan, Standard for Structural Calculation of Reinforced Concrete Structures, Chapter 2, AIJ, 1985, pp. 8-11.
- Carrasquillo R.L, Nilson A.H. and Slate F.O., Properties of High-Strength Concrete Subjected to Short-Term Loads. ACI Journal Proceedings, Vol.78, No.3, May-June 1981, pp.171-178.
- Rashid M. A.; Mansur M. A and Paramasivam P., Correlations between Mechanical Properties of High-Strength Concrete. Journal of Materials in Civil Engineering / May-June 2002, pp.230-238
- Paultre P. and Mitchell D., Code Provision for High-Strength Concrete- An International Perspective. May 2003, Concrete International.pp.76-90.
- Noguchi T., Tomosawa F., Nemati M., Chiaia M., Fantilli P, A practical equation of elastic modulus of concrete, ACI Structural Journal, Vol.106, No.5, 2009, pp. 690-696.
- Al-Ameeri A., The effect of steel fiber on some mechanical properties of self compacting concrete. American Journal of Civil Engineering.Vo.1, No.3, 2013. pp.102-110.
- Khalil I. and Abdulrazaq A., Mechanical properties of high performance carbon fiber concrete, Engineering and Technology Journal, Vol.29, No.5, 2011
- Al-Azzawi A.A., Ali S.A., Risan H.K., Behavior of ultra high performance concrete Structures. ARPN Journal of Engineering and Applied Science, Vol.6, No.5, May 2011. Pp.95-109.

Al-Baghdadi H.M., Al-Ameeri A.S., Using different types of fine aggregate to produce high strength concrete. Babil University Journal for Pure and Applied Science.Vol.2, No.18, 2010.pp.713-723

Khalil W.I, Gorgis I.N., Mahdi Z.R., Mechanical properties of high performance of fiber reinforced concrete. Engineering and Technology Journal, Vol.31, Part A, No.7, 2013.pp.1365-1387.

Salih A., Rejeb K., Najim B., Improving the modulus of elasticity of high performance concrete by using steel fibers. Anbar Journal for Engineering Science, Pp.205-216.

Sarsam K. and Al-Azzawi Z.K., Mechanical properties of high-strength fiber reinforced concrete. Engineering and Technology Journal, Vol.28, No.12, 2010.

Hassan M., Stiffness and strength of concrete in-filled steel frame with and without opening subjected to vertical loading, MSc Dissertation, College of Engineering, University of Sulaimani, 2002.

Aziz Q. and Taha O., Mechanical properties of high-strength concrete HSC with and without chopped carbon fiber CCF. International Journal of Civil Engineering (IJCE).Vol.2, Issue 1, 2013.pp.1-12.

Al-Khafaji F., Predicting of Mechanical properties of high performance concrete by using non-destructive tests. MSc. Dissertation, College of Building and Construction, University of Technology, 2008.

Shah P., and Ahmad A., High –Performance Concrete and Applications, Edward Arnol, London, UK, 1994, 403pp.

Iravani S., Mechanical Properties of High-Performance Concrete. ACI Materials, No.93 (5), Pp.416-426.

5. NOTATIONS

E_{cm} =modulus of elasticity by EC2-02

E_c =modulus elasticity by ACI 318-14

E_{ci} =a direct measurement of modulus of elasticity

$E_{c,150}$ and $E_{c,100}$ = modulus of elasticity for concrete casted in cubes with 150mmx150mmx150mm and 100mmx100mmx100mm, respectively.

f_{cm} = the mean charcteristic compression strength by EC2-02

f_{ck} = the characteristic cylinder compression strength of the concrete, *MPa* .

$f'_{c,150}$ and $f'_{c,100}$ = the characteristic compression strength for cubes with 150mmx150mmx150mm and 100mmx100mmx100mm, respectively.

f_y = they yield strength of steel, *MPa* .

k_1 and k_2 = factors for the types of aggregates and mineral admixtures respectively

α_e = factor of to take account of different types of aggregate (by MC90)

γ_c = the concrete unit weight