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MODELING PROPERTIES OF INTERVAL VALUED ANTI-NEUTROSOPHIC FUZZY IDEALS IN NEAR-RINGS

K. Lenin Muthu Kumaran[1] and A.Rajalakshmi[2]

[1] Associate Professor, Department of Mathematics, Shanmuga Industries Arts & Science College, Affiliated to Thiruvalluvar University, Tiruvannamalai, Tamilnadu, India – 606603. Email: leninmaths@shanmugacollege.edu.in

[2] Research Scholar, Department of Mathematics, Shanmuga Industries Arts & Science College, Affiliated to Thiruvalluvar University, Tiruvannamalai, Tamilnadu, India – 606603. Email: pearlakshmi03@gmail.com

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ABSTRACT

This paper characterizes interval-valued anti-neutrosophic fuzzy ideals (IVANFIs) within near-rings, addressing the gap in algebraic theory related to their behavior in such structures. We define the direct product, complement, and factor group of IVANFIs and explore their properties through examples. The study combines interval-valued fuzzy sets with anti-neutrosophic logic, offering a flexible framework for handling uncertainty. By examining key properties such as homomorphisms, union and intersection behaviors, and relationships with other fuzzy ideals, the paper enhances the understanding of these structures and their applications in decision-making under uncertainty.

KEYWORDS

Near-Rings, Interval-Valued Anti-Neutrosophic Fuzzy Ideals (IVANFIs), Fuzzy Set Theory, Algebraic Structures, Fuzzy Ideals.

1. INTRODUCTION

Recently, the development of fuzzy set theory has had a significant impact on various Mathematical structures, providing tools to deal with uncertainty, vagueness, and imprecision (Abou-Zaid, 1991; Biwaz, 1990; Hemabala & Srinivasa Kumar, 2022; Kim et al., 2005). In all the available mathematical structures, the idea of Near-Rings is a key algebraic system in applied mathematics in different sectors such as cryptography, coding theory, and theoretical computer science. However, the traditional methods to deal uncertainties in Near-Rings are not sufficient for complex problems that are having higher degrees of indetermination. To combat this issue. Various advanced fuzzy sets such as neutrosophic sets, anti-neutrosophic sets, and interval-valued fuzzy sets are introduced (Lenin Muthu Kumaran & Rajalakshmi, 2023; Makamba, 1992; Pilz, 1977; Salama et al., 2014).

Salama et al., 2014 introduced a neutrosophic fuzzy sets that generalize fuzzy set theory through the notions of truth, indeterminacy, and falsity in Interval-valued fuzzy sets. They allow for a range of membership values rather than a single fixed value. Interval-Valued Anti-Neutrosophic Fuzzy Sets (IVANFS) proves as an efficient method for addressing uncertainty with the combination of interval-valued and neutrosophic sets. Because of the valued characteristic of the model, the inconsistent and contradictory information can be avoided (Solairaju & Thiruveni, 2018; Thillaigovindan et al., 2015). In this hybrid nature, the interval-valued fuzzy sets and anti-neutrosophic logic are carried out in an efficient manner. Near-Rings utilize the application of fuzzy sets and they are adopted widely to encompass standard ideal theory with the help of fuzziness concept.

The interval-valued anti-neutrosophic fuzzy ideals in Near-Rings have further enriched the theory by examining the uncertainties and inconsistencies in real-world scenarios. (Zadeh, 1965; Zadeh, 1975). Many advancements have been familiarized over the interval-valued neutrosophic soft sets in resource constrained environment to report the uncertainties in multi-attribute decision-making processes (Dong et al., 2021; Zhou et al., 2019; Thong et al., 2019). Moreover, the applications such as stock trending and other financial analysis for decision-making process make utilize the idea of near rings in interval neutrosophic sets (Wang et al., 2012; Sudan et al., 2019; Jha et al., 2020; Rashno et al., 2022; Bui et al., 2023).

The proposed work plan to offer a novel idea into the algebraic structure and applications of IVANFSs. The properties of homomorphisms, union and intersection behaviors are presented in the work to address the uncertainties in near rings. The work also investigate the relationship between IVANFSs and different kinds of fuzzy ideals

1.1. Research Contributions

The characteristics of fuzzy ideals and their role in Near-Rings and associated algebraic structures and the main contributions are given below:

1. Formal Definition of Fuzzy Ideals: The formal definitions of fuzzy ideals are specified in terms of the concept of Near-Rings which is helpful in the work to clarify the base concepts and linking the gaps between theory and practical applications.
2. Development of Interval-Valued Anti-Neutrosophic Fuzzy Ideals (IVANFIs): The interval-valued anti-neutrosophic fuzzy ideals is introduced which can be used as a hybrid framework to integrate the methods such as interval-valued fuzzy sets and anti-neutrosophic logic. This new model effectively address the uncertainty and inconsistency in algebraic structures.
3. Mathematical Properties and Theorems: The key properties involved in IVANFIs, such as homomorphisms, union and intersection, and complementarity behaviors are exposed. These are helpful to analyze the behavior of IVANFIs within Near-Rings.
4. Applications in Decision-Making: The proposed fuzzy ideals are applied in various application in decision-making process that highlights how the theoretical framework supports real-world problems in addressing the uncertainties.
5. Algorithmic Validation: The proposed fuzzy ideals are validated through real world examples which is used to assess the performance of this model in many practical scenarios.

2. PRELIMINARIES

In this section, all the mathematical definitions related to Near-Rings, fuzzy sets, interval-valued fuzzy sets, neutrosophic sets, and anti-neutrosophic sets are defines which forms the basis for IVANFI in Near-Rings.

2.1. Definition: Near-Ring

Let 'N' be a non-empty set. There are two binary operations such as addition (+) and multiplication (·) in near ring (N, +, N.,) satisfies the following:

1. In addition, a group for (N,+) is formed, where ++ is associative, but not essentially commutative.
2. In multiplication, a group for (N,·) is formed, where ·· is associative.
3. The right distributive law holds:

$$a \cdot (b + c) = (a \cdot b) + (a \cdot c), \quad \forall a, b, c \in N.$$

- The addition operation (+) is not required to be commutative, so (N,+) is not necessarily an abelian group.
- Multiplication (·) does not need to distribute over addition from the left.

2.2. Definition: Fuzzy Ideal in a Near-Ring

Let N be a Near-Ring with operations + and ·. μ: N→[0,1] is a fuzzy subset which is often said to be a fuzzy ideal of N if the following conditions are satisfied:

1. Closure under Addition:

For all $a, b \in N$,

$$\mu(a+b) \geq min\{\mu(a), \mu(b)\}.$$

2. Multiplicative Closure:

For all $a \in N$ and $b \in N$,

$$\mu(a \cdot b) \geq \mu(b).$$

3. Neutral Element:

The relationship value of 0∈*N* is the maximum, i.e.,

$\mu(0) = 1$.

The definition discussed in the section helps to establish all the important properties required to model uncertainty and vagueness in algebraic structures such as Near-Rings. This definition is further valued to identify the behavior of near rings under different operations and brings a strong framework for many applications in real-world scenarios.

2.3. Definition: Interval-Valued Fuzzy Set

Let 'N' be a non-empty set. $\bar{\mu}: N \rightarrow D([0,1])$ is considered as a mapping, where D([0,1]) signifies the set of all closed subintervals of [0,1], is said to be an interval-valued fuzzy set (IVFS) on N. For every element, $a \in N, \bar{\mu}(a) = [\mu^L(a), \mu^U(a)]$ is an interval, where μ^L(a) and $\mu^U(a)$ indicates the lower and upper limits of the notch of participation in the fuzzy set, which satisfies $0 \leq \mu^L(a) \leq \mu^U(a) \leq 1$.

2.4. Definition: Interval-Valued Anti-Neutrosophic Set

Let 'N' be a non-empty set. An interval-valued anti-neutrosophic set (IVANS) is defined by three mappings:

- $T_A: N \rightarrow D([0,1])$, ,
- $I_A: N \rightarrow D([0,1])$,
- $F_A: N \rightarrow D([0,1])$.

For each a∈N, these functions assign intervals $T_A(b) = T_A^L(b), T_A^U(b)$, $I_A(b) = I_A^L(b), I_A^U(b)$, and $F_A(b) = F_A^L(b), F_A^U(b)$, where , $0 \leq T_A^L(b) \leq T_A^U(b) \leq 1.0 \leq I_A^L(b) \leq I_A^U(b) \leq F_A^L(b) \leq F_A^U(b) \leq 1$, with the condition:

$T_A^L(a) + F_A^L(a) \leq 1, \forall a, b \in N.$

2.5. Definition: Interval-Valued Anti-Neutrosophic Fuzzy Ideal

Let (N,+,·) be a Near-Ring, and 'A' is an interval-valued anti-neutrosophic fuzzy set in N, categorized by the relationship functions T_A, I_A, F_A. The interval-valued anti-neutrosophic fuzzy ideal (IVANFI) is represented by A of N if the following conditions hold:

For all $a, b \in N$:

$T_A\ (x + y) \geq min\ (T_A(\text{x}), T_A(\text{y}))$, $F_A\ (x + y) \leq\ max\ (F_A\ (x)\ , F_A\ (y)$

For all $x, y \in N$:

$T_A\ (x. y) \geq T_A(\text{y})$, $F_A\ (x. y) \geq F_A(\text{y})$

The above conditions make sure that the set gratifies the properties of both the Near-Ring structure and the interval-valued anti-neutrosophic fuzzy set, hence establishing a strong framework for addressing uncertainties in near rings.

3. CHARACTERIZATION OF IVANF

In this section, the key properties of IVANFI in Near-Rings are exposed in the study. The main aim of this section is to demonstrate the behavior of such ideals with the expression of mathematical analysis under Near-Rings operations, and how membership functions in IVANFI for truth, indeterminacy, and falsity behave.

Let (N,+,·) be a Near-Ring and 'A' belongs to an interval-valued anti-neutrosophic fuzzy ideal (IVANFI) in N. They are categorized with the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A. These ideals are characterized by the following properties with the results.

3.1. Theorem 1 (Additive Property)

Let A be an IVANFI in N. For all a,b∈N, the membership values of the sum a+b satisfy:

$T_A\ (x + y) \geq min\ (T_A(x), T_A(y))$,

$F_A\ (x + y) \leq\ max\ (F_A\ (x)\ , F_A\ (y))$.

This theorem will ensure that the T_A is non-decreasing and the $F_A$is non-increasing for the addition operation. This property is very important to demonstrate that the ideal performs well with respect to the additive structure of the Near-Ring.

3.2. Theorem 2 (Multiplicative Property)

For all a,b∈N, the membership values of the product a·b satisfy:

$T_A\ (x. y) \geq T_A(\text{y})$,

$F_A\ (x. y) \geq F_A(\text{y})$

This theorem is used to confirm that the truth-membership function of the product is at least as high as the T_A of the second operand, and the $F_A$is non-increasing. The multiplicative structure of the near rings are investigated and confirmed.

3.3. Theorem 3 (Scalar Multiplication Property)

Let λ∈N and a∈N. The membership values of the scalar multiplication λ·a satisfy:

$T_A\ (\lambda. a) \geq T_A(\text{a})$, $F_A\ (\lambda. a) \geq F_A(\text{a})$

The result of the above theorem confirms that the truth-membership value augmented under scalar multiplication, and the falsity-membership value is thereby reduced. To ensure the stability under ideal in the operations, this property is crucial for scalar operations within Near-Rings.

3.4. Theorem 4 (Complement Property)

Let A be an IVANFI in N. The complement A¯ of A is defined by:

$T_{\bar{A}}(x+y) \geq min\ (T_{\bar{A}}(x), T_{\bar{A}}(y),$
$F_{\bar{A}}(x+y) \leq max\ (F(x), F_{\bar{A}}(y)$

IVANFI is constructed with the complement of existing properties and is useful for constructing more flexible algebraic manipulations.

3.5. Theorem 5 (Union and Intersection Properties)

Let A and B be two IVANFIs in N. The union and intersection of these ideals are defined by the following membership functions:

$$T_{AUB}(a) = ma\,x\big(T_A(a), T_B(a)\big),$$
$$F_{AUB}(a) = \ min\ (F_A(a), F_B(a)),$$
$$T_{AnB}(a) = min\,x\big(T_A(a), T_B(a)\big),$$
$$F_{AnB}(a) = \ max\ (F_A(a), F_B(a)),$$

for all a∈N. IVANFIs is also considered as the union and intersection of two IVANFIs. The result discussed here brings the clear observation that the set of IVANFIs is closed under various operations with the lattice structure which makes the various combinations of ideals.

3.6. Theorem: Homomorphism of IVANFIs

Let $f: N \rightarrow M$ be a Near-Ring homomorphism, and let A be an IVANFI in N. The copy of A under f, designated by f(A), is an IVANFI in M, and the membership functions is given as:

$$T_{f(A)}\left(f(x)\right) = \ T_A(x), I_{f(A)}\left(f(x)\right) = \ I_A(x), F_{f(A)}\left(f(x)\right) = \ F_A(x)$$

for all x∈N. The notable thing in this theorem is that that IVANFIs are well-maintained under homomorphisms, founding their invariance under Near-Ring morphisms.

The performance of interval-valued anti-neutrosophic fuzzy ideals is captured with the above characterization in seizing uncertainty, indeterminacy, and falsity in Near-Rings' algebraic structure. By providing these formal properties, the important framework to deal with IVANFIs is exposed which is specially applied in algebraic systems.

Example 3.1. Let $N_1, N_2, N_3 = \{0,1,2\}$. Addition modulo 3 and $\cdot_{N_1}, \cdot_{N_2}, \cdot_{N_3}$ be defined as follows,

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

$\cdot_{N_1}$	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2

$\cdot_{N_2}$	0	1	2
0	0	0	0
1	0	1	2
2	0	1	2

$\cdot_{N_3}$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Then clearly, $(N, +, \cdot_{N_1}), (N, +, \cdot_{N_2}), (N, +, \cdot_{N_3})$ are left near-rings. Let

$P_{\bar{f}_1}: N_1 \rightarrow D[0,1], P_{\bar{f}_2}: N_2 \rightarrow D[0,1]$, and $P_{\bar{f}_3}: N_3 \rightarrow D[0,1]$ be an i-v neutrosophic fuzzy subsets of N_1, N_2 and N_3 respectively defined by,

$P_{\bar{S}_1}(0) = [0.2, 0.3], P_{\bar{S}_1}(1) = P_{\bar{S}_1}(2) = [0.3,0.4]$
$P_{\bar{S}_2}(0) = [0.4, 0.5], P_{\bar{S}_2}(1) = P_{\bar{S}_2}(2) = [0.5,0.6]$
$P_{\bar{S}_3}(0) = [0.5, 0.6], P_{\bar{S}_3}(1) = P_{\bar{S}_3}(2) = [0.7,0.8]$.

Clearly, $P_{\bar{S}_1}$, $P_{\bar{S}_2}$and $P_{\bar{S}_3}$are i-v neutrosophic fuzzy ideals of N_1, N_2 and N_3. Let $N = N_1 \times N_2 \times N_3$ and $P_{\bar{S}} = P_{\bar{S}_1} \times P_{\bar{S}_2} \times P_{\bar{S}_3}$. Let $\bar{S}: N \rightarrow D[0,1]$ defined by $P_{\bar{S}}(l_1, l_2, l_3) = max\ \{P_{\bar{S}_1}(l_1),\ P_{\bar{S}_2}(l_2),\ P_{\bar{S}_3}(l_3)\}$ for all $f_1 \in N_1, s_2 \in N_2$ and $l_3 \in N_3$,

$P_{\bar{S}}(0,0,0) = [0.5,0.6]$	$P_{\bar{S}}(0,0,1) = [0.7,0.8]$	$P_{\bar{S}}(0,0,2) = [0.7,0.8]$
$P_{\bar{S}}(0,1,0) = [0.5,0.6]$	$P_{\bar{S}}(0,1,1) = [0.7,0.8]$	$P_{\bar{S}}(0,1,2) = [0.7,0.8]$
$P_{\bar{S}}(0,2,0) = [0.5,0.6]$	$P_{\bar{S}}(0,2,1) = [0.7,0.8]$	$P_{\bar{S}}(0,2,2) = [0.7,0.8]$
$P_{\bar{S}}(1,0,0) = [0.5,0.6]$	$P_{\bar{S}}(1,0,1) = [0.7,0.8]$	$P_{\bar{S}}(1,0,2) = [0.7,0.8]$
$P_{\bar{S}}(1,1,0) = [0.5,0.6]$	$P_{\bar{S}}(1,1,1) = [0.7,0.8]$	$P_{\bar{S}}(1,1,2) = [0.7,0.8]$
$P_{\bar{S}}(1,2,0) = [0.5,0.6]$	$P_{\bar{S}}(1,2,1) = [0.7,0.8]$	$P_{\bar{S}}(1,2,2) = [0.7,0.8]$
$P_{\bar{S}}(2,0,0) = [0.5,0.6]$	$P_{\bar{S}}(2,0,1) = [0.7,0.8]$	$P_{\bar{S}}(2,0,2) = [0.7,0.8]$
$P_{\bar{S}}(2,1,0) = [0.5,0.6]$	$P_{\bar{S}}(2,1,1) = [0.7,0.8]$	$P_{\bar{S}}(2,1,2) = [0.7,0.8]$
$P_{\bar{S}}(2,2,0) = [0.5,0.6]$	$P_{\bar{S}}(2,2,1) = [0.7,0.8]$	$P_{\bar{S}}(2,2,2) = [0.7,0.8]$

Let$Q_{\bar{S}_1}: N_1 \rightarrow D[0,1], Q_{\bar{S}_2}: N_2 \rightarrow D[0,1]$ and $Q_{\bar{S}_3}: N_3 \rightarrow D[0,1]$ be an i-v neutrosophic fuzzy subsets of N_1, N_2 and N_3 respectively defined by,

$Q_{\bar{S}_1}(0) = [0.4, 0.5] \quad Q_{\bar{S}_1}(1) = Q_{\bar{S}_1}(2) = [0.2, 0.3]$
$Q_{\bar{S}_2}(0) = [0.7, 0.8] \quad Q_{\bar{S}_2}(1) = Q_{\bar{S}_2}(2) = [0.5,0.6]$
$Q_{\bar{S}_3}(0) = [0.8, 0.9] \quad Q_{\bar{S}_3}(1) = Q_{\bar{S}_3}(2) = [0.6,0.7]$.

Clearly, $Q_{\bar{S}_1}$, $Q_{\bar{S}_2}$and $Q_{\bar{S}_3}$are i-v neutrosophic fuzzy ideals of N_1, N_2 and N_3. Let $N = N_1 \times N_2 \times N_3$ and $Q_{\bar{S}} = Q_{\bar{S}_1} \times Q_{\bar{S}_2} \times Q_{\bar{S}_3}$. Let $\bar{S}: N \rightarrow D[0,1]$ defined by $Q_{\bar{S}}(l_1, l_2, l_3) = min\ \{Q_{\bar{S}_1}(l_1),\ Q_{\bar{S}_2}(l_2),\ Q_{\bar{S}_3}(l_3)\}$ for all $l_1 \in N_1, l_2 \in N_2$ and $l_3 \in N_3$ is shown below,

$Q_{\bar{S}}(0,0,0) = [0.4,0.5]$	$Q_{\bar{S}}(0,0,1) = [0.4,0.5]$	$Q_{\bar{S}}(0,0,2) = [0.4,0.5]$
$Q_{\bar{S}}(0,1,0) = [0.4,0.5]$	$Q_{\bar{S}}(0,1,1) = [0.4,0.5]$	$Q_{\bar{S}}(0,1,2) = [0.4,0.5]$
$Q_{\bar{S}}(0,2,0) = [0.4,0.5]$	$Q_{\bar{S}}(0,2,1) = [0.4,0.5]$	$Q_{\bar{S}}(0,2,2) = [0.4,0.5]$
$Q_{\bar{S}}(1,0,0) = [0.2,0.3]$	$Q_{\bar{S}}(1,1,0) = [0.2,0.3]$	$Q_{\bar{S}}(1,0,2) = [0.2,0.3]$
$Q_{\bar{S}}(1,1,0) = [0.2,0.3]$	$Q_{\bar{S}}(1,1,1) = [0.2,0.2]$	$Q_{\bar{S}}(1,1,2) = [0.2,0.3]$
$Q_{\bar{S}}(1,2,0) = [0.2,0.3]$	$Q_{\bar{S}}(1,2,1) = [0.2,0.3]$	$Q_{\bar{S}}(1,2,2) = [0.2,0.3]$
$Q_{\bar{S}}(2,0,0) = [0.2,0.3]$	$Q_{\bar{S}}(2,0,2) = [0.2,02]$	$Q_{\bar{S}}(2,0,2) = [0.2,0.3]$
$Q_{\bar{S}}(2,1,0) = [0.2,0.3]$	$Q_{\bar{S}}(2,1,1) = [0.2,0.3]$	$Q_{\bar{S}}(2,1,2) = [0.2,0.3]$
$Q_{\bar{S}}(2,2,0) = [0.2,0.3]$	$Q_{\bar{S}}(2,2,1) = [0.2,0.3]$	$Q_{\bar{S}}(2,2,2) = [0.2,0.3]$

Let$R_{\bar{S}_1}: N_1 \rightarrow D[0,1], R_{\bar{S}_2}: N_2 \rightarrow D[0,1]$ and $R_{\bar{S}_3}: N_3 \rightarrow D[0,1]$ be an i-v neutrosophic fuzzy subsets of N_1, N_2 and N_3 respectively defined by,

$R_{\bar{S}_1}(0) = [0.3, 0.4] \quad R_{\bar{S}_1}(1) = R_{\bar{S}_1}(2) = [0.2, 0.3]$
$R_{\bar{S}_2}(0) = [0.5, 0.6] \quad R_{\bar{S}_2}(1) = R_{\bar{S}_2}(2) = [0.4,0.5]$
$R_{\bar{S}_3}(0) = [0.7, 0.8] \quad R_{\bar{S}_3}(1) = R_{\bar{S}_3}(2) = [0.6,0.7]$.

Clearly, $R_{\bar{S}_1}$, $R_{\bar{S}_2}$ and $R_{\bar{S}_3}$ and an i-v neutrosophic fuzzy ideals of N_1, N_2 and N_3. Let $N = N_1 \times N_2 \times N_3$ and $R_{\bar{S}} = R_{\bar{S}_1} \times R_{\bar{S}_2} \times R_{\bar{S}_3}$. Let $\bar{S}: N \to D[0,1]$ defined by $R_{\bar{S}}(l_1, l_2, l_3) = min\ \{R_{\bar{S}_1}(l_1),\ R_{\bar{S}_2}(l_2),\ R_{\bar{S}_3}(l_3)\}$ for all $l_1 \in N_1, l_2 \in N_2$ and $l_3 \in N_3$,

$R_{\bar{S}}(0,0,0) = [0.3,0.4]$	$R_{\bar{S}}(0,0,1) = [0.3,0.4]$	$R_{\bar{S}}(0,0,2) = [0.3,0.4]$
$R_{\bar{S}}(0,1,0) = [0.3,0.4]$	$R_{\bar{S}}(0,1,1) = [0.3,0.4]$	$R_{\bar{S}}(0,1,2) = [0.3,0.4]$
$R_{\bar{S}}(0,2,0) = [0.3,0.4]$	$R_{\bar{S}}(0,2,1) = [0.3,0.4]$	$R_{\bar{S}}(0,2,2) = [0.3,0.4]$
$R_{\bar{S}}(1,0,0) = [0.2,0.3]$	$R_{\bar{S}}(1,0,1) = [0.2,0.3]$	$R_{\bar{S}}(1,0,2) = [0.2,0.3]$
$R_{\bar{S}}(1,1,0) = [0.2,0.3]$	$R_{\bar{S}}(1,1,1) = [0.2,0.3]$	$R_{\bar{S}}(1,1,2) = [0.2,0.3]$
$R_{\bar{S}}(1,2,0) = [0.2,0.3]$	$R_{\bar{S}}(1,2,1) = [0.2,0.3]$	$R_{\bar{S}}(1,2,2) = [0.2,0.3]$
$R_{\bar{S}}(2,0,0) = [0.2,0.3]$	$R_{\bar{S}}(2,0,1) = [0.2,0.3]$	$R_{\bar{S}}(2,0,2) = [0.2,0.3]$
$R_{\bar{S}}(2,1,0) = [0.2,0.3]$	$R_{\bar{S}}(2,1,1) = [0.2,0.3]$	$R_{\bar{S}}(2,1,2) = [0.2,0.3]$
$R_{\bar{S}}(2,2,0) = [0.2,0.3]$	$R_{\bar{S}}(2,2,1) = [0.2,0.3]$	$R_{\bar{S}}(2,2,2) = [0.2,0.3]$

3.7. Theorem 7 (Product of IVANFIs)

Let $\bar{S}_1$ and $\bar{S}_2$ be an i-v anti neutrosophic fuzzy ideal of N_1, N_2 respectively. Then $\bar{S}_1 \times \bar{S}_2$ is an i-v anti neutrosophic fuzzy ideal of $N_1 \times N_2$.

Proof. Let $\bar{S}_1$ and $\bar{S}_2$ be an i-v anti neutrosophic fuzzy ideal of N_1, N_2 respectively. Let $(k_1, k_2), (l_1, l_2), (v_1, v_2) \in N_1 \times N_2$.

$$P_{\bar{f}_1\times\bar{f}_2}\big((f_1, f_2) - (z_1, z_2)\big) = P_{\bar{f}_1\times\bar{f}_2}\big((v_1 - m_1), (v_2 - m_2)\big)$$
$$= max\big\{P_{\bar{S}_1}(z_1 - m_1), P_{\bar{S}_2}(z_2 - y_2)\big\}$$
$$\leq$$
$$max\ \{max\big\{P_{\bar{S}_1}(z_1), P_{\bar{S}_1}(v_1)\big\}, max\ \{P_{\bar{S}_2}(l_2), P_{\bar{S}_2}(y_2)\}\}$$
$$=$$
$$max\ \{max\big\{P_{\bar{S}_1}(z_1), P_{\bar{S}_2}(z_2)\big\}, max\ \{n_{\bar{S}_1}(z_1), n_{\bar{S}_2}(z_2)\}\}$$
$$= max\ \{P_{\bar{S}_1\times\bar{S}_2}(l_1, l_2), P_{\bar{S}_1\times\bar{S}_2}(z_1, z_2)\}$$

$$Q_{\bar{f}_1\times\bar{f}_2}\big((f_1, f_2) - (z_1, z_2)\big) = Q_{\bar{f}_1\times\bar{f}_2}\big((v_1 - z_1), (v_2 - z_2)\big)$$
$$= min\big\{Q_{\bar{S}_1}(v_1 - z_1), Q_{\bar{S}_2}(v_2 - z_2)\big\}$$
$$\geq$$
$$min\ \{min\big\{Q_{\bar{S}_1}(l_1), Q_{\bar{S}_1}(z_1)\big\}, min\ \{z_{\bar{S}_2}(l_2), z_{\bar{S}_2}(z_2)\}\}$$
$$=$$
$$min\ \{min\big\{Q_{\bar{S}_1}(l_1), Q_{\bar{S}_2}(l_2)\big\}, min\ \{z_{\bar{S}_1}(z_1), z_{\bar{S}_2}(z_2)\}\}$$
$$= min\ \{Q_{\bar{S}_1\times\bar{S}_2}(y_1, y_2), Q_{\bar{S}_1\times\bar{S}_2}(z_1, z_2)\}$$

$$R_{\bar{f}_1\times\bar{f}_2}\big((f_1, f_2) - (z_1, z_2)\big) = R_{\bar{f}_1\times\bar{f}_2}\big((y_1 - y_1), (v_2 - y_2)\big)$$
$$= min\big\{R_{\bar{S}_1}(y_1 - z_1), R_{\bar{S}_2}(y_2 - z_2)\big\}$$
$$\geq$$
$$min\ \{min\big\{R_{\bar{S}_1}(l_1), R_{\bar{S}_1}(k_1)\big\}, min\ \{z_{\bar{S}_2}(l_2), z_{\bar{S}_2}(k_2)\}\}$$
$$=$$
$$min\ \{min\big\{R_{\bar{S}_1}(v_1), R_{\bar{S}_2}(v_2)\big\}, min\ \{z_{\bar{S}_1}(k_1), z_{\bar{S}_2}(k_2)\}\}$$
$$= min\ \{R_{\bar{S}_1\times\bar{S}_2}(v_1, v_2), R_{\bar{S}_1\times\bar{S}_2}(k_1, k_2)\}$$

$$P_{\bar{f}_1\times\bar{f}_2}\big((f_1,f_2)(z_1,z_2)\big) = P_{\bar{f}_1\times\bar{f}_2}\big((l_1z_1),(l_2z_2)\big)$$
$$= max\big\{P_{\bar{S}_1}(l_1k_1),P_{\bar{S}_2}(l_2k_2)\big\}$$
$$\leq$$
$$max\,\{max\big\{P_{\bar{S}_1}(l_1),P_{\bar{S}_1}(z_1)\big\},max\,\{P_{\bar{S}_2}(l_2),P_{\bar{S}_2}(z_2)\}\}$$
$$=$$
$$max\,\{max\big\{P_{\bar{S}_1}(l_1),P_{\bar{S}_2}(l_2)\big\},max\,\{P_{\bar{S}_1}(k),P_{\bar{S}_2}(k_2)\}\}$$
$$= max\,\{P_{\bar{S}_1\times\bar{S}_2}(v_1,v_2),P_{\bar{S}_1\times\bar{S}_2}(m_1,m_2)\}$$
$$Q_{\bar{f}_1\times\bar{f}_2}\big((l_1,l_2)(k_1,k)\big) = Q_{\bar{f}_1\times\bar{f}_2}\big((l_1k_1),(l_2k_2)\big)$$
$$= min\big\{Q_{\bar{S}_1}(y_1z_1),Q_{\bar{S}_2}(z_2z_2)\big\}$$
$$\geq$$
$$min\,\{min\big\{Q_{\bar{S}_1}(l_1),Q_{\bar{S}_1}(z_1)\big\},min\,\{Q_{\bar{S}_2}(l_2),Q_{\bar{S}_2}(z_2)\}\}$$
$$=$$
$$min\,\{min\big\{Q_{\bar{S}_1}(l_1),Q_{\bar{S}_2}(l_2)\big\},min\,\{Q_{\bar{S}_1}(z_1),Q_{\bar{S}_2}(z_2)\}\}$$
$$= min\,\{Q_{\bar{S}_1\times\bar{S}_2}(v_1,v_2),Q_{\bar{S}_1\times\bar{S}_2}(z_1,z_2)\}$$
$$R_{\bar{f}_1\times\bar{f}_2}\big((l_1,l_2)(z_1,z_2)\big) = R_{\bar{f}_1\times\bar{f}_2}\big((n_1z_1),(n_2z_2)\big)$$
$$= min\big\{R_{\bar{S}_1}(l_1z_1),R_{\bar{S}_2}(l_2z_2)\big\}$$
$$\geq$$
$$min\,\{min\big\{R_{\bar{S}_1}(l_1),R_{\bar{S}_1}(k_1)\big\},min\,\{R_{\bar{S}_2}(y_2),R_{\bar{S}_2}(z_2)\}\}$$
$$=$$
$$min\,\{min\big\{R_{\bar{S}_1}(l_1),R_{\bar{S}_2}(l_2)\big\},min\,\{R_{\bar{S}_1}(m_1),R_{\bar{S}_2}(m_2)\}\}$$
$$= min\,\{R_{\bar{S}_1\times\bar{S}_2}(y_1,y_2),R_{\bar{S}_1\times\bar{S}_2}(z_1,z_2)\}$$
$$P_{\bar{f}_1\times\bar{f}_2}\big((x_1,x_2)+(v_1,v_2)-(n_1,n_2)\big) = P_{\bar{f}_1\times\bar{f}_2}\big((k_1+v_1-k_1),(k_2+v_2-k_2)\big)$$
$$= max\big\{P_{\bar{S}_1}(z_1+l_1-z_1),P_{\bar{S}_2}(z_2+l_2-z_2)\big\}$$
$$\leq max\big\{P_{\bar{S}_1}(y_1),P_{\bar{S}_2}(y_2)\big\}$$
$$= P_{\bar{S}_1\times\bar{S}_2}(l_1,l_2)$$
$$Q_{\bar{S}_1\times\bar{S}_2}\big((x_1,x_2)+(v_1,v_2)-(n_1,n_2)\big) = Q_{\bar{f}_1\times\bar{f}_2}\big((z_1+y_1-k_1),(z_2+v_2-z_2)\big)$$
$$= min\big\{Q_{\bar{S}_1}(n_1+l_1-q_1),Q_{\bar{S}_2}(q_2+l_2-q_2)\big\} \quad \geq min\big\{Q_{\bar{S}_1}(l_1),Q_{\bar{S}_2}(l_2)\big\}$$
$$= Q_{\bar{S}_1\times\bar{S}_2}(l_1,l_2)$$

$$R_{\bar{S}_1\times\bar{S}_2}\big((x_1,x_2)+(v_1,v_2)-(x_1,x_2)\big) = R_{\bar{S}_1\times\bar{S}_2}\big((x_1+l_1-x_1),(x_2+l_2-x_2)\big)$$
$$= min\big\{R_{\bar{S}_1}(n_1+l_1-n_1),R_{\bar{S}_2}(n_2+l_2-n_2)\big\}$$
$$\geq min\big\{R_{\bar{S}_1}(l_1),R_{\bar{S}_2}(l_2)\big\}$$
$$= R_{\bar{S}_1\times\bar{S}_2}(l_1,l_2)$$
$$P_{\bar{f}_1\times\bar{f}_2}\big((v_1,v_2)(k_1,k_2)\big) = P_{\bar{f}_1\times\bar{f}_2}\big((y_1z_1),(y_2z_2)\big)$$
$$= max\big\{P_{\bar{S}_1}(x_1m_1),P_{\bar{S}_2}(x_2m_2)\big\}$$
$$\leq max\big\{P_{\bar{S}_1}(z_1),P_{\bar{S}_2}(z_2)\big\}$$
$$=P_{\bar{f}_1\times\bar{f}_2}(k_1,k_2)$$
$$Q_{\bar{f}_1\times\bar{f}_2}\big((y_1,l_2)(z_1,z_2)\big) = Q_{\bar{f}_1\times\bar{f}_2}\big((l_1z_1),(l_2z_2)\big)$$

$$= min\{Q_{\bar{S}_1}(x_1m_1), Q_{\bar{S}_2}(x_2m_2)\}$$
$$\geq min\{Q_{\bar{S}_1}(z_1), Q_{\bar{S}_2}(z_2)\}$$
$$=Q_{\bar{S}_1\times\bar{S}_2}(k_1,k_2)$$
$$R_{\bar{f}_1\times\bar{f}_2}\big((f_1,f_2)(k_1,k_2)\big) = R_{\bar{f}_1\times\bar{f}_2}\big((l_1k_1),(l_2k_2)\big)$$
$$= min\{R_{\bar{S}_1}(x_1z_1), R_{\bar{S}_2}(x_2z_2)\}$$
$$\geq min\{R_{\bar{S}_1}(x_1z_1), R_{\bar{S}_2}(x_2z_2)\}$$
$$= R_{\bar{S}_1\times\bar{S}_2}(z_1,z_2)$$
$$P_{\bar{S}_1\times\bar{S}_2}\Big(\big((x_1,x_2)+(c_1,c_2)\big)(x_1,x_2)-(f_1,f_2)(x_1,x_2)\Big)$$
$$= P_{\bar{S}_1\times\bar{S}_2}\big(((v_1+x_1)k_1-l_1z_1),((v_2+n_2)z_2-l_2z_2)\big)$$
$$= max\{P_{\bar{S}_1}((v_1+m_1)n_1-y_1z_1), P_{\bar{S}_2}((v_2+n_2)z_2-l_2z_2)\}$$
$$\leq max\{P_{\bar{S}_1}(n_1), P_{\bar{S}_2}(n_2)\}$$
$$= P_{\bar{S}_1\times\bar{S}_2}(n_1,n_2)$$
$$Q_{\bar{S}_1\times\bar{S}_2}\Big(\big((f_1,f_2)+(y_1,y_2)\big)(k_1,k_2)-(h_1,h_2)(k_1,k_2)\Big)$$
$$= Q_{\bar{S}_1\times\bar{S}_2}\big((v_1+n_1)z_1-v_1z_1),((v_2+n_2)z_2-n_2z_2)\big)$$
$$= min\{Q_{\bar{S}_1}((v_1+n_1)m_1-l_1m_1), Q_{\bar{S}_2}((v_2+n_2)m_2-l_2m_2)\}$$
$$\geq min\{Q_{\bar{S}_1}(n_1), Q_{\bar{S}_2}(n_2)\}$$
$$= Q_{\bar{S}_1\times\bar{S}_2}(n_1,n_2)$$
$$R_{\bar{S}_1\times\bar{S}_2}\Big(\big((f_1,f_2)+(n_1,n_2)\big)(z_1,z_2)-(f_1,f_2)(m_1,m_2)\Big)$$
$$= R_{\bar{S}_1\times\bar{S}_2}\big((y_1+z_1)y_1-v_1z_1),((v_2+n_2)z_2-v_2z_2)\big)$$
$$= min\{R_{\bar{S}_1}((l_1+n_1)z_1-l_1z_1), R_{\bar{S}_2}((v_2+n_2)k_2-v_2z_2)\}$$
$$\geq min\{R_{\bar{S}_1}(n_1), R_{\bar{S}_2}(n_2)\}$$
$$= R_{\bar{S}_1\times\bar{S}_2}(b_1,b_2)$$

Therefore $\bar{S}_1 \times \bar{S}_2$ is an IVAN fuzzy ideal of $N_1 \times N_2$.

3.8. Definition (Complement of an IVAN Set)

The complement of an IVAN set $\bar{S}$ is denoted by $\bar{S}^C$ and defined as follows,

$$(1)\ P_{\bar{S}^C}(l) = 1-\{P_{\bar{f}}(l)\} = [1-P_{S^+}(l), 1-P_{S^-}(l)]$$
$$(2)\ Q_{\bar{S}^C}(l) = 1-\{Q_f(l)\} = [1-Q_{S^+}(l), 1-Q_{S^-}(l)]$$
$$(3)\ R_{\bar{S}^C}(l) = 1-\{R_{\bar{f}}(l)\} = [1-R_{S^+}(l), 1-R_{S^-}(l)]$$

Example 3.2. Let $Z_3 = \{0,1,2\}$, the set of integers modulo 3. The operations of addition and multiplication modulo 3 are defined as follows:

Addition Table (modulo 3):

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

Multiplication Table (modulo 3):

·	0	1	2
0	0	0	0
1	0	1	2
2	0	1	2

In this left near-ring structure, the set N is Z_3={0,1,2}. Interval-valued anti-neutrosophic fuzzy ideals P_s, R_s, Q_s are defined on N. For P_s, the membership intervals for elements of N are assigned such that the membership value for one element differs significantly from others, reflecting the properties of anti-neutrosophic fuzzy ideals. Similarly, for $R_s$and Q_s, membership intervals are assigned for each element of N.

3.9. Definition (Complement of Fuzzy Ideals)

The complement of each fuzzy ideal is constructed by reversing the truth values of the membership intervals while maintaining the structure required for interval-valued fuzzy systems. For example:

The complement of P_s assigns new intervals for elements 0,1,2 in Z3, ensuring consistency with the complement definition in fuzzy theory.

A similar process is applied to $R_s$and Q_s, with their respective membership intervals adjusted accordingly.

It can be verified that these complements, denoted P'_s, R'_s, Q'_s, also form valid interval-valued anti-neutrosophic fuzzy ideals within the near-ring structure $(Z_3,+,\cdot)$ These complements preserve the necessary properties of such ideals in algebraic structures.

3.10. Theorem 8 (Complement as an IVAN Fuzzy Ideal)

The IVANF of $\bar{S}$ in N is IVAN of N if and only if its complement $\bar{S}^C$ is an i-v neutrosophic fuzzy ideal of N.

Proof. Let $\bar{S}$ is an IVANF of N. For $g, h, i \in N$,

$$Y_{\bar{S}^C}(g-h) = \bar{1} - Y_{\bar{S}}(g-h)$$
$$\geq \bar{1} - max\ \{Y_{\bar{S}}(l), Y_{\bar{S}}(m)\}$$
$$= min\ \{\bar{1} - Y_{\bar{S}}(g), \bar{1} - Y_{\bar{S}}(h)\}$$
$$= min\ \{P_{\bar{S}^C}(g), P_{\bar{S}^C}(h)\}$$
$$Z_{\bar{S}^C}(g-h) = \bar{1} - Z_{\bar{S}}(g-h)$$
$$\leq \bar{1} - min\ \{z_{\bar{S}}(g), z_{\bar{S}}(h)\}$$
$$= max\ \{\bar{1} - z_{\bar{S}}(g), \bar{1} - z_{\bar{S}}(h)\}$$
$$= max\ \{z_{\bar{S}^C}(g), z_{\bar{S}^C}(h)\}$$
$$k_{\bar{S}^C}(g-h) = \bar{1} - k_{\bar{S}}(g-h)$$
$$\leq \bar{1} - min\ \{k_{\bar{S}}(g), k_{\bar{S}}(h)\}$$
$$= max\ \{\bar{1} - k_{\bar{S}}(g), \bar{1} - k_{\bar{S}}(h)\}$$
$$= max\ \{k_{\bar{S}^C}(g), k_{\bar{S}^C}(h)\}$$
$$Y_{\bar{S}^C}(gh) = \bar{1} - Y_{\bar{S}}(gh)$$
$$\geq \bar{1} - max\ \{Y_{\bar{S}}(g), Y_{\bar{S}}(h)\}$$
$$= min\ \{\bar{1} - Y_{\bar{S}}(g), \bar{1} - Y_{\bar{S}}(h)\}$$
$$= min\ \{Y_{\bar{S}^C}(g), Y_{\bar{S}^C}(h)\}$$
$$Z_{\bar{S}^C}(gh) = \bar{1} - Z_{\bar{S}}(gh)$$
$$\leq \bar{1} - min\ \{Z_{\bar{S}}(g), Z_{\bar{S}}(h)\}$$

$= max\ \{\bar{1} - Z_{\bar{S}}(g), \bar{1} - Z_{\bar{S}}(h)\}$

$= max\ \{Z_{\bar{S}^c}(g), Z_{\bar{S}^c}(h)\}$

$R_{\bar{S}^c}(gh) = \bar{1} - R_{\bar{S}}(gh)$

$\leq \bar{1} - min\ \{R_{\bar{S}}(g), R_{\bar{S}}(h)\}$

$= max\ \{\bar{1} - R_{\bar{S}}(g), \bar{1} - R_{\bar{S}}(h)\}$

$= max\ \{R_{\bar{S}^c}(g), R_{\bar{S}^c}(h)\}$

$P_{\bar{S}^c}(x + k - x) = \bar{1} - P_{\bar{S}}(x + g - x)$

$\geq \bar{1} - P_{\bar{S}}(g)$

$= P_{\bar{S}^c}(g)$

$Q_{\bar{S}^c}(x + k - x) = \bar{1} - Q_{\bar{S}}(x + g - x)$

$\leq \bar{1} - Q_{\bar{S}}(g)$

$= Q_{\bar{S}^c}(g)$

$R_{\bar{S}^c}(x + g - x) = \bar{1} - R_{\bar{S}}(x + g - x)$

$\leq \bar{1} - R_{\bar{S}}(g)$

$= R_{\bar{S}^c}(g)$

$P_{\bar{S}^c}(gh) = \bar{1} - P_{\bar{S}}(gh)$

$\geq \bar{1} - P_{\bar{S}}(h)$

$= P_{\bar{S}^c}(h)$

$Q_{\bar{S}^c}(gh) = \bar{1} - Q_{\bar{S}}(gh)$

$\leq \bar{1} - Q_{\bar{S}}(h)$

$= Q_{\bar{S}^c}(h)$

$R_{\bar{S}^c}(gh) = \bar{1} - R_{\bar{S}}(gh)$

$\leq \bar{1} - R_{\bar{S}}(h)$

$= R_{\bar{S}^c}(h)$

$P_{\bar{S}^c}((x + n)c - gh) = \bar{1} - P_{\bar{S}}((x + n)c - gh)$

$\geq \bar{1} - P_{\bar{S}}(n)$

$= P_{\bar{S}^c}(n)$

$Q_{\bar{S}^c}((x + n)c - gh) = \bar{1} - Q_{\bar{S}}((x + n)c - gh)$

$\leq \bar{1} - Q_{\bar{S}}(n)$

$= Q_{\bar{S}^c}(n)$

$R_{\bar{S}^c}((x + n)c - gh) = \bar{1} - R_{\bar{S}}((x + n)c - gh)$

$\leq \bar{1} - R_{\bar{S}}(n)$

$= R_{\bar{S}^c}(n)$

Therefore $\bar{S}^C$ is an IVAN ideal of N.

Le $\bar{S}^C$is an i-v neutrosophic fuzzy ideal of N. For $g, h, i \in N$. Then

$\bar{1} - P_{\bar{S}}(g - h) = P_{\bar{S}^c}(g - h)$

$\geq min\{P_{\bar{S}^c}(l), P_{\bar{S}^c}(n)\}$

$= \bar{1} - max\ \{P_{\bar{S}}(v), P_{\bar{S}}(n)\}$

which is infers that, $\bar{1} - P_{\bar{S}}(x - y) \geq\ \bar{1} - max\ \{P_{\bar{S}}(g), P_{\bar{S}}(h)\}$.

Thus $P_{\bar{S}}(k - m) \leq\ max\ \{P_{\bar{S}}(g), P_{\bar{S}}(h)\}$

$$\bar{1} - Q_{\bar{S}}(k - m) = Q_{\bar{S}^c}(k - m)$$
$$\leq max\{Q_{\bar{S}^c}(v), Q_{\bar{S}^c}(n)\}$$
$$= \bar{1} - min\ \{Q_{\bar{S}}(v), Q_{\bar{S}}(n)\}$$

which is infers that, $\bar{1} - Q_{\bar{S}}(k - m) \leq \bar{1} - min\ \{Q_{\bar{S}}(l), Q_{\bar{S}}(m)\}$.

Thus $Q_{\bar{S}}(k - m) \geq min\ \{Q_{\bar{S}}(g), Q_{\bar{S}}(h)\}$

$$\bar{1} - R_{\bar{S}}(g - h) = R_{\bar{S}^c}(g - h)$$
$$\leq max\{R_{\bar{S}^c}(l), R_{\bar{S}^c}(v)\}$$
$$= \bar{1} - min\ \{R_{\bar{S}}(v), R_{\bar{S}}(n)\}$$

which is infers that, $\bar{1} - R_{\bar{S}}(x - m) \leq \bar{1} - min\ \{R_{\bar{S}}(g), R_{\bar{S}}(h)\}$.

Thus $z_{\bar{S}}(k - m) \geq min\ \{z_{\bar{S}}(g), R_{\bar{S}}(h)\}$

$$\bar{1} - Y_{\bar{S}}(lm) = Y_{\bar{S}^c}(lm)$$
$$\geq min\{Y_{\bar{S}^c}(l), Y_{\bar{S}^c}(x)\}$$
$$= \bar{1} - max\ \{Y_{\bar{S}}(v), Y_{\bar{S}}(n)\}$$
$$\bar{1} - Y_{\bar{S}}(lm) \geq \bar{1} - max\ \{Y_{\bar{S}}(v), Y_{\bar{S}}(n)\}.$$

Therefore $P_{\bar{S}}(lm) \leq max\ \{P_{\bar{S}}(l), P_{\bar{S}}(m)\}$

and
$$\bar{1} - Q_{\bar{S}}(lm) = Q_{\bar{S}^c}(lm)$$
$$\leq max\{Q_{\bar{S}^c}(v), Q_{\bar{S}^c}(x)\}$$
$$= \bar{1} - min\ \{Q_{\bar{S}}(l), Q_{\bar{S}}(m)\}$$
$$\bar{1} - Q_{\bar{S}}(xv) \leq \bar{1} - min\ \{Q_{\bar{S}}(x), Q_{\bar{S}}(x)\}.$$

Therefore $Q_{\bar{S}}(lm) \geq min\ \{Q_{\bar{S}}(v), Q_{\bar{S}}(x)\}$

and
$$\bar{1} - z_{\bar{S}}(lm) = z_{\bar{S}^c}(lm)$$
$$\leq max\{z_{\bar{S}^c}(l), z_{\bar{S}^c}(m)\}$$
$$= \bar{1} - min\ \{z_{\bar{S}}(l), z_{\bar{S}}(m)\}$$
$$\bar{1} - R_{\bar{S}}(lm) \leq \bar{1} - min\ \{z_{\bar{S}}(l), z_{\bar{S}}(m)\}.$$

Therefore $z_{\bar{S}}(lm) \geq min\ \{z_{\bar{S}}(l), z_{\bar{S}}(m)\}$

and
$$\bar{1} - P_{\bar{S}}(c + k - x) = P_{\bar{S}^c}(c + l - x)$$
$$\geq P_{\bar{S}^c}(l)$$
$$= \bar{1} - P_{\bar{S}}(l)$$

Therefore $P_{\bar{S}}(m + g - m) \leq P_{\bar{S}}(l)$

$$\bar{1} - Q_{\bar{S}}(x + k - v) = Q_{\bar{S}^c}(x + k - v)$$
$$\leq Q_{\bar{S}^c}(g)$$
$$= \bar{1} - Q_{\bar{S}}(g)$$

Therefore $Q_{\bar{S}}(m + g - m) \geq Q_{\bar{S}}(g)$.

$$\bar{1} - R_{\bar{S}}(x + k - m) = R_{\bar{S}^c}(x + k - m)$$
$$\leq R_{\bar{S}^c}(g)$$
$$= R_{\bar{S}}(g)$$

Therefore $R_{\bar{S}}(x + k - m) \geq R_{\bar{S}}(l)$.

Next
$$\bar{1} - P_{\bar{S}}(gh) = P_{\bar{S}^c}(gh)$$
$$\geq P_{\bar{S}^c}(h)$$
$$= \bar{1} - P_{\bar{S}}(h)$$

Thus $P_{\bar{S}}(lm) \leq P_{\bar{S}}(m)$

$$\bar{1} - Q_{\bar{S}}(gh) = Q_{\bar{S}^c}(gh)$$
$$\leq Q_{\bar{S}^c}(m)$$
$$= \bar{1} - Q_{\bar{S}}(m)$$

Thus $Q_{\bar{S}}(gh) \geq Q_{\bar{S}}(h)$.

$$\bar{1} - R_{\bar{S}}(gh) = R_{\bar{S}^c}(gh)$$
$$\leq R_{\bar{S}^c}(h)$$
$$= R_{\bar{S}}(h)$$

Thus $R_{\bar{S}}(gh) \geq R_{\bar{S}}(h)$.

Further, $\bar{1} - P_{\bar{S}}((x+v)c - lm) = P_{\bar{S}^c}\big((x+v)c - lm\big)$
$$\geq P_{\bar{S}^c}(n)$$
$$= \bar{1} - P_{\bar{S}}(n)$$

Thus $P_{\bar{S}}((k+n)m - lm) \leq P_{\bar{S}}(n)$

$$\bar{1} - Q_{\bar{S}}((g+h)m - lm) = Q_{\bar{S}^c}\big((x+n)c - lc\big)$$
$$\leq Q_{\bar{S}^c}(y)$$
$$= \bar{1} - Q_{\bar{S}}(y)$$

Thus $Q_{\bar{S}}\big((v+m)c - lx\big) \geq Q_{\bar{S}}(y)$.

$$\bar{1} - R_{\bar{S}}((x+n)m - lm) = R_{\bar{S}^c}\big((l+n)m - lm\big)$$
$$\leq R_{\bar{S}^c}(n)$$
$$= R_{\bar{S}}(n)$$

Thus $R_{\bar{S}}((v+n)m - lm) \geq R_{\bar{S}}(n)$.

Therefore $\bar{S}$ is an i-v anti neutrosophic fuzzy ideal of N.

4. IMPLICATIONS OF THE PROPERTIES OF INTERVAL-VALUED ANTI-NEUTROSOPHIC FUZZY IDEALS (IVANFIS)

The properties of Interval-Valued Anti-Neutrosophic Fuzzy Ideals (IVANFIs) present profound implications across various fields in both theoretical and applied mathematics, as well as in real-world scientific and engineering problems. These implications primarily stem from the unique ability of IVANFIs to model uncertainty, indeterminacy, and inconsistency in ways that traditional fuzzy set theories cannot. The major implications of the basic properties of IVANFIs in various domains are discussed below:

4.1. Signal Denoising

The ability of removing erroneous data from the set yet retaining the essential information is very crucial in signal processing systems. Signal denoising can make use of IVANFIs with help of their unique properties in order to handle uncertainties. The wavelet transforms and Fourier methods in denoising work on static membership values and the higher degree of certainty about the data is presumed. But the data here is basically incomplete and imprecise in different experiments, and the traditional models could not able to offer a guaranteed working function to the basic structure.

IVANFIs provides authorization for the modeling of a range of possible values in signal processing with the involvement of IVANFI to address all the uncertainties associated with them. This is used to design more flexible filters to acclimate different levels of noise and is helpful for producing more accurate denoising results where noise levels are not static. The performances of homomorphisms and intersection in IVANFIs is helpful to categorize the accumulation of information amidst various data sources for efficient signal recovery process.

4.2. Similarity Measures in Data Mining

The applications such as pattern recognition, clustering, and classification need similarity measures for enhancement. The performance of similarity measures depends on the elimination of uncertainties of the data. In most of the datasets used in the image analysis, bioinformatics, and social network analysis, the relationships among data points are frequently ambiguous, incomplete, or imprecise.

The traditional fuzzy set-based similarity measures cannot perform well than the properties of IVANFIs. Through the utilization of interval-valued fuzzy membership functions and anti-neutrosophic logic, IVANFIs can compute and integrate several types of uncertainty—such as truth, indeterminacy, and falsity—in similarity calculations. This is helpful to attain more accurate assessments between data points, particularly in situations concerning contradictory or conflicting information. IVANFI can be used to enhance the accuracy of clustering algorithms and anomaly detection techniques, because these algorithms are severely depend on proper similarity measures between data points.

4.3. Handling Missing Data in Large Datasets

The integrity of datasets is mainly affected by the availability of missing data in techniques such as machine learning and statistical analysis. Various issues such as improper predictions, biased models and unreliable conclusions may be aroused because of incomplete data. The methods currently used for handling missing data are mostly depends on the assumptions that the advanced techniques like regression or expectation-maximization should be used to distribute the data.

IVANFIs presented here are helpful for handling incomplete data through the representation of uncertainty in missing values. Through the utilization of the interval-valued fuzzy membership, IVANFIs can model the uncertainty about missing information, which is used to get more accurate predictions. The inconsistencies in data sources can be modeled through IVANFI which supports for the improvement to infer missing values in complex datasets. This characteristics is beneficial in various areas such as healthcare, finance, and sensor networks to ensure reliable analysis and decision-making by handling missing data.

5. ROBUSTNESS IN DECISION-MAKING AND PROBLEM SOLVING

Decision-makers need to deal with vague, inconsistent information due to missing values to address uncertainties. IVANFIs can be utilized in this context to combat uncertainties in a more efficient way. A nuanced and flexible approach for higher degrees of accuracy could be modeled with the help of proposed hybrid method in decision-making process for more accurate representations of the problem at hand.

For instance, the choices that are chosen by the decision makers are properly analyzed due to the incomplete data available in risk management and strategic planning. Such complexities could be identified with the help of the properties of IVANFI for more informed and strong decisions without the known problem of uncertainties. Moreover, the combined nature of IVANFI from different sources enhances the overall decision-making process which helps to create resilient to contradictions and inconsistencies.

6. APPLICATIONS IN COMPLEX SYSTEMS AND MULTI-AGENT SYSTEMS

Many complex systems such as distributed networks include multiple agents or components interrelating in uncertain and dynamic environments. The uncertainty in the model should be avoided which is very crucial in the situations when there is a chance of making decisions under incomplete information.

Different types of uncertainty can be represented by applying IVANFIs in the system for more accurate modeling of the communications amongst agents in dealing with inconsistencies. For example, sensor data may be unreliable or incomplete in area such as vehicular network or Internet of Things. IVANFIs is used for the integration of data inorder to ensure that system-level decisions should be handled through consistent information.

7. ENHANCING COGNITIVE AND BEHAVIORAL MODELING

For the modeling of human reasoning and decision-making, cognitive science carries the application of IVANFIs. Cognitive models are useful in processing the information, assessment of risk and decision making by addressing the uncertainties in them. Through the use of IVANFIs, the cognitive processes are categorized more precisely to gain higher degree of accuracy. More realistic models of human behavior can be created in various domains such as psychology, behavioral economics, and artificial intelligence

8. CONCLUSIONS

Interval-Valued Anti-Neutrosophic Fuzzy Ideals (IVANFIs) introduced in the work provides a significant theoretical innovation and generate a plethora of opportunities for real-world applications in the field of engineering and science. From improving signal denoising and similarity metrics to addressing issues with missing data and strong decision-making, the adaptability of IVANFIs and expressiveness permit them to handle real-world problems more precisely. These consequences lead to further study and application in complex uncertain situations with the help of IVANFIs for proper handling of uncertainties in the near rings. IVANFIs could be applied to increasingly intricate algebraic structures and expanding their applicability to larger mathematical frameworks is the key idea of future research. Additionally, the work will be extended to create computational algorithms that process IVANFIs inventively although examining their potential for use in optimization problems and real-time decision-making systems.

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