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# STAGED NONLINEAR STATIC ANALYSIS OF SHRINKAGE IN REINFORCED CONCRETE ONE-WAY SLABS

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## ABSTRACT

The cracking due to restrained shrinkage at early ages of concrete is one of the shortcomings of concrete, which may reduce its sustainability. Staged nonlinear static analysis adopted by SAP2000 software is used to analyze one-way slabs applied to restrained shrinkage. Staged nonlinear static analysis has been improved using the concept of cracked element and the proposed model showed good correlation with test results. Creep strain calculated by CEB-FIP cannot be used correctly with shrinkage at early ages of concrete (first three months), and a modification factor has been suggested in the present research. The modification factor is dependent on the steel ratio. Shrinkage relaxed due to creep has been found slightly affected by the steel ratio, which may be considered constant and equals to 60% approximately. Steel reinforcement will control crack width by reducing the restrained length of shortened concrete due to shrinkage, which is equal to the distance between multiple cracks.

## KEYWORDS

Shrinkage, concrete, cracking control, one-way slab, creep, staged analysis

# التحليل الستاتيكي اللاخطي المتدرج للانكماش في السقوف الخرسانية المسلحة أحادية الاتجاه

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**الخلاصة**

تعد التشققات الناتجة عن إسناد الانكماش من نقاط الضعف الأساسية في الخرسانة والتي تعمل على تقليل استخدامها. طريقة التحليل الستاتيكي اللاخطي المتدرج التي يتبناها برنامج الكومبيوتر ساب2000 تم اعتمادها في تحليل السقوف أحادية الاتجاه المعرضة إلى انكماش مسند. تم تطوير طريقة التحليل الستاتيكي اللاخطي المتدرج باستعمال فكرة العنصر المتشقق وقد أعطت الطريقة نتائج قريبة على نتائج الفحص المختبري. انفعالات الزحف المحسوبة بطريقة CEB-FIP لا يمكن تطبيقها مباشرة مع الانكماش في الاعمار المبكرة الخرسانة (الأشهر الثلاثة الأولى ) لذلك تم اقتراح معامل تعديل يتغير مع نسبة حديد التسليح المستعملة في تسليح السقوف. تم التوصل إلى ان الانكماش المخفف بسبب الزحف نسبته تتغير بشكل طفيف مع نسبة حديد التسليح ويمكن اعتبارها ثابتة وتساوي 60%. يقوم حديد التسليح بالسيطرة على التشققات الناتجة من الانكماش المسند من خلال تقليل طول التقلص الذي يحصل بسببه التشقق والذي يساوي للمسافة بين تلك التشققات المتعددة.

## 1. INTRODUCTION

The cracking due to restrained shrinkage at early ages of concrete is one of the shortcomings of concrete, which may reduce its sustainability. As the cracking could not be prevented, design codes tried to control the problem through limiting the crack width to allowable values. Use of steel reinforcement is one of the well-known methods adopted by design codes. The previous researches may be divided into two categories, the first studied the problem experimentally and the second analytically and numerically. Kheder (1997) tested walls and Nejade and Gilbert (2004) tested one-way slabs under the effect of restrained shrinkage. Gilbert (2001) and Lee and Seo (2014) tried to derive an analytical method to predict crack width and spacing due to restrained shrinkage of reinforced concrete. Acarcan et al. (2008) and Attiyah et al. (2013) used nonlinear finite element analysis to model the shrinkage problem, where material nonlinearity was used in the analysis. All the mentioned researches focused on the effect of reinforcement on the cracking pattern and crack width at the end of shrinkage process. More depth understanding of the reinforcement role in crack control requires step-by-step model in the time domain.

Previous researchers also tested uncracked concrete in order to study the interaction between creep and shrinkage, where the concrete was applied to cycles of compressive stresses to prevent cracking due to restrained shrinkage [Kovler (1994), Altoubat and Lang (2001a), and Altoubat and Lang (2001b)]. The creep strain was found from the difference between strains measured in two similar specimens, one applied to free shrinkage and the other applied to restrained shrinkage. It was found that the tensile creep reduced the stress development in the retrained concrete by 50% [Altoubat and Lang (2001b)]. Recently, the aforementioned method of testing was applied to reinforced concrete specimens, and nearly 16% of concrete shrinkage stress was reduced due to use of 2% reinforcement before cracking [Xin et al. (2015)]. However, the uncracked reinforced concrete was studied and there is a need to estimate the creep strain in cracked reinforced concrete.

The main objective of the present research is to use staged analysis to trace the stress evolution in steel and concrete after cracking during the shrinkage process. Staged nonlinear static analysis adopted by SAP2000 software will be used in an iterative way to account for cracking. The interactive effect of creep and shrinkage in reinforced concrete will be discussed to estimate the creep relaxation of restrained shrinkage for different steel ratios after cracking.

## 2. STAGED NONLINEAR STATIC ANALYSIS

Staged Analysis is used in some software to analyze the structures under the effect of staged construction loads. It allows defining a sequence stages with removing or adding part of the structure or its loads. Moreover, time-dependent material behavior can be studied using the staged analysis, such as the effects of shrinkage and creep [Computers and Structures (2013)].

Stages are defined in the time domain to consider the change in the modulus of elasticity, shrinkage and creep strains. At each stage, the material properties for elements are updated and the structure is analyzed under the effect of stresses due to restrained shrinkage and creep strains.

In the present research, the software SAP2000 will be used to model the shrinkage effect on one-way slabs. The software adopted the CEB-FIP model to calculate the development of shrinkage and creep strains with time [Computers and Structures (2013)]. The main shortcoming in using SAP2000 to study the shrinkage effect on concrete is crack modeling, where the software used elastic analysis without considering crack. Hence, a method will be proposed to consider the effect of cracking on the structural behavior.

The proposed method is iterative and can be summarized in the following steps, which are shown also in Fig. 1:

1- Analyze the structure as elastic uncracked under the effect of shrinkage and creep strains.
2- Assess the time step where the tensile stress in one element or more exceeds the tensile strength of concrete at that age. Those elements will be the cracked elements.
3- Modify the properties of the cracked elements using the stress release in the direction of tensile stress.
4- Repeat the analysis from the first time step assuming that the cracked elements will lose its tensile strength at time step mentioned in step 2.
5- Repeat the previous steps until the end of shrinkage and creep strains or the end specified by user.

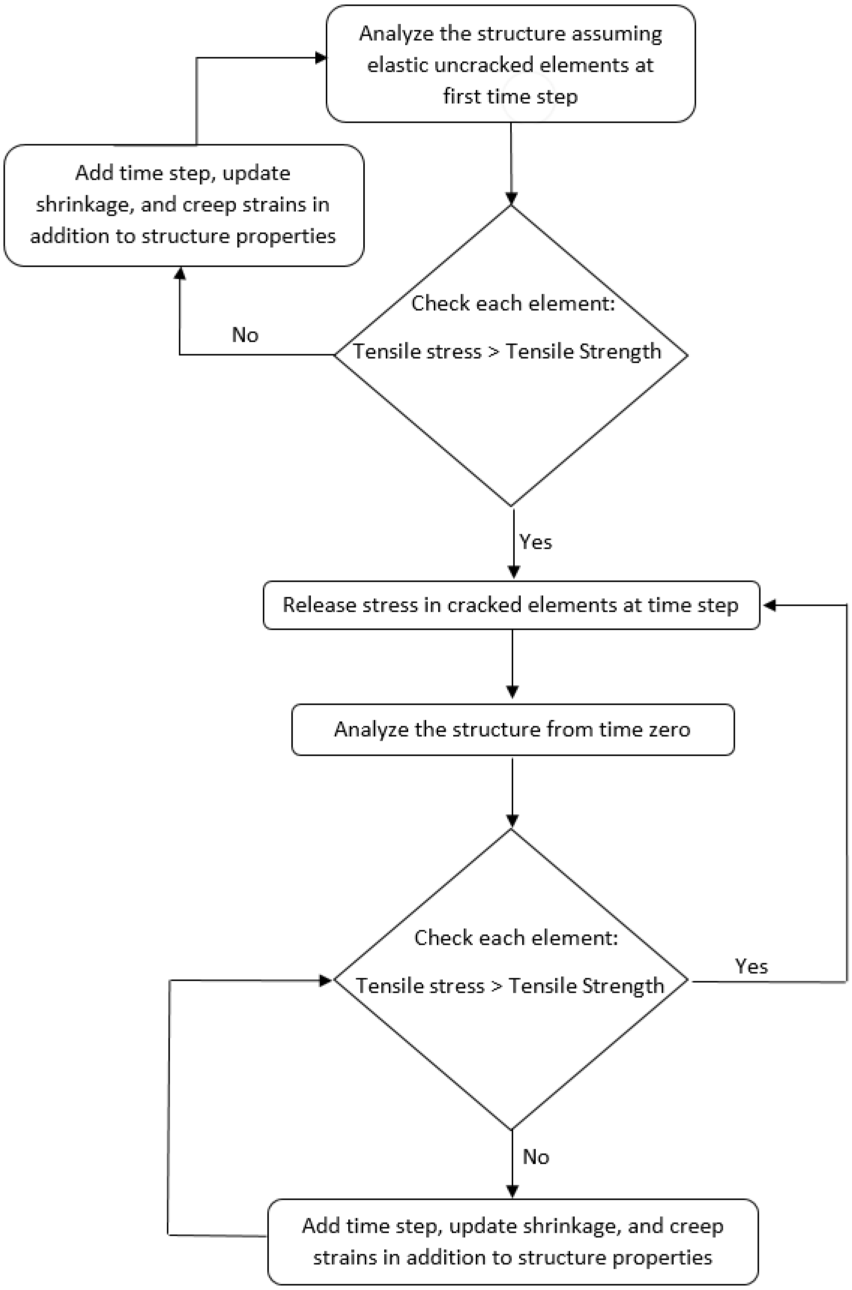


**Fig. 1. Flow chart of the proposed model**

## 3. VERIFICATION OF THE PROPOSED METHOD OF ANALYSIS

### 3.1. Nejadi and Gilbert tests of one-way slabs

Nejadi and Gilbert (2004) tested one-way slabs under the effect of restrained shrinkage to understand the behavior of steel bars in controlling the shrinkage induced cracks. Crack widths and strains in concrete and steel were measured for various steel ratios at different times along four months. The experimental results will be used to verify the analysis model proposed in the present research.

Four specimens had the same geometry were reinforced with different steel ratios as shown in Fig. 2 and Table 1. Those slabs are modeled with finite elements using thin shell elements and the steel bars are modeled with frame elements as shown in Fig. 3. Two elements are removed from the middle of the slab to model the grooves used in the test to encourage the start of shrinkage crack at that location.

Nejade and Gilbert (2004) tested unreinforced slab of dimensions 600x600x100 mm subjected to the same environment to measure the evolution of shrinkage strains with time. Fig. 4 shows the shrinkage strains calculated by the software SAP2000 according to CEB-FIP compared to that of Nejade and Gilbert test results. It is seen that the shrinkage strains calculated by SAP2000 can model the tested specimens accurately.

The staged analysis is based on updating the material properties at each time step, and the evolution of concrete modulus of elasticity with time calculated by SAP2000 is adopted. Hence, Fig. 5 shows a comparison between the adopted values of modulus of elasticity and that measured in the test. A difference of 33% is seen at early age, which is reduced to zero gradually with time. However, this difference will be ignored and the modulus of elasticity values calculated by SAP2000 will be used for simplicity.

The slab S1a is analyzed under the effect of shrinkage, where initial strain is applied on the elements at each time increment. The time step used in the analysis is one day, and Fig. 6 shows that all concrete elements are applied to tensile stresses due to restrained shrinkage. As mentioned earlier, the tensile stresses should be compared with concrete strength to predict the threshold of cracking as shown in Fig. 1. Nejade and Gilbert (2004) found that the tensile strength of concrete ranges from 0.08 to 0.1 of the compressive strength at each time, where cylinder splitting test was used to assess their values. As expected, the maximum tensile stress is seen at the elements close to the groove as shown in Fig. 7. As a result, stress will be released at those elements and after few cycles, there will be a line of cracked elements in the middle of slab at the first day after placing of concrete as shown in Fig. 7. Later, other cracks arise in the elements near the support and after several cycles, two symmetrical lines of cracked elements will be seen at both sides at the second day as shown in Fig. 8. Test results also showed that all cracks happened at the first three days after placing of concrete [Nejade and Gilbert (2004)].

It should be mentioned here that stress concentration is seen in the zone of intersection between the cracked elements and the steel bars as shown in Fig. 8. It is fake stress concentration, because the stress at that zone should be the least as concrete stresses increases gradually from zero at crack face to maximum far from it. So, concrete stresses should be found from elements far from steel bars. Fig. 9 exhibits the distribution of stresses in both concrete and steel after the first cracking [Gilbert (2001)]. Immediately after cracking, the concrete cannot resist tension anymore and all the tension will be carried by steel at the crack face. Away from the crack, the concrete tension started to increase and the steel tension will reduce rapidly to zero. Moreover, region 2 in Fig. 9 shows that the steel stresses change from tension to compression, which is found similar to the results of the proposed staged static analysis as seen from Fig. 10.

Steel strains were measured near the crack using different strain gauges [Nejade and Gilbert (2004)], and those test results will be converted to steel stresses to be compared with the proposed staged analysis method. Test results showed that all cracks happened at the first three days after placing of concrete, and the steel strains and stresses increased rapidly after that time as seen from Fig. 11. Moreover, finite element results shows that the creep should be taken into account even that it is long term effect. The interaction between creep and shrinkage at early age of concrete will be discussed in the next section.

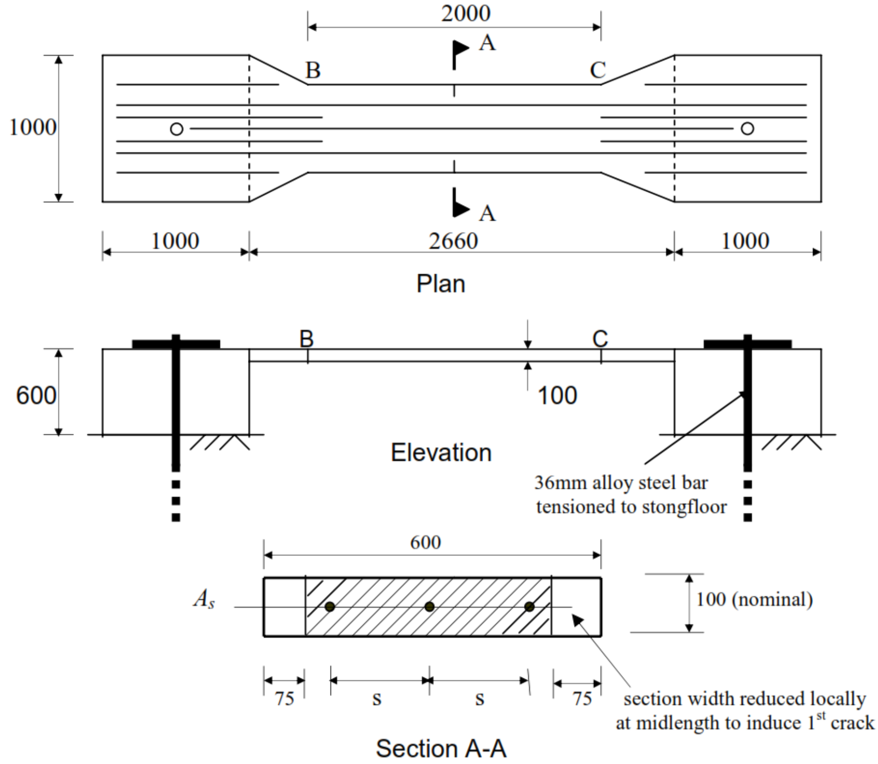


**Fig. 2. One-way slab S1a test specimen, Nejadi and Gilbert (2004)**

**Table 1. Details of test specimens, Nejadi and Gilbert (2004)**

| Specimen | No. of Bars | Bar Dia. (mm) | *s* (mm) | Steel Ratio |
|---|---|---|---|---|
| Slab S1a | 3 | 12 | 185 | 0.565 |
| Slab S2a | 3 | 10 | 185 | 0.393 |
| Slab S3a | 2 | 10 | 300 | 0.262 |
| Slab S4a | 4 | 10 | 120 | 0.523 |

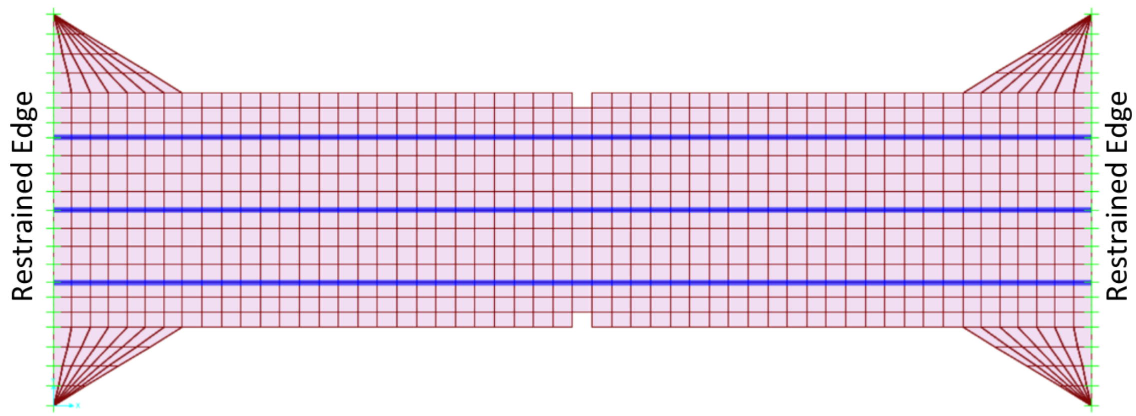


**Fig. 3. Finite element mesh of specimen S1a**

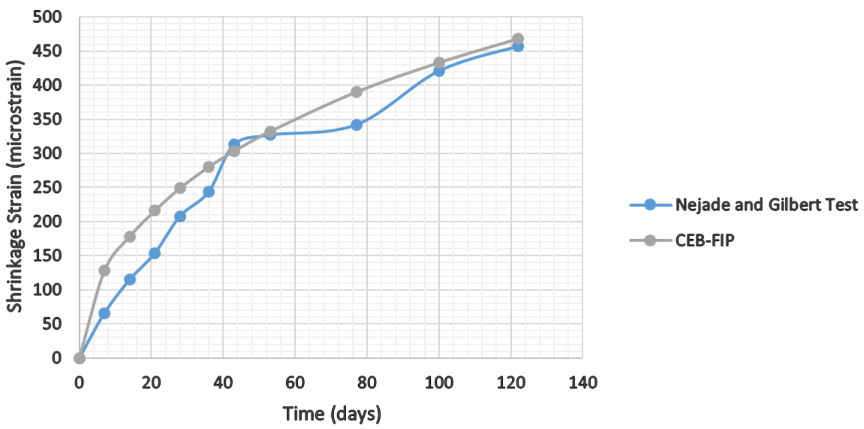


**Fig. 4. Evolution of shrinkage strain with time**

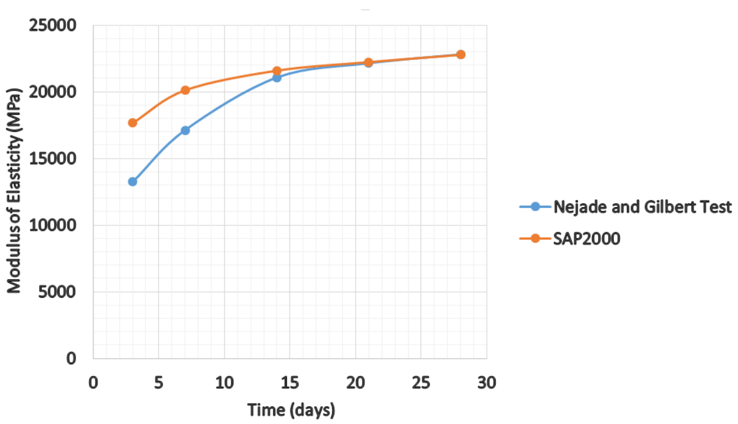


**Fig. 5. Evolution of modulus of elasticity with time**

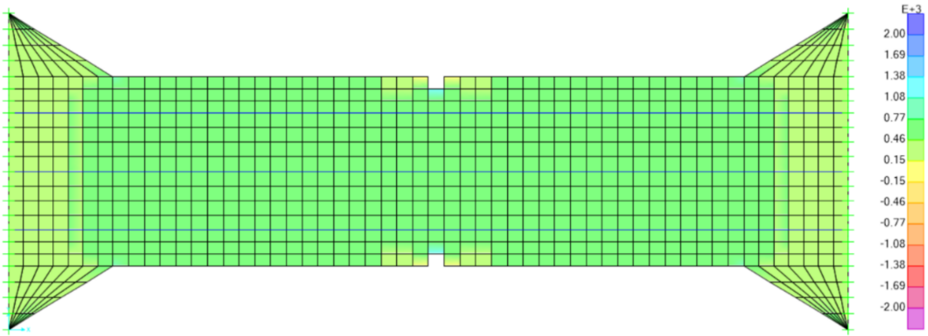

Fig. 6. Concrete tensile stresses (kPa) at the first day after placing and before cracking

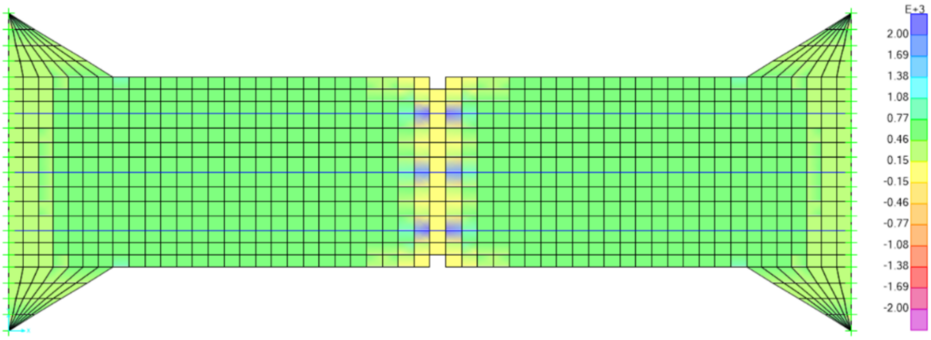

Fig. 7. Concrete tensile stresses (kPa) at the second day after placing

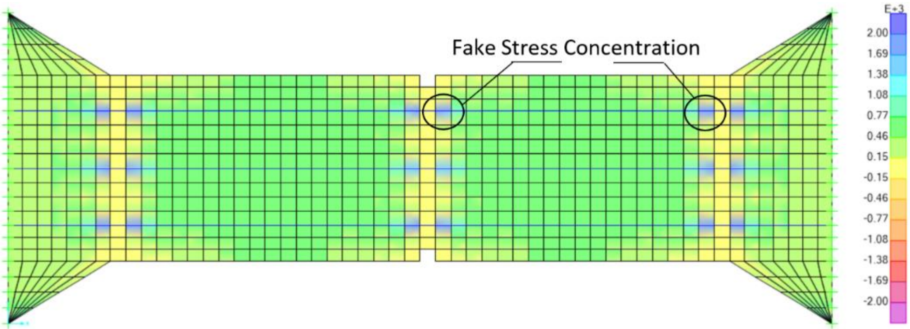


Fig. 8. Concrete tensile stresses at the third day after placing

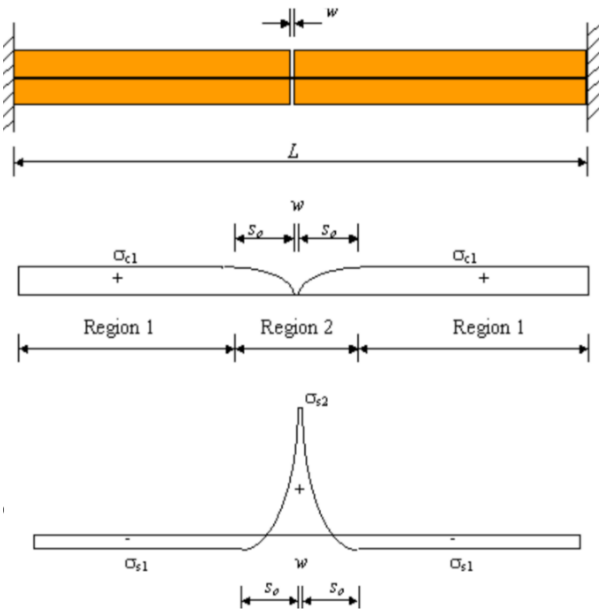


**Fig. 9. Distribution of stresses along steel bars after first crack, Gilbert (2001)**

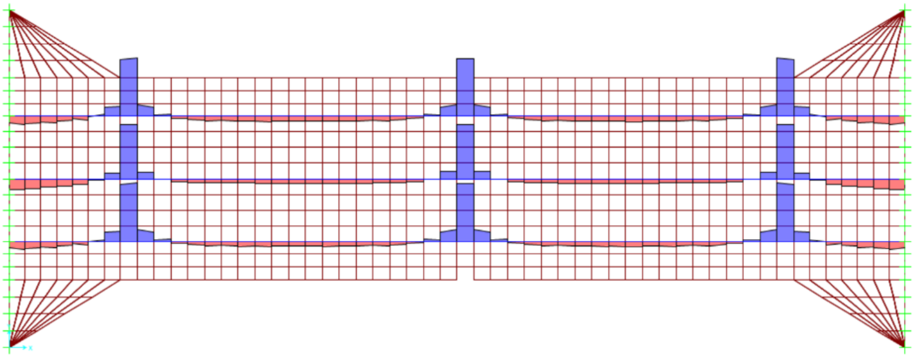

**Fig. 10. Distribution of stresses along steel bars at time 22 days**

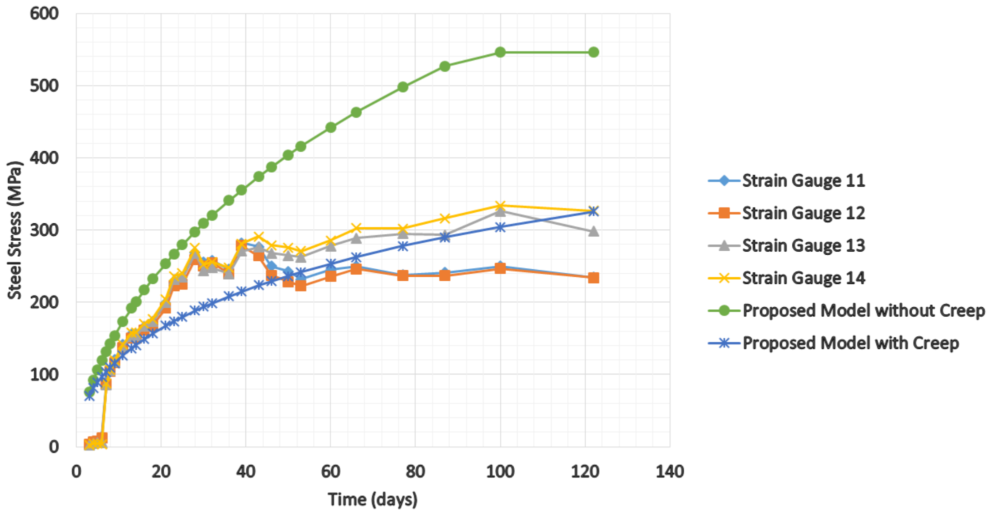


**Fig. 11. Comparison of steel stresses of slab S1a between Nejade and Gilbert (2004) test results at different strain gauges and the proposed model**

### 3.2. Creep effect on restrained shrinkage

Nejade and Gilbert (2004) tested concrete cylinders applied to constant stress to measure the creep coefficients at different times up to four months. The measured creep under constant stress is highly different from that calculated by SAP2000 as seen from Fig. 12. The difference is attributed to the type of stress inducing creep, where the drying creep is caused by gradually increasing stress rather than constant stress. Therefore, it may be concluded that superposition cannot be used to calculate or measure creep strains at early ages of concrete.

Creep strain has beneficial effect, because it will relax the shrinkage restraining in what was commonly known as Pickett effect or drying creep. In the present research, the proposed staged analysis model is used to estimate the relaxation of restrained shrinkage stress due to creep and reinforcement after cracking. The slab specimen of Nejade and Gilbert (2004) is assumed unreinforced and analyzed under the effect of shrinkage and creep using the proposed method. Thereafter, the specimens are analyzed with the proposed method assuming them as reinforced with the same ratios used in the experimental work of Nejade and Gilbert, which was mentioned in Table 1. The creep strain due to rebars will be the difference between the strains of unreinforced and reinforced specimens.

SAP2000 adopted the CEB-FIP method to calculate creep strain at each time, but it cannot be used directly for all reinforcement ratios. Figs. 13, 14, and 15 show that the creep strains in SAP2000 should be modified to have good correlation with the experimental results for reinforcement ratios 0.393%, 0.262%, and 0.523%, respectively. A modification factor P is suggested in the present research, which is dependent on the reinforcement ratio $\rho$ as shown in Fig. 16. The modification factor of creep strain was found from the best-fit curve used in Excel software as follows:

$$P = -2.69\,\rho + 2.67 \qquad (1)$$

where $\rho$ in % and ranges between 0.262% and 0.565%

Hence, the creep coefficient can be rewritten as follows:

$$\Phi_{modified} = P.\ \Phi_{CEB} \qquad (2)$$

where $\Phi_{CEB}$ is the creep coefficient calculated according to CEB-FIP method.

The analysis results of the plain concrete specimen and the four reinforced concrete specimens are shown in Fig. 17.

To understand how the creep strains relax the stresses due to restrained shrinkage after cracking, a restrained prism is assumed to be cracked in the middle. The creep stress will be reduced to zero after cracking in the unreinforced concrete prism and the crack continue to widening as the shrinkage strain C increases as shown in Fig. 18b. In the reinforced prism, the bars will restrain the shrinkage strain to some extent after cracking and the creep strain will not vanish as seen in Fig. 18c. The total strain will be:

$$\varepsilon_{tot.} = \varepsilon_{sh.} = C \text{ for unreinforced concrete} \qquad (3)$$

$$\varepsilon_{tot.} = \varepsilon_{sh.} + \varepsilon_{cr.} = (1\text{-}x).C \text{ for reinforced concrete} \qquad (4)$$

Then, the creep strain is found from the difference between Eqs. (3) and (4).

$$\varepsilon_{cr.} = x.C \qquad (5)$$

The parameter x can be defined as the ratio creep / free shrinkage, which is an important index used to reflect the reduction of tensile strain in the restrained concrete and consequently the degree of stress relaxation [Altoubat and Lang (2001a)]. The parameter x will be called in the present research "Shrinkage Relaxation Index SRI". Fig. 19 shows that the SRI is slightly affected by steel ratio after cracking and its value ranges between 0.57 and 0.64. This range of values is close to that found previously for uncracked concrete (i.e 0.5) (Altoubat and Lang 2001a). It means that the shrinkage strains reduced due to reinforcement by 7%-14% for steel ratios 0.262%-0.565%, which may be compared to that found recently by Xin et al. 2015 (i.e 16% for steel ratio 2%).

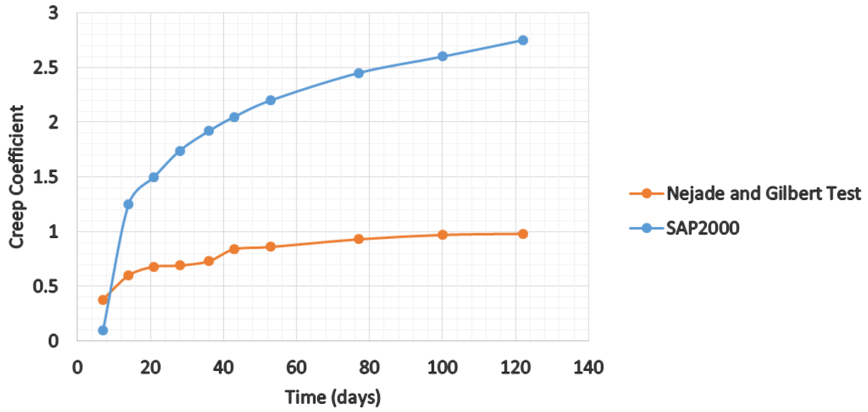


**Fig. 12. Comparison of creep coefficient between Nejade and Gilbert (2004) test results at and SAP2000**

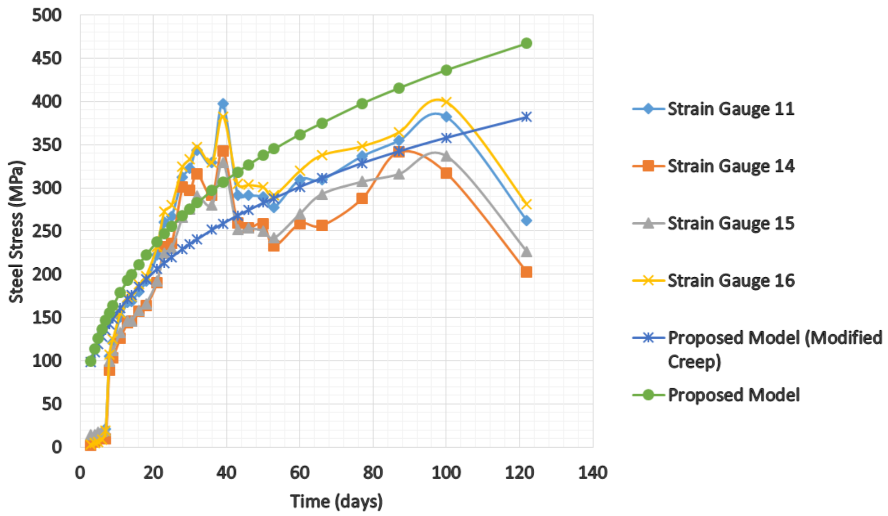


**Fig. 13. Comparison of steel stresses of slab S2a between Nejade and Gilbert (2004) test results at different strain gauges and the proposed model**

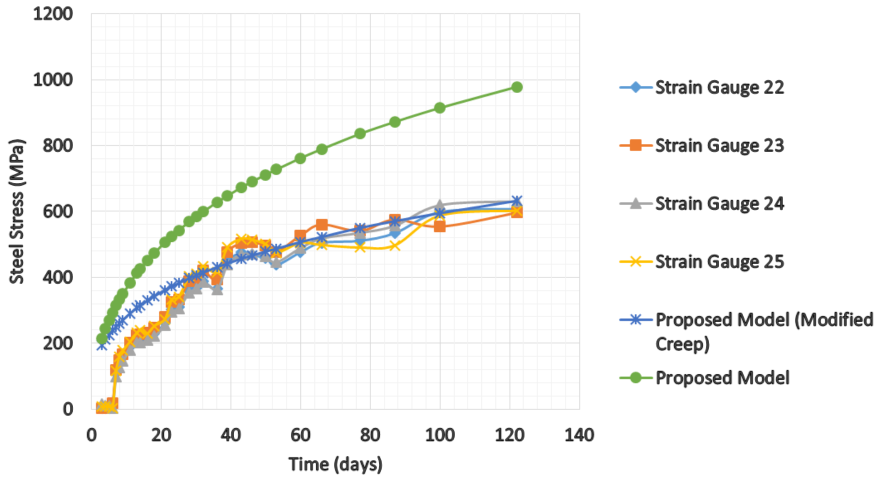


**Fig. 14. Comparison of steel stresses of slab S3a between Nejade and Gilbert (2004) test results at different strain gauges and the proposed model**

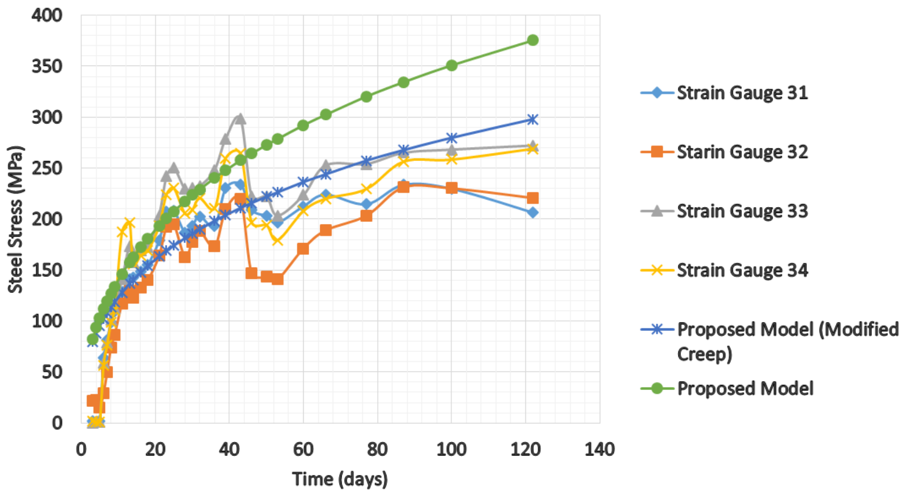


**Fig. 15. Comparison of steel stresses of slab S4a between Nejade and Gilbert (2004) test results at different strain gauges and the proposed model**

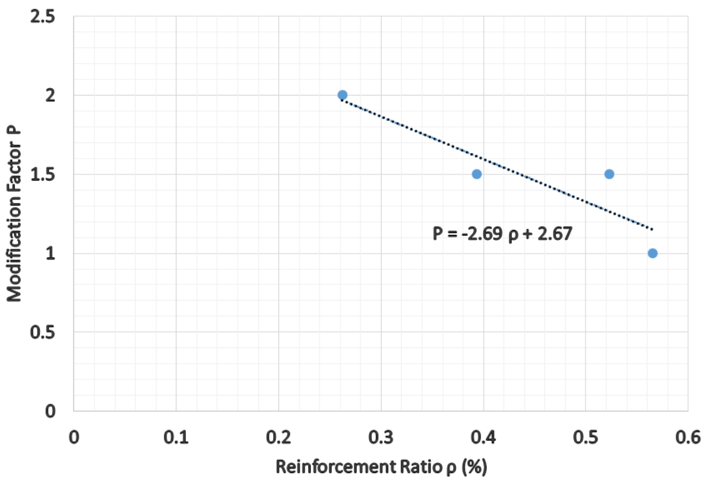


**Fig. 16. Modification factor for CEB-FIP creep**

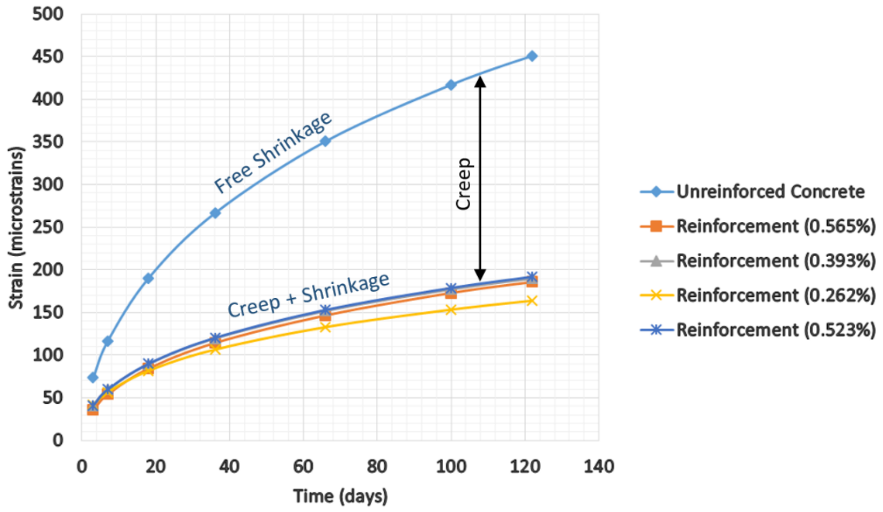


**Fig. 17. Shrinkage and creep strains for different reinforcement ratios**

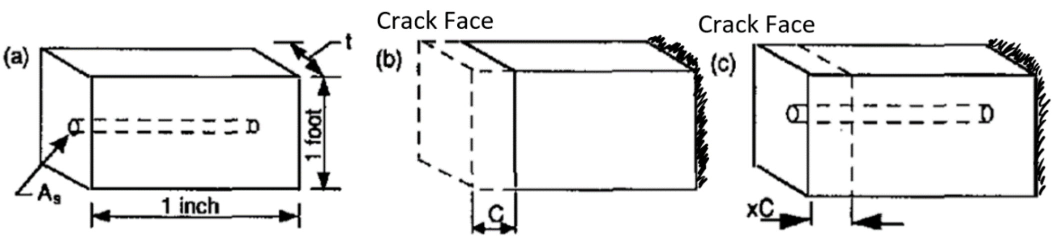


**Fig. 18. (a) reinforced section, (b) shrinkage of unreinforced section, and (c) shrinkage of reinforced section**

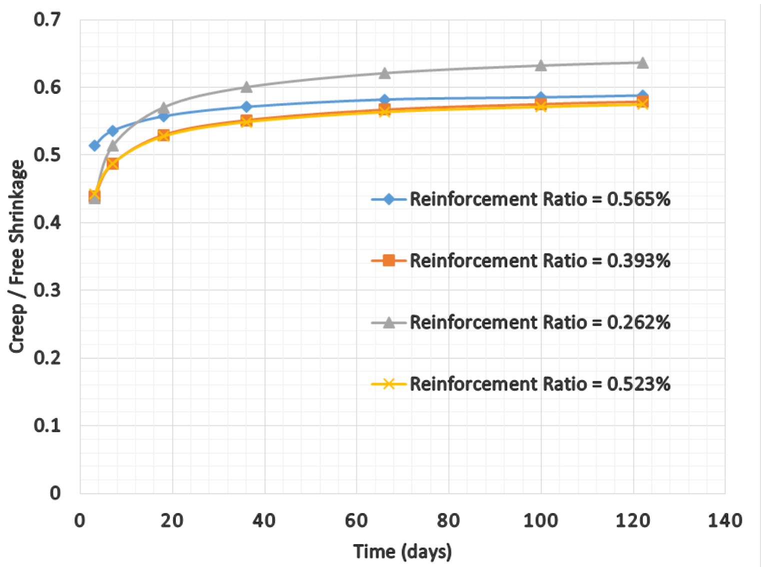


**Fig. 19. Creep / free shrinkage for different reinforcement ratios**

## 3.3. Control of Crack width

The design codes adopted the crack width as an index to control the tensile stresses due to restrained shrinkage. So, the crack width will be calculated using the proposed staged nonlinear static analysis. The sum of the displacements of the cracked element edges will be the crack width as shown from the deformed shape in Fig. 20. Table 2 exhibit that the proposed method of analysis yields crack widths close to that seen in the experimental work of Nejade and Gilbert (2004).

From the analysis of four one-way slab specimens, the reinforcement role in crack width control after cracking can be understood from the following simplified equations. From Fig. 18, the crack width $w_{crack}$ for prism of length L will be:

$$w_{crack} = C. L \text{ for unreinforced concrete} \tag{6}$$

$$w_{crack} = (1\text{-}x).C. S \text{ for reinforced concrete} \tag{7}$$

where S is the spacing between cracks arise due to restrained shrinkage (S=L for the case of one crack), and x is the Shrinkage Relaxation Index. Table 3 shows the calculated values of crack widths compared with that found form test results [Nejade and Gilbert (2004)] and the proposed method. From the above, crack control can simply described as follows:

When the steel reinforcement ratio is small, only one crack will arise and the tensile stress carried by the bars at the crack face will increase rapidly with time, because the shrinkage shortening of the whole length is restrained. However, the crack width in this case is 40% less than that of unreinforced concrete due to the effect of creep. For high steel ratio, multiple cracks form beside the first one and the steel stress will not increase as in the first case, because the shrinkage shortening of a part of the whole length is restrained (which is equal to the crack spacing). Therefore, more number of cracks mean less restrained shortening.

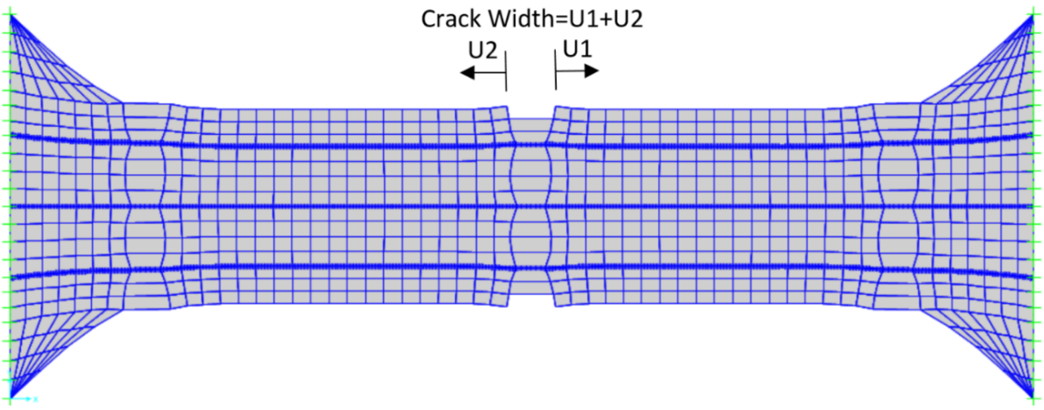


**Fig. 20. Crack width measured in the proposed model**

**Table 2. Number of cracks and crack width compared to Nejadi and Gilbert 2001 test**

| Specimen | No. of cracks | | Crack width (mm) | |
|---|---|---|---|---|
| | Test results | Proposed method | Test results | Proposed method |
| S1a | 4 | 3 | 0.21 | 0.24 |
| S2a | 3 | 3 | 0.3 | 0.26 |
| S3a | 1 | 1 | 0.84 | 0.44 |
| S4a | 3 | 3 | 0.23 | 0.26 |

**Table 3. Crack width for different reinforcement ratios**

| Specimen | Reinforcement ratio (%) | No. of cracks | Crack width (mm) | | | SRI (%) | Steel stress (MPa) |
|---|---|---|---|---|---|---|---|
| | | | Test | Proposed | Simplified | | |
| Unreinforced | 0 | 1 | - | 1.2 | 1.2 | - | - |
| S3a | 0.262 | 1 | 0.84 | 0.44 | 0.43 | 64 | 632 |
| S2a | 0.393 | 3 | 0.3 | 0.26 | 0.19 | 58 | 382 |
| S4a | 0.523 | 3 | 0.23 | 0.26 | 0.19 | 57 | 298 |
| S1a | 0.565 | 3 | 0.21 | 0.24 | 0.18 | 59 | 326 |

## 4. CONCLUSIONS

The following points are concluded in the present research:

1- Staged nonlinear static analysis can be used to study the effect of restrained shrinkage on the behavior of one-way slabs at early ages. The model has been improved using the concept of cracked element, so that software adopting elastic analysis can be used such as SAP2000. The proposed model shows good correlation with test results of Nejade and Gilbert (2004).

2- Creep strain calculated by CEB-FIP cannot be used correctly with shrinkage at early ages of concrete, and a modification factor is suggested in the present research. The modification factor is dependent on the steel ratio.

3- Shrinkage relaxed due to creep is slightly affected by the steel ratio, and it may be considered as constant equals to 60% approximately.

4- Steel reinforcement will control crack width by reducing the restrained length of shortened concrete due to shrinkage.

## 5. REFERENCES

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