



# **FLEXURAL CAPACITY FOR RC BEAMS WITH EXPOSED REINFORCEMENT**

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## **ABSTRACT**

The currently available approaches on the effects of the exposed reinforcement on the flexural strength capacity of beams by other researchers are critically reviewed. These methods for estimating the flexural-compression strengths of beams with main steel exposed along all or part of the span do not give good predictions for the ordinary reinforced member. In this paper, the simple shear-compression theory introduced in earlier study is modified to the treatment of unbonded beams and a very simplistic empirical equation is proposed. The accuracy of the proposed equation is examined by comparing with results of 44 beams from literature. The comparison showed that the predictions by the proposed equation are between 0.85-1.25 to those of test results. The ratio of the experimental ultimate moments to the calculated ultimate moments by the proposed equation gives an average of 1.06 and C.O.V=0.21. The method of calculation proposed here is relatively successful in predicting the ultimate moment resistances but not in predictions of the physical behavior of the beams with exposed reinforcements.

**KEYWORDS:** Flexural capacity, Neutral axis depth, Exposed reinforcement, Exposed length, Load pattern and Reinforcement ratio.

## 1. INTRODUCTION

There are many research works were carried out to study the flexural failure due to debonding between the steel reinforcement and the surrounding concrete. However, the majority of these works intended to evaluate the residual flexural resistance of the elements with corroded steel reinforcement and their physical behaviour but they did not propose any method of analysis (Al-Sulaimani et al. (1990), Almusallam et al. 1996, Rodriguez 1997, Raof and Lin 1997, Mangat and Elgarf 1999, El Maaddawy et al 2005, Du et al 2007, Azad et al. 2010).

The first theoretical approach is a work on unbonded post-tensioned concrete, where the subject is normally expressed in terms of stress increment of the unbonded tendons due to the external loading. If the initial prestress is zero, the member becomes an unbonded. Researches on unbonded tendons have been divided into two schools of thoughts based on deformation were used to analyze the tendon stress at flexural failure (Au and Du 2004). The first school is based on the type of loads, load arrangement and span depth ratio and the other thought based on the neutral axis depth of a section at the ultimate moment (Loretsen 1958 and Loretsen 1964, Pannell 1969, Mattock et al. 1971 and Tam and Pannell 1976).

It is well known; in a reinforced concrete member rotational equilibrium requires that

$$Nz = M \quad 1$$

where  $N$  =longitudinal force (tension in main steel=compression in concrete)

$z$  = internal lever arm

$M$  = External bending moment

Differentiation of the equation gives

$$N \frac{dz}{dx} + z \frac{dN}{dx} = \frac{dM}{dx} = V \quad 2$$

where  $x$  = length measured along the beam

$V$  = shear force

In Normal beam action  $z$  is approximately constant and

$$\frac{dN}{dx} = \frac{V}{z} \quad 3$$

However, if the bond forces required for this magnitude of  $dN/dx$  are not realizable, due for example to the weakening of bond by spalling of concrete as a result of corrosion of main bars or cutting a part of the concrete cover for repairment purpose, the equation (2) requires a variation of  $z$ . In more directly physical terms, a part of the load is resisted by an arching action. In the extreme condition, if  $dN/dx = 0$ , the beam behaves entirely as an arch, with the main steel functioning as a tie, anchored at the ends if the beam is simply supported. The strain of the steel at any section of the beam is no longer equal to that of the surrounding concrete. In terms of overall deformations, the elongation of the main steel is greater than that in beam action while the shortening of the extreme fibre of the concrete is less. The elastic neutral axis depth is reduced and crushing of the concrete prior to yielding of the steel may reduce the member's flexural capacity. This is the case if the depth of the compression zone in the arching action is less than that required for the yield of the reinforcement and cause flexural failure of the member.

The analysis for RC beams with exposed steel bars had been commenced by researchers when they observed that there are similarities between prestressed concrete with unbonded tendons and steel reinforced concrete beams with exposed (unbonded) bars (Cairns 1993). This thought led to adopt Pannell's model as the basis of BS8110 and the Canadian Code A23.3-94. Very limited researches have been focused on the behavior and the evaluation of the flexural capacity of RC beams with an exposed length of steel bars and various methods of analysis by means of experimental works and analytical models have been developed. (Cairns and Zhao 1993, Zhang and Raoof 1995, Wang and Liu 2009, Jnaid and Aboutaha 2014).

Cairns (1993) believed that the adoption of Pannell's approach for RC beam analysis is imprudent by extrapolating results from prestressed concrete to reinforced concrete when prestress is zero in reinforcement. He gave some restriction evidences for prestressed beams with tendons in the middle of the surrounding concrete and anchored externally while the unbonded steel bars lies outside the concrete. Also, the tendons' yielding is limited up to 70% of their characteristic strength which is sensible high strength tendons. Therefore he first merited to conduct further investigations in this area.

It is worth to appraise the thought by Pannell (1969) and Tim and Pannell (1976), which was based on the neutral axis depth of a section at the ultimate moment. They suggested that the ratio ( $\phi$ ) of equivalent length of plastic zone to depth of the neutral axis at ultimate moment ( $c$ ) is constant and equal to 10 for design purpose, this means the deformation of the concrete to be concentrated within a plastic zone of length  $10c$ , or more accurately the depth of the centre

of rotation due to loading. The value of  $\phi$  had been further studied by assessment of experimental results of 148 simply supported beams from literature and the mean values of the  $\phi$  showed there is obvious variation from the value of 10 by Pannell to other values by others (Au and Du 2004). Therefore, Au and du (2004) examined different values of  $\phi$  equal to 9.3, 10 and 16.1 using Pannell's equation (eq.1) and they found  $\phi = 9.3$  is on the safe side for practical purpose. However, in the analysis of RC beams, Wang and Liu (2009) used the constant values of  $\phi$  by Pannell or Au and Du. Recently, Jnaid and Aboutaha (2014) have replaced  $\phi$  with a symbol of  $\Psi$ , using its values between 3-13 in their FEA analysis to give a good account for other parameters like steel reinforcement ratio, the span-depth ratio, concrete compression ratio and loading type have. This raises a question of whether the other values have better predictions for the flexural capacity of RC beams. This uncertainty has been taken as one of turning points in adopting the Pannell's method along with other shortcomings as explained in the following part of this section.

## 2. REIVEW OF PREVIOUS STUDIES

Pannell (1969) considered the deformation of the concrete to be concentrated within a plastic zone of length  $10c$ , where  $c$  is the depth of the neutral axis at ultimate moment, or more accurately the depth of the centre of rotation due to loading. Within this length the main strain of the extreme fibre in compression is  $\varepsilon_{cu}$ , and the strain at the level of the prestressed reinforcement is:

$$\varepsilon_{cp} = \varepsilon_{cu} \frac{d_p - c}{c} \quad 4$$

where  $d_p$  = the effective depth of the prestressed reinforcement

Pannell proposed an equation for prestressed concrete and if it is applied to unbonded reinforced concrete, It gives

$$\frac{c}{d} = 10 \frac{\varepsilon_{cu} E_s \rho}{\alpha f'_c} \left( \frac{d}{l_t} \right) / \left[ 1 + 10 \frac{\varepsilon_{cu} E_s \rho}{\alpha f'_c} \left( \frac{d}{l_t} \right) \right] \quad 5$$

where  $\rho = A_s / b d$ ,  $\alpha =$  the ratio of the average compression stress of concrete to the compression strength of concrete,  $f'_c =$  cylinder strength of concrete and  $l_t =$  the length between anchorages.

This approach predicts  $c/d$  and thence  $M/f'_c b d^2$  to be highly dependent on  $l_i/d$  and independent of the pattern of loading.

Method by Lorentsen (1963) also primarily concerned with prestressed concrete seem to have a stronger physical basis and did specifically consider unbonded reinforced concrete as a limiting case. He did not treat the deformation of the concrete in the same way as Pannell. Instead of considering it to be concentrated to a limited plastic length, he integrated strains along the span in order to define a “bond factor”  $F$ . The  $F$  is defined as the ratio of the steel strain, which is constant along the span, to the strain of the concrete at the level of the steel at midspan, see Fig. 1.

$$F = \frac{\varepsilon_s}{\varepsilon_{csm}} = \frac{\varepsilon_s}{\varepsilon_{cm}} \frac{(x_m x_m / d)}{(1 - x_m x_m / d)} \quad 6$$

Where, as shown in Fig. 1,  $\varepsilon_s$  = the steel strain,  $\varepsilon_{csm}$  = the strain of the concrete at the level of the steel at midspan,  $\varepsilon_{cm}$  = the extreme fibre compressive strain of the concrete at midspan,  $x_m$  = the neutral axis depth at midspan and  $d$  is the effective depth of the section.

The midspan moment is given by:

$$\frac{M}{bd^2} = \alpha f_c \left( \frac{x_m}{d} \right) \left( 1 - \beta \left( \frac{x_m}{d} \right) \right) = \frac{\rho \sigma_s}{f_c} \left( 1 - \frac{\beta}{\alpha} \frac{\rho \sigma_s}{f_c} \right) \quad 7$$

where  $\sigma_s$  = the stress of the reinforcement

$$\sigma_s = \frac{\alpha f_c}{\rho} \frac{1}{1 + \varepsilon_s / F \varepsilon_{cm}} \quad 8$$

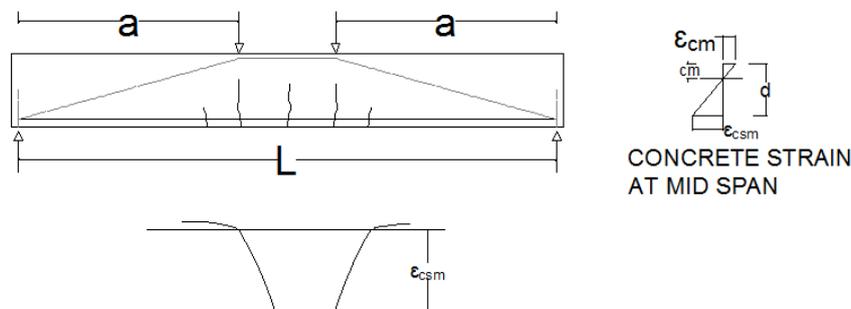


Fig. 1. Basic parameters of Lorentsen's theory.

Lorentsen carried out a programme of tests which was largely concerned with prestressed concrete but included five tests of ordinary reinforced concrete beams. Details and test results are given in [Table 1](#).

[Cairn and Zhao \(1993\)](#) followed an approach similar to Lorentsen's, with a realistic stress-strain relationship for concrete, and with the addition of consideration of bond slip between the steel and concrete. The numerical model was implemented in a program and the predictions are compared with the results of 17 tests of rectangular and nearly rectangular beams in which the main steel was exposed over varying lengths symmetrical about midspan. The beams were simply supported and loaded by pairs of concentrated loads situated symmetrically with respect to the midspan. In the majority of case, the exposed lengths were about 90% of the span.

The predictions from the computer modeling agree very well with the experimental ultimate loads-mean  $M_{calc}/M_{test}=1.01$ , standard deviation=0.06 for 14 beams with exposed steel. The other three beams were the control specimens with fully bonded reinforcement. The failures were predominantly flexural. Details and test results are given in [Table 1](#).

[Raooof and Lin \(1997\)](#) reports fully 88 tests of beams with a cross-section 150x300mm and very summarily on some tests of smaller specimens. In the main tests, the tension reinforcement was exposed over various lengths in 3.0 m simply supported spans subjected to a variety of loading patterns. The main interest in the work appears to have been in shear resistance but some beams failed in flexure and could be due to the use of main reinforcement with a yield stress of only  $363 \text{ N/mm}^2$  (presumably plain round bars). This low steel strength makes theoretical flexural resistance low, even if the main bars are not exposed. In a number of cases, the experimental ultimate moments were well above the theoretical flexural resistance, presumably showing that the reinforcement strain hardened significantly, and this makes the analysis of the results difficult. Some of the beams contained shear reinforcement, but its spacing was equal to  $1.8d$ , which makes its efficiency rather dubious, especially because the positions of links relative to those of bar exposure are unknown. The paper does not propose any method of analysis.

**Table 1. Summary of Test Data and Results.**

Author	Beam No.	$f_{cu}$ , N/mm <sup>2</sup>	$\rho$ , %	$d$ , mm	$\frac{l}{d}$	$\frac{l_e}{l}$	$\left( \frac{M_u}{f_{cu} b d^2} \right)$
Lorentsen	3	49.5	0.52	260	15.38	1.000	0.069
	15	39.5	2.16	180	22.22	1.000	0.164
	18	42.8	2.18	180	22.22	1.000	0.156
	21	53.3	0.54	260	15.38	1.000	0.069
	22	52.0	2.06	180	22.22	1.000	0.154
Regan	2	49.0	24.0	300	8.53	0.890	0.187
	3	52.3	2.18	300	8.53	0.890	0.136
	4	59.2	2.18	300	8.53	0.890	0.124
	6	33.1	1.99	210	12.19	0.890	0.201
	7	49.6	1.99	210	12.19	0.890	0.152
	8	51.3	1.99	210	12.19	0.890	0.112
Cairns	S2	25.0	0.75	372	7.26	0.925	0.124
	S3	31.2	0.73	380	7.11	0.630	0.111
	S4	38.2	1.08	253	10.67	0.933	0.109
	S4B	24.9	1.27	225	12.00	0.933	0.165
	S5	35.4	1.40	195	13.85	0.941	0.093
	S7	30.3	0.77	358	7.54	0.859	0.153
	S8	29.6	1.93	340	7.94	0.859	0.179
	S9	32.4	0.50	350	7.71	0.948	0.076
	S10	29.9	0.74	200	13.50	0.948	0.100
	S11	34.9	0.74	200	13.50	0.600	0.093
	W1	25.5	1.94	245	11.02	0.911	0.181
W2	25.4	1.94	250	10.80	0.696	0.202	

Wang and Liu (2009) studied tests results of RC beams with a partially unbonded length of steel in tension zone. They proposed a model representing a combination of compatibility condition of deformation with equilibrium condition of forces. The calculation of shortening of the concrete as  $9.3x E_c$  with no influence from the inclination of the trust line is rather unrealistic,

but any method simple enough to be usable is going to involve some off assumptions. The other point about this paper is its many references of corrosion. The real beams other than lintels contain stirrups and, so long as some of them survive, the bars are held in contact with the concrete core. In this situation, the ribs of the bars should ensure a significant bond resistance is restrained. This increases the flexural compression capacity but increases the risks of shear and anchorage failures. However, they refer to further research which will aim to address this.

Jnaid and Aboutaha (2014) conducted an FEA model to predict the residual flexural capacity of reinforced concrete beams with unbonded reinforcement. They found that the reinforcement ratio and span-to-depth ratio and the debonding length over the span have large effects and the load patterns have less effect on the ultimate flexural capacity. They only considered the case loss of bonding between the reinforcement and the surrounding of concrete and neglect the other concern of losing of bar cross section due to corrosion as considered by Wang and Chen (2011) in FEA programme. Their equation to calculate the ultimate strength of unbonded concrete beams including a factor of ( $\psi$ ) is rather too long and it would be very difficult for practical use.

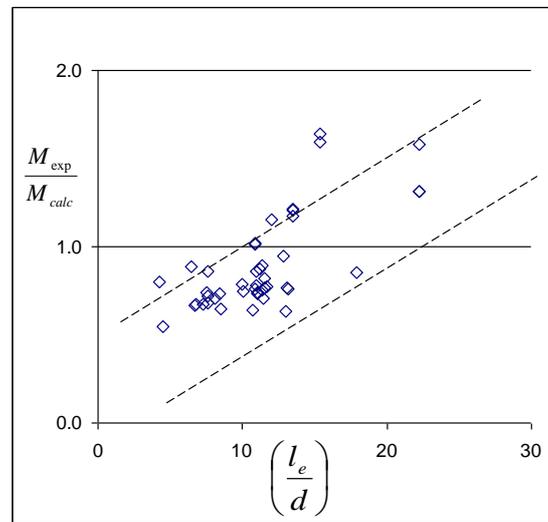
### 3. DISSCUSSION AND ANALYSIS

Pannell'All of the analytical methods explained in the previous section have some basic consequences in terms of predictions and were theoretically formulated in different equations. The equations were initially based on the strain and stress compatibility between the reinforcement and concrete in a cracked section.

Pannell used the ratio of ( $l_e/d$ ) as a major factor and he found the calculated ultimate moment decreases as the ratio increases. Test results in Table 1 were used to shows  $M_{exp}/M_{calc}$  plotted against  $l_e/d$ , with  $M_{calc}$  determined via Pannell's equation (5) as shown in Fig. 2. There is a clear tendency for ultimate moments to be overestimated when  $l_e/d$  is small and underestimated when the ratio is large. Thus the influence predicted for  $l_e/d$  is incorrect and would seem that as a first approximation the ratio is not an influencing parameter.

Except the Pannell's approach, the methods of Lorentsen and all the others reviewed, predict that the pattern of loading had a major effect on the ultimate moment, with the moment being very considerably increased if there is a constant moment region rather than a peak moment at a single concentrated load. Tests by Regan show that this is not the case and indeed that the reverse is true. The reason is that in the case of a concentrated load, applied from above, the concrete compression zone at the section of maximum moment is restrained by the load with

the result that failure either occurs to the side of the load where the depth of the compressive zone is not a minimum or at the loaded section but higher concrete stresses sustained.



**Fig. 2. Comparison of experimental ultimate moments with prediction by Pannell's method.**

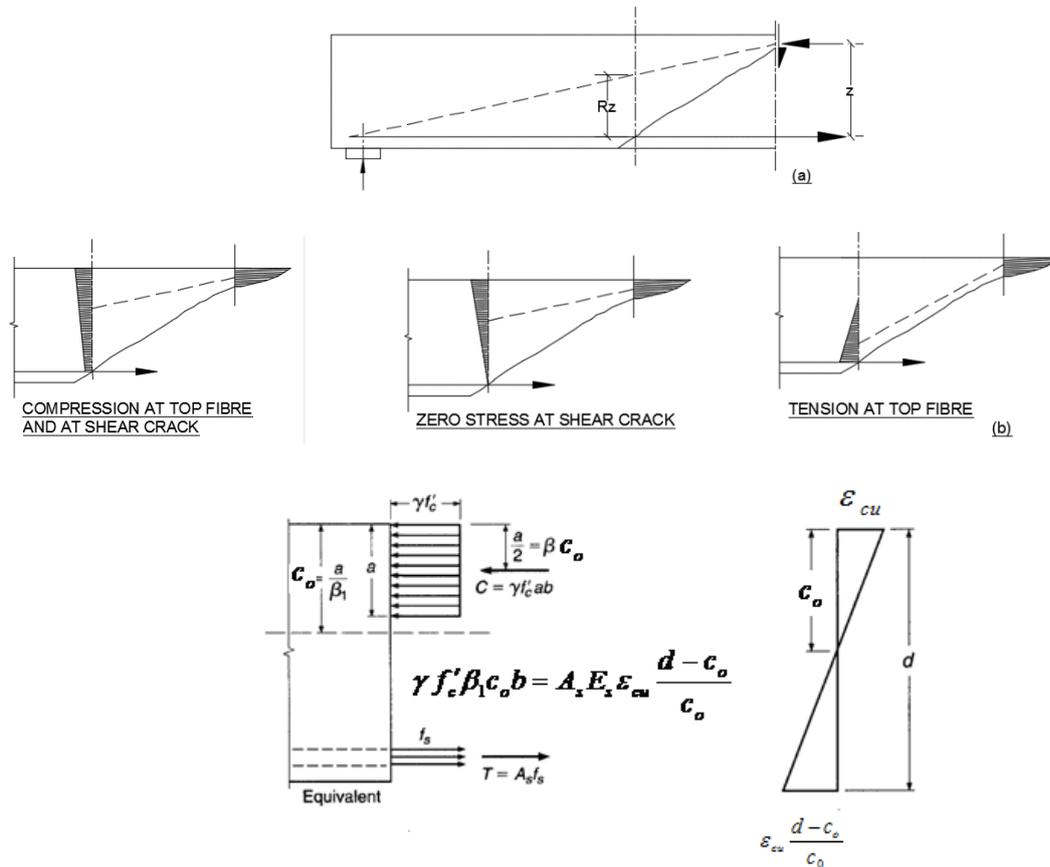
The calculation of shortening of the concrete as  $9.3xE_c$  by Wang and Liu do not account for the influence of some parameters (Jnaid and Aboutaha 2014). Also, the assumption of a linear relationship between the strain in the bar and the length of the exposed zone conflicts with the non-linear relationship according to Jnaid and Aboutaha FEA outcomes.

In view of the shortcomings of the approaches by Pannell, Lorentsen, Wang and Jnaid and of the difficulty and complexity of deriving a truly rational theory, it is sensible to seek a rather simplistic theory. The simple shear-compression theory by Regan (1967) gives a rather simplistic solution.

#### **4. TRANSFER OF THE SHEAR-COMPRESSION MODEL TO THE TREATMENT OF UNBONDED BEAMS**

In shear-compression failure where the debonding occurs not by design but is caused by shear cracks. Regan (1967) analyses the behaviour of a region in which the compression zone and the tensile reinforcement are separated by shear crack. He considered three situations in regard to the relationship between the thrust line and the shear crack and three possibilities are considered in terms of the concrete's stress-strain relationship as shown in Fig. 3.

Part (a) shows a shear span in a beam without shear reinforcement with an inclined crack at its right-hand end. Because of the lack of stirrups, the triangle of concrete below the inclined crack is isolated from the rest of the beam by the inclined crack and a vertical crack at the section of maximum moment leaving just a point of contact at the top of the inclined crack.



(c) Derivation of the reference neutral axis depth  $C_o$

Fig. 3. Regan's Shear-Compression Theory.

The force in the flexural reinforcement is therefore constant along the length from the right – hand end to the section where the inclined crack intersects the reinforcement. The same is true for the compression force in the concrete, which has a horizontal component equal to the force in the reinforcement. Equilibrium with the applied moments requires that the lever arm should reduce shown from  $z$  to  $Rz$  in the length between the sections at the two ends of the crack. The part of the broken line above the inclined crack represents the thrust line in the concrete. The extension to the support is drawn so as to define the ratio  $R$ , but what actually happens in the region depends on the bond of the reinforcement but is not of much relevance so long as there is no bond/anchorage failure.

Part (b) of the figure shows possible distributions of the compression stresses in the concrete above the inclined crack drawn on the assumptions that tension in the concrete is negligible. The distribution of the top fibre strain of the concrete can be calculated for any of the situations shown in part (b), and for any stress-strain relationship for concrete as a function of the horizontal component of the force in the concrete, the neutral axis depth “ $c$ ” at the right-hand end and the ratio  $R$  between the lever arms at the end sections of the length considered (note:

the calculation and its results depend on R but are independent of the length between the lever arms at the end section)

The actual depth of the neutral axis at the right-hand end can be found using a compatibility condition in terms of the shortening  $\Delta_c$  of the top surface and the lengthening of the reinforcement  $\Delta_s$  between the two end sections

$$(c/d)/(1-c/d) = \Delta_c / \Delta_s \quad 9$$

$\Delta_c$  is calculated by integration of the top fibre strains and  $\Delta_s$  is equal to the constant steel strain times the length.

For practical purposes simplification is necessary and one need only consider conditions when the compression zone above the top of the crack reaches its failure state.

Part(c) shows the conditions at failure in a normal beam failing in flexural compression where the compatibility condition is  $\varepsilon_{cu}/\varepsilon_s = c/(d-c)$

From the first equation in part (c) substitution of  $K = \rho E_s \varepsilon_{cu} / \alpha f'_c$  leads to  $(c_o/d) = K[(d/c_o) - 1]$  from which

$$\left(\frac{c_o}{d}\right) = \frac{K}{2} \left[ \sqrt{1 + \frac{4}{K}} - 1 \right] \quad 10$$

With a horizontal thrust line ( $R=1$ ) in Fig. 3(b), the situation in respect to the neutral axis depth at compression failure is the same as that leading to the last equation above. Given this and the fact that the distribution of strain along the top is a function of R suggests the possibility of a simplification making  $(c/d)$  a function of  $(c_o/d)$  and R.

Then it could be assumed

$$\left(\frac{c}{d}\right) = \left(\frac{c_o}{d}\right) \frac{(c_o/d)}{(1-R) + (c_o/d)} \quad 11$$

Which makes  $(c/c_o) = 1$  when  $R = 1$  and makes it reduce as  $R$  decreases, to reach a minimum of  $(c_o/d)/[1 + (c_o/d)]$  for  $R = 0$

However, there are some initial assumptions should be taken into account in the derivation of a possible expression to determine the neutral axis depth at the section of maximum moment.

1. The thrust lines in the concrete are straight between the centers of compression at the section of maximum moment and those at the supports or the ends of the unbonded lengths.

2. There is a relationship between the neutral axis at the section of maximum moment and what would be the neutral axis depth in a normal beam failing by flexural compression. and
3. The cases to be considered exclude beams failing by yielding of the flexural reinforcement, and shear or anchorage failure.

In order to explore the empirical value of  $R$  in equation (11), data in Table 1 are used along with other tests by Regan (1989), Rodriguez (1997), Wang (2001) and Li et al. as shown in Table 3.

All tests were conducted on simply supported reinforced concrete beams with rectangular sections except beams 3 and 21 by Lorentsen were T-sections. Tests by Cairns and Zhao, Rodriguez, Li, Wang et al. and beams 2, 6 and 7 by Regan were conducted with two-point loading. The rest of the tests of which by Lorentsen and beams 3, 4 and 8 by Regan were with a single load.

To obtain the theoretical neutral axis depth according to ACI, the value of  $K$  in equation (11) is calculated as  $K = \rho E_s \varepsilon_{cu} / 0.85 \beta_1 f'_c$

Considering values for  $\varepsilon_{cu}=0.003$  and  $E_s=200 \times 10^3 \text{ N/mm}^2$  for different strength of concrete as given in Table 2 below.

**Table 2. Value of K.**

	$f'_c$ (MPa)				
	$\leq 28$	35	42	49	$\geq 56$
$\beta_1$	0.85	0.80	0.75	0.70	0.65
(MPa) $K(\rho / f'_c)$	830	882	941	1008	1085

For Li tests, the following  $K$  is used due to the appointed  $E_s$  by the authors.

**Table 3. Value of  $K$  for Li. et al. tests.**

$E_s \times 10^5$ (MPa)	1.75	1.85	1.88	2.00	2.01	2.06	2.10	2.18	2.27
$K(\rho / f'_c)$ (MPa)	726	768	780	830	872	855	871	905	942

Then the experimental value of the neutral axis depth at failure could be inferred from the ultimate moment:

$$\left(\frac{c}{d}\right) \cong 1 - \sqrt{1 - 3m}$$

12 14

For sake of simplification, the value of  $\beta_1$  is taken as an average of 0.80.

Where  $m = M_u / f'_c b d^2$  and  $M_u$  is the ultimate moment

Fig. 4 shows the relationship between the  $(c_o/d)$  and  $(c/d)$  to obtain the empirical prediction of the neutral axis depth of unbonded beam at a section of maximum moment.

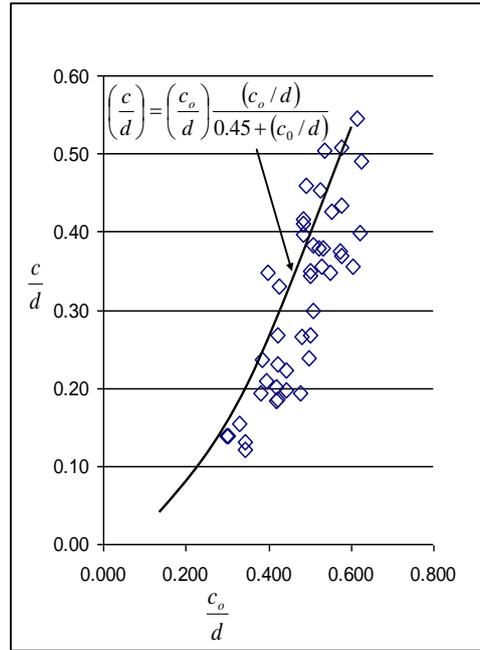


Fig. 4. Relationships between the  $(c/d)$  and  $(c_o/d)$ .

There is inevitably a scatter relationship in Fig.4; due partly to particular features of various tests, but there does appear to be a general relationship between the two neutral axis depths, which may be approximated by

$$\left(\frac{c}{d}\right) = \frac{\left(\frac{c_o}{d}\right)}{0.45 + \left(\frac{c_o}{d}\right)}$$

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i.e. in effect equation (8) with  $R=0.55$ .

For the case of unbonded reinforcement, the calculated  $(c/d)$  from equation (10) is replaced in the ACI-318 expression for moment capacity as shown in equation (11)

$$M_{calc} = 0.85\beta_1 f'_c b d^2 \frac{c}{d} \left(1 - 0.5\beta_1 \frac{c}{d}\right)$$

In most of the tests in Table 4, the main reinforcement was exposed for almost the full span but there are a few instances where this was not the case and such instances can certainly arise in practice if debonding occurs as the result of corrosion of the main bars. If the steel is bonded in significant parts of the span toward the supports, the elongation of the steel must be reduced and this can be allowed for by the modifying equation (14) to

$$\left(\frac{c}{d}\right) = \left(\frac{c_o}{d}\right) \frac{(c_o/d)}{0.45(l_e/l) + (c_o/d)}$$

The modification assumes not merely that there is some bond but also that it doesn't fail. This is unlikely to be the case in the absence of stirrups and  $l_e$  should be taken as equal to  $l$  for the plain-webbed members. Where stirrups are present, the effectiveness of the bond will depend on the numbers of stirrups and on the positions of the main bars relative to the stirrup corners. For the present analysis, it has been assumed that bond was effectively effective in the tests by Cairns and Zhao (1993). 15

## 5. RESULTS FOR THE PROPOSED EQUATION

Table 4 summarizes the results of calculations using equations 10, 13 and 14 and compares them with the tests results. The ultimate flexural capacity of beams is given in terms of the case for the bonded beam  $M_b$  (the superscript "b" denotes the perfect bonded steel),  $M_{exp}$  for experimental results and  $M_{ub, calc}$  the calculated moment according to the proposed equation (14). The comparison is shown graphically in Fig. 5 between the  $M_{exp}$  and  $M_{ub, calc}$  for four parameters of  $f'_c$ ,  $l/d$ ,  $l_e/l$  and load patterns. Each part of which contains the same points, but with different symbols used to distinguish between different groups of tests.

The overall agreement between calculated and experimental ultimate moments is fairly good and none of the factors considered in Fig. 5 (a-d) seems to lead to any very systematic trend to errors. Fig. 5 (a-d) shows prediction of ultimate  $M_{ub, calc}$  by the proposed equation were between  $0.80 M_{exp}$  (or  $M_{exp}=1.25 M_{ub, calc}$ ) and  $1.20 M_{exp}$  (or  $M_{exp}=0.85 M_{ub, calc}$ ). The comparison is noticeably better than that for Pannell's method as shown in Fig. 2. The results of this analysis show the mean value of  $M_{exp}/M_{ub, calc}$  equals to 1.06, STDV=0.22 and C.O.V. =0.21. The value of C.O.V may indicate to some results on the unsafe or conservative side. One of the reasons may be due to the lower reinforcement ratio as the steel yield completely before any crush in concrete at compression fibre.

The method of calculation by the proposed equation is relatively successful in predicting ultimate moment resistance but neither Table 2 and Fig. 5 shows whether or not it predicts

actual behavior. Two comparisons were demonstrated in Fig. 6 and Fig. 7 to give the behavior of the proposed equation with the tests results.

**Table 4. Comparison between calculated and experimental results.**

Beam Name	$f'_c$ $\left(\frac{N}{mm^2}\right)$	$\rho$ %	$d$ (mm)	$\frac{l}{d}$	$\frac{l_e}{l}$	$\frac{c_o}{d}$	$\frac{c}{d}$	$\left(\frac{M_b}{f'_c b d^2}\right)$	$\left(\frac{M_{exp}}{f'_c b d^2}\right)$	$\left(\frac{M_{ub,calc}}{f'_c b d^2}\right)$	$\frac{M_{exp}}{M_b}$	$\frac{M_{exp}}{M_{ub,calc}}$
<b>Cairns and Zhao</b>												
S2	20.0	0.75	372.0	7.26	0.93	0.424	0.214	0.175	0.155	0.140	0.88	1.10
S3	25.0	0.73	380.0	7.11	0.63	0.386	0.223	0.141	0.139	0.146	0.99	0.95
S4	30.6	1.08	253.0	10.67	0.93	0.424	0.213	0.165	0.136	0.132	0.83	1.03
S4B	19.9	1.27	225.0	12.00	0.93	0.509	0.279	0.269	0.206	0.178	0.77	1.16
S5	28.3	1.40	195.0	13.85	0.94	0.477	0.253	0.219	0.116	0.163	0.53	0.71
S7	24.2	0.77	358.0	7.54	0.86	0.398	0.202	0.150	0.191	0.133	1.27	1.43
S8	23.7	1.93	340.0	7.94	0.86	0.551	0.324	0.304	0.224	0.202	0.74	1.11
S9	25.9	0.5	350.0	7.71	0.95	0.328	0.143	0.096	0.095	0.097	0.99	0.98
S10	23.9	0.74	200.0	13.50	0.95	0.394	0.189	0.145	0.125	0.126	0.86	0.99
S11	27.9	0.74	200.0	13.50	0.60	0.381	0.223	0.126	0.116	0.146	0.92	0.80
W1	20.4	1.94	245.0	11.02	0.91	0.577	0.338	0.337	0.226	0.209	0.67	1.08
W2	20.3	1.94	250.0	10.80	0.70	0.578	0.375	0.337	0.253	0.228	0.75	1.11
<b>Lorentsen</b>												
3	39.6	0.52	260.0	15.38	1.00	0.304	0.122	0.110	0.086	0.074	0.79	1.16
15	31.6	2.16	180.0	22.22	1.00	0.531	0.288	0.229	0.205	0.173	0.89	1.18
18	34.2	2.18	180.0	22.22	1.00	0.530	0.287	0.216	0.195	0.173	0.90	1.13
21	42.6	0.54	260.0	15.38	1.00	0.299	0.119	0.108	0.086	0.073	0.80	1.19
22	41.6	2.06	180.0	22.22	1.00	0.500	0.263	0.175	0.193	0.151	1.10	1.27
<b>Regan</b>												
2	39.2	2.4	300.0	8.53	0.89	0.524	0.297	0.251	0.234	0.168	0.93	1.39
3	41.8	2.18	300.0	8.53	0.89	0.508	0.284	0.220	0.170	0.162	0.77	1.05
4	47.4	2.18	300.0	8.53	0.89	0.500	0.277	0.199	0.155	0.149	0.78	1.04
6	26.5	1.99	210.0	12.19	0.89	0.537	0.308	0.292	0.251	0.193	0.86	1.30

7	39.7	1.99	210.0	12.19	0.89	0.502	0.279	0.214	0.190	0.159	0.89	1.19
8	41.0	1.99	210.0	12.19	0.89	0.496	0.275	0.208	0.140	0.157	0.67	0.89

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**Rodriguez**

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126	28.0	1.51	171.0	11.99	1.00	0.492	0.257	0.227	0.236	0.165	1.04	1.43
313	29.6	1.51	171.0	13.45	1.00	0.483	0.250	0.217	0.217	0.153	1.00	1.42
314	29.6	1.51	171.0	13.45	1.00	0.483	0.250	0.217	0.220	0.153	1.01	1.44
316	29.6	1.51	171.0	13.45	1.00	0.483	0.250	0.217	0.212	0.153	0.98	1.39

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**Wang et al.**

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L-1	18.2	0.67	160.0	13.13	1.00	0.421	0.203	0.110	0.112	0.134	1.02	0.84
L-2	18.2	0.66	167.0	12.57	0.67	0.419	0.244	0.102	0.111	0.158	1.09	0.70
L-3	18.2	0.66	165.0	12.73	0.33	0.419	0.309	0.102	0.120	0.194	1.18	0.62
L-8	18.2	1.39	161.0	13.04	1.00	0.426	0.207	0.209	0.184	0.137	0.88	1.35

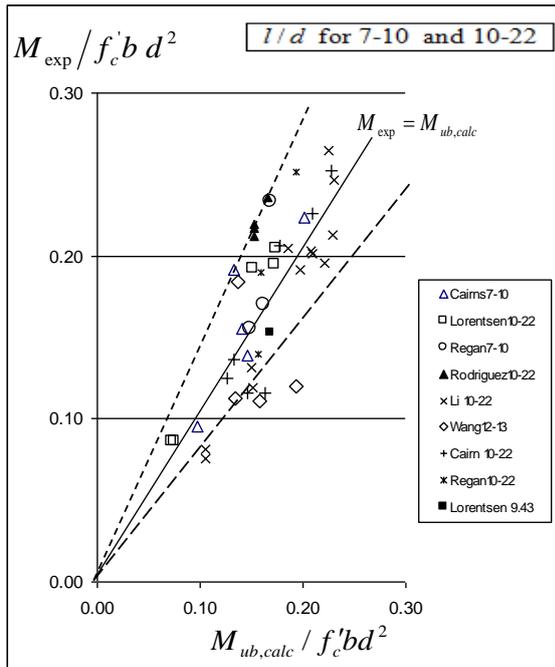
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**Li**

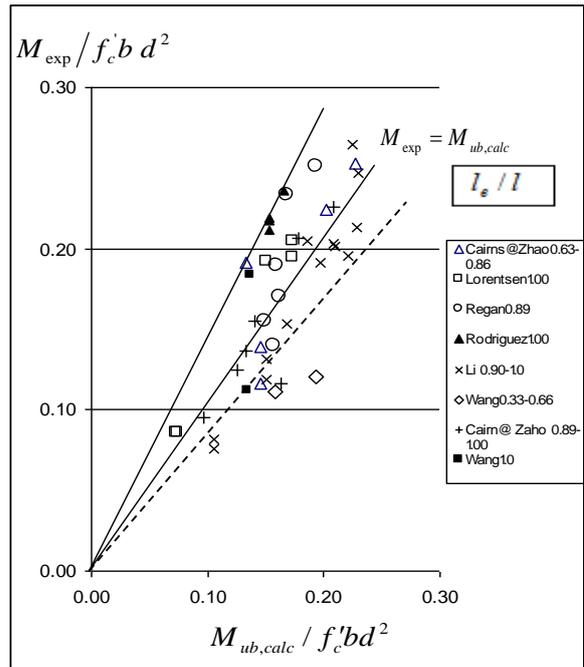
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L-1a	27.1	0.51	164.6	12.75	0.90	0.343	0.157	0.078	0.082	0.106	1.04	0.77
L-1b	27.1	0.51	163.0	12.88	0.90	0.342	0.156	0.078	0.076	0.105	0.97	0.72
L-2a	27.1	1.14	167.3	12.55	0.90	0.442	0.230	0.127	0.137	0.150	1.04	0.88
L-2b	27.1	1.12	166.3	12.63	0.90	0.444	0.231	0.122	0.119	0.151	0.98	0.79
L-3a	27.1	2.01	167.0	12.57	0.90	0.522	0.293	0.201	0.205	0.185	1.02	1.11
L-3b	27.1	2.01	165.0	12.73	0.90	0.550	0.316	0.217	0.192	0.198	0.88	0.97
L-4a	27.1	2.46	172.7	12.16	0.90	0.578	0.339	0.215	0.201	0.210	0.93	0.96
L-4b	27.1	2.46	172.3	12.19	0.90	0.575	0.336	0.195	0.203	0.208	1.04	0.97
L-5a	27.1	2.95	177.7	11.82	0.90	0.606	0.362	0.222	0.195	0.222	0.88	0.88
L-6a	27.1	3.64	173.7	12.09	0.90	0.614	0.369	0.299	0.264	0.225	0.88	1.18
L-6b	27.1	3.64	173.3	12.12	0.90	0.626	0.379	0.315	0.247	0.230	0.79	1.07
L-7	27.1	1.46	222.7	9.43	0.90	0.482	0.261	0.146	0.153	0.168	1.05	0.91
L-8	27.1	3.20	106.3	19.75	0.90	0.623	0.377	0.300	0.213	0.229	0.71	0.93

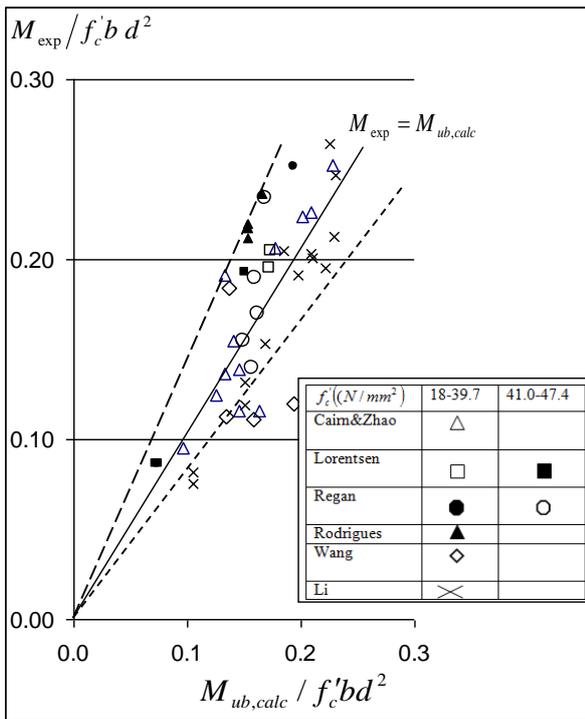
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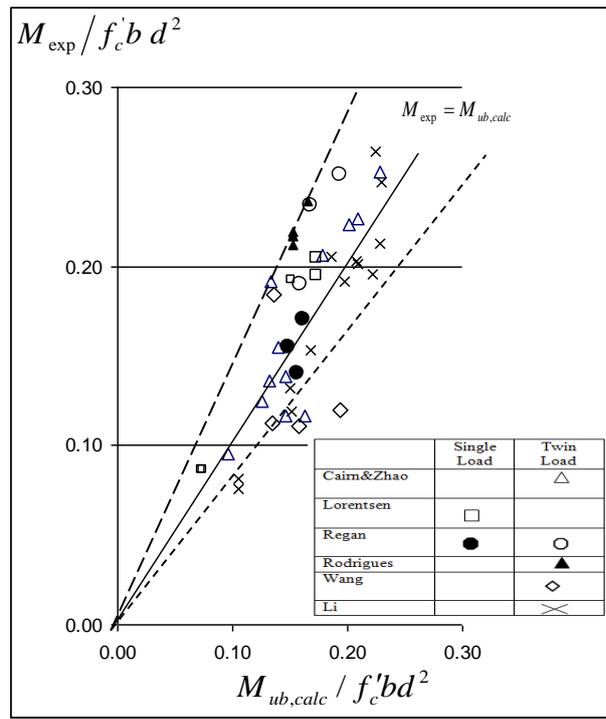
(a)



(b)



(c)



(d)

$$M_{exp} = 1.25 M_{ub,calc} \quad \text{-----} \quad M_{exp} = 0.85 M_{ub,calc} \quad \text{-----}$$

**Fig. 5. Comparison of experimental and calculated ultimate moments.**

The tests by Cairns and Zhao included one test (T1) in which the depth of main steel exposed was increased in stages by breaking out successively more concrete. In each condition, the loading applied was such as to produce the same maximum moment. The recorded neutral axis depths are plotted against  $(l_e/l)$  in Fig.6, where they are compared with values calculated by

equation (13) and Pannell's predictions. The experimental and calculated values are slightly higher, which is reasonable since the experimental ones do not correspond to the ultimate limit state and the neutral depth to increase near failure.

In overall terms, the rather limited data on neutral axis depth shows that the values calculated from equation (13) are in good agreement but not by Pannell's equation.

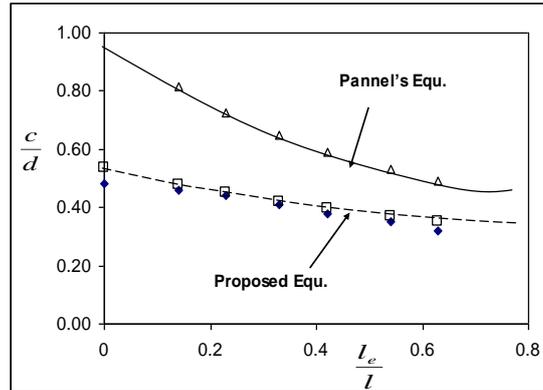


Fig. 6. Variation of neutral axis depth with exposed length Beam T1 by Cairns and Zhao.

Pannell, Wang and Jnaid confirmed the importance of the ratio of exposed length to depth of the section. In Fig. 7, the data from Table 4 were classified to four ranges of  $(l_e/d)$  and showed against the  $M_{exp}/M_{ub,calc}$ . The figure shows a greater reduction in more highly reinforcement ratio and where there is larger exposed reinforcement.

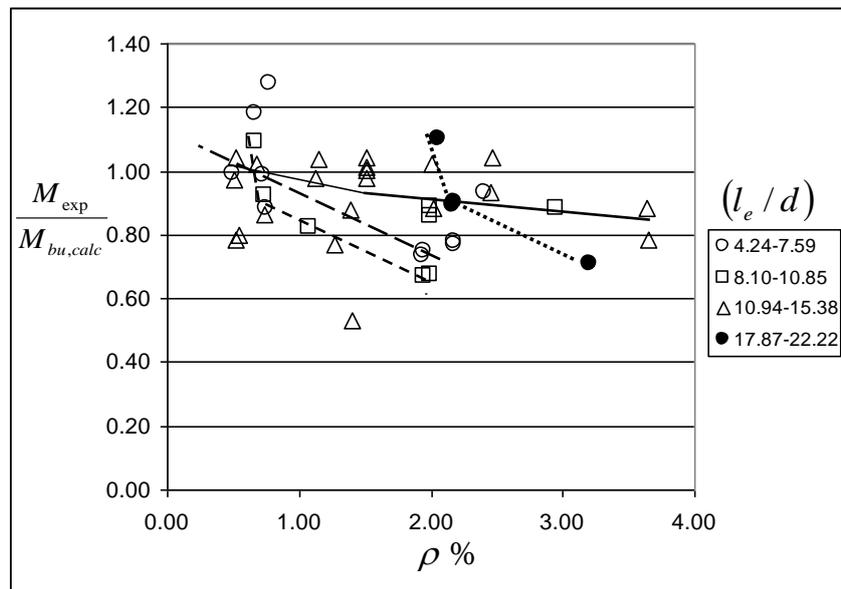


Fig. 7. Reduction of flexural strength in beams with exposed reinforcement.

## 6. CONCLUSIONS

1. Pannell's methods in which the deformation of the concrete is taken to depend only on its ultimate strain, the depth of the neutral axis predicts a strong inverse relationship between the ultimate moment and the span/depth ratio. No such relationship is evident in the test results. Other methods, such as Lorentsen's, which determine concrete deformation by integration along the span, predict that the ultimate should increase with an increase in the length of the constant moment region. The tests do not show this.
2. The other works on the effects of debonding due to the effects of corrosion and/or the cutting away of concrete in preparation for repair work by Cairns and Zhao, Zhang and Raoof. These analytical approaches appear to be successful but the papers do not provide a practicable method of calculation.
3. There are FEA and proposed models for estimating the residual strength by Wang and Liu and Jnaid and Aboutaha. Their models are based on simple assumptions and they concentrated on the physical behavior of the unbonded steel through the influenced parameters.
4. A very simplistic empirical equation is proposed to predict the flexural capacity of unbonded RC beam. The efficiency of the proposed equation is examined by comparing its predictions with the results of 44 tests from literature. The comparisons showed that the predictions by the proposed equation are in good agreements with test results. The method of calculation proposed here is relatively successful in predicting ultimate moment resistances but not in predictions of their physical behavior. The ratio of the experimental ultimate moments to the calculated ultimate moments gives an average of 1.06, STDV=0.22 and C.O.V=0.21.
5. It is worth to add some of a very relative issue for further considerations in future:
  - (i) Corrosion can affect both stirrups, which are particularly vulnerable due to their small cover, where they are outside the main bars and worst of all at their at the corners of beams. Corrosion of main bars is unlikely to completely remove their bond so long as some stirrups remain intact and can hold the bars in contact with the concrete core of the beam. The major types of failure are shear and loss of anchorages.
  - (ii) In its present form, the proposed equation does not treat continuous beams.
  - (iii) It does not treat other possible modes of failure such as loss of end anchorage.

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